The geometrization of the electromagnetic field

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Abstract

Einstein used the term 'unified field theory' in a title of a publication for the first time in 1925. Somewhat paradoxically, an adequate historical, physical and philosophical understanding of the dimension of Einstein's unification program cannot be understood without fully acknowledging one of Einstein's philosophical principles. Despite many disappointments, without finding a solution besides of the many different approaches along the unified field theory program and in ever increasing scientific isolation, Einstein insisted on *the unity of objective reality as the foundation of the unity of science*. Einstein's engagement along his unification program was burdened with a number of difficulties and lastly in vain. Nevertheless, a successful geometrization of the gravitational and the electromagnetic fields within the framework of the general theory of relativity is possible. Thus far, it is a purpose of the present contribution to geometrize the electromagnetic field within the framework of the general theory of relativity.

Keywords

Quantum theory, Relativity theory, Unified field theory, Causality

1. Introduction

It is very easy to get lost in the many [1] and conceptually somewhat very different attempts at the unified field theories. Lastly, the progress [2] at unification has been very slow. Therefore, in this paper in order to "geometrize" the electromagnetic field I will follow neither the scalar gravitational theory with electromagnetism and its introduction of an additional (four spatial and one time dimension) space dimension (Nordström [3], Kaluza [4]), nor Weyl's trial for generalising Riemannian geometry and his concept of "gauging" (Weyl [5]), nor will I use of an asymmetric Ricci tensor (Eddington [6]), nor will I try to add an antisymmetric tensor to the metric (Bach [7], Einstein [8]), nor will I use the framework of quantum field theory et cetera as the point of departure to "geometrize" the electromagnetic field. Theoretically, it seems indeed possible to approach unification in the framework of quantum field theory. Still, a satisfactory inclusion of gravitation into the scheme of quantum field theory is not in sight. From this point of view, Finsler [9] geometry introduced by Randers [10], as a kind of a generalization of Riemann geometry, is another and alternative approach to the geometrization of electromagnetism and gravitation. Taken all together, the point of departure for including the electromagnetic field into a geometric setting will be general relativity. In this context, at least one point has to be considered. Taken Einstein for granted, we must give up general relativity theory. Einstein himself in his hunt for progress at the unification went so far to force us to give up his own general theory of relativity and the successful geometrization of the gravitational field. According to Einstein, we must go beyond the general theory of relativity. At the end, a generalization of the theory of the gravitational field is necessary. In this context, Einstein's position concerning the unified field theory is very clear and strict.

> "The theory we are looking for must therefore be a generalization of the theory of the gravitational field. The first question is: What is the natural generalization of the symmetrical tensor field? ... What generalization of the field is going to provide the most natural theoretical system? The answer ... is that the symmetrical tensor field must be replaced by a non-symmetrical one. This means that the condition $g_{ik} = g_{ki}$ for the field components must be dropped. " [11]

Figure 1. Einstein and the problem of the unified field theory.

Anyhow, if we follow Einstein's proposal at this point to account for a classical unified field theory of the gravitational and electromagnetic fields with the conceptual unification of the gravitational and electromagnetic field into one single and unique *hyper-field* [12], it appears to be necessary and justified on a foundational level to concentrate at the heart of general relativity, the crucial mathematical concept of the metric tensor field $g_{\mu\nu}$. The following paper can be characterized as follows. The attempt to develop some new, basic and fundamental insights is grounded on a deductive-hypothetical methodological approach. In the section *material and methods* the basic mathematical objects and tensor calculus rules needed to achieve the "geometrization" of the electromagnetic field will be defined and described. In this context, physicists should be able to follow the technical aspects of this paper without any problems, while reader without prior knowledge of general relativity or of the mathematics of tensor calculus might gain an insight into the new methods and the scientific background involved. In general, it is necessary to decrease the amount of notation needed. Thus far, I will restrict myself as much as possible to *contra-variant* second rank tensors. I apologize for the shortcoming. Especially, to enable the fusion of quantum theory and relativity theory into a new and single conceptual formalism the starting point of all theorems in the section results is axiom I or *+1 = +1* (*lex identitatis*). The same axiom I possess the strategic capacity serves as common ground for relativity and quantum theory with regard to unified field theory. The section discussion examines some the consequences of the theorems proved. In this context, from the conceptual point of view of the unified field theory, it is the purpose of this publication to in find a convincing formulation of *a geometrization of the electromagnetic fields* within the framework of the general theory of relativity.

2. Material and Methods

2.1. Definitions

Einstein's general theory of relativity

Definition: Einstein's field equations

Einstein field equations (EFE), originally [13] published [14] without the extra 'cosmological' term $\Lambda \times g_{\mu\nu}$ may be written in the form

$$G_{\mu\nu} + \Lambda \times g_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} \times g_{\mu\nu} + \Lambda \times g_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} \right) = \frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{\mu\nu} \quad (1)$$

where $G_{\mu\nu}$ is the Einsteinian tensor, $T_{\mu\nu}$ is the stress-energy tensor of matter (still a field devoid of any geometrical significance), $R_{\mu\nu}$ denotes the Ricci tensor (the curvature of space), R denotes the Ricci scalar (the trace of the Ricci tensor), Λ denotes the cosmological "constant" and $g_{\mu\nu}$ denotes the metric tensor (a 4×4 matrix) and where π is Archimedes' constant (π = 3.14159265358979323846264338327950288419716939937510582 09…), γ is Newton's gravitational "constant" and the speed of light in vacuum is c = 299 792 458 [m/s] in S. I. units.

Scholium.
The stress-energy tensor $T_{\mu\nu}$, still a tensor devoid of any geometrical significance, contains all forms of energy and momentum which includes all matter present but of course any electromagnetic radiation too. Originally, Einstein's universe was spatially closed and finite. In 1917, Albert Einstein modified his own field equations and inserted the cosmological constant Λ (denoted by the Greek capital letter lambda) into his theory of general relativity in order to force his field equations to predict a stationary universe.

"Ich komme nämlich zu der Meinung, daß die von mir bisher vertretenen Feldgleichungen der Gravitation noch einer kleinen Modifikation bedürfen …" [15]

By the time, it became clear that the universe was expanding instead of being static and Einstein abandoned the cosmological constant Λ. "Historically the term containing the 'cosmological constant' λ was introduced into the field equations in order to enable us to account theoretically for the existence of a finite mean density in a static universe. It now appears that in the dynamical case this end can be reached without the introduction of λ" [16] But lately, Einstein's cosmological constant is revived by scientists to explain a mysterious force counteracting gravity called dark energy. In this context it is important to note that neither Newton's gravitational "constant" big G [17], [18] nor Einstein's cosmological constant Λ [19] is a constant.

Definition: General tensors

Independently of the tensors of the theory of general relativity, we introduce by definition the following covariant second rank tensors of preliminary unknown structure whose properties we leave undetermined as well. We define the following covariant second rank tensors of yet unknown structure as

$$A_{\mu\nu},\ B_{\mu\nu},\ C_{\mu\nu},\ D_{\mu\nu},\ {}_{R}U_{\mu\nu},\ {}_{R}\underline{U}_{\mu\nu},\ {}_{0}W_{\mu\nu},\ {}_{0}\underline{W}_{\mu\nu},\ {}_{R}W_{\mu\nu} \tag{2}$$

while the tensors $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}$, $D_{\mu\nu}$ may equally denote something like the four basic fields of nature. Especially, the Ricci tensor $R_{\mu\nu}$ itself can be decomposed in many different ways. In the following of this publication we define the following relationships. We decompose the Ricci tensor $R_{\mu\nu}$ by definition as

$$R_{\mu\nu} \equiv A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} + D_{\mu\nu} \equiv {}_{R}U_{\mu\nu} + {}_{R}\underline{U}_{\mu\nu} \equiv {}_{0}W_{\mu\nu} + {}_{0}\underline{W}_{\mu\nu} \equiv {}_{R}W_{\mu\nu} \tag{3}$$

to assure that both gravitation and electromagnetism is geometrized simultaneously. Since everything is expressed in terms of curvature tensor, the electromagnetic field itself is completely geometrized from the beginning. By the following definition, the electromagnetic stress energy tensor, denoted as $B_{\mu\nu}$, appears as part of Einstein's stress-energy tensor $T_{\mu\nu}$, while the tensor $A_{\mu\nu}$, also part of curvature, denotes the stress energy tensor of 'ordinary' matter. Thus far, we obtain

$${}_{R}E_{\mu\nu} \equiv A_{\mu\nu} + B_{\mu\nu} \equiv {}_{R}U_{\mu\nu} \equiv \frac{4\times 2\times\pi\times\gamma}{c\times c\times c\times c}\times T_{\mu\nu} \tag{4}$$

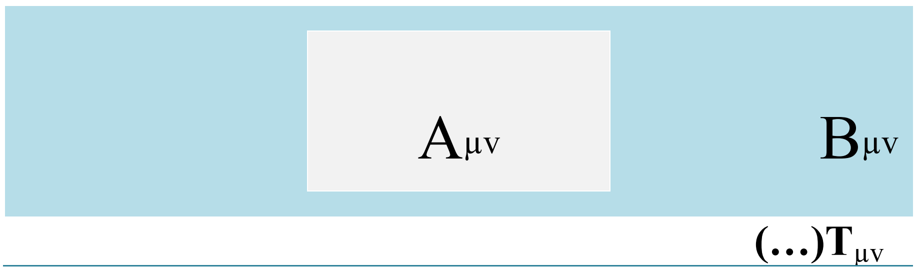
Scholium.
By this definition, we are following *Vranceanu* in his claim that the energy tensor T_{kl} can be treated as the sum of two tensors one of which is due to the electromagnetic field.

"On peut aussi supposer que le tenseur d'énergie T_{kl} soit la somme de deux tenseurs dont un dû au champ électromagnétique …" [20]

In English:
"One can also assume that the energy tensor T_{kl} be the sum of two tensors one of which is due to the electromagnetic field"

In other words, the stress-energy tensor of the electromagnetic field $B_{\mu\nu}$ is equivalent to the portion of the stress-energy tensor of energy $T_{\mu\nu}$ which is determined by the electromagnetic field. The *table 1* may illustrate this relationship in more detail.

Table 1. The relationship between ordinary matter $A_{\mu\nu}$ and the electromagnetic field $B_{\mu\nu}$.



Einstein himself was demanding something similar.

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld' und 'Materie' in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie' bezeichnet wird, also nicht nur *die 'Materie' im üblichen Sinne*, sondern auch *das elektromagnetische Feld*." [14]

To get an *anti tensor* [2] of Einstein's stress energy tensor $T_{\mu\nu}$, we define

$$ {}_{R}\underline{E}_{\mu\nu} \equiv C_{\mu\nu} + D_{\mu\nu} \equiv {}_{R}\underline{U}_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} \tag{5} $$

while Einsteinian tensor $G_{\mu\nu}$ is represented by

$$ G_{\mu\nu} \equiv A_{\mu\nu} + C_{\mu\nu} \equiv {}_{0}W_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} \times g_{\mu\nu} \tag{6} $$

where $A_{\mu\nu}$ is the known tensor of ordinary matter. One consequence of this definition is that the tensor of ordinary matter $A_{\mu\nu}$ is *a joint tensor* since the same tensor is a determining part of the Einstein's stress energy tensor $T_{\mu\nu}$ and equally of Einsteinian tensor $G_{\mu\nu}$. In probability theory, such a tensor would represent a joint distribution function. Finally, to obtain the *anti-tensor* $\underline{G}_{\mu\nu}$ of Einsteinian tensor $G_{\mu\nu}$, we define

$$ \underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} \equiv {}_{0}\underline{W}_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu} $$

(7)

Scholium.

The following 2x2 table (**Table 2**) may illustrate the basic relationships above

Table 2. The decomposition of the Ricci tensor $\mathbf{R}_{\mu\nu}$.

		Curvature		
		yes	**no**	
Energy / momentum	**yes**	$A_{\mu\nu}$	$B_{\mu\nu}$	${}_{R}U_{\mu\nu}$
	no	$C_{\mu\nu}$	$D_{\mu\nu}$	${}_{R}\underline{U}_{\mu\nu}$
		${}_{0}W_{\mu\nu}$	${}_{0}\underline{W}_{\mu\nu}$	${}_{R}W_{\mu\nu} \equiv R_{\mu\nu}$

The unified field under conditions of the general theory of relativity.

The tensors $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}$, $D_{\mu\nu}$ may have different meanings depending upon circumstances. In our attempt to reach a common representation of all four fundamental interactions, the unified field ${}_{R}W_{\mu\nu}$ or the Ricci tensor $R_{\mu\nu}$ is decomposed into several (sub-) fields $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}$, $D_{\mu\nu}$ in order to achieve unification between general relativity theory and quantum (field) theory from the beginning. The unification of the fundamental interactions is assured by the (sub-) fields $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}$, $D_{\mu\nu}$ which denote *the four basic fields of nature*. Quantum field theory itself is describing particles as a manifestation of an (abstract) field. In this context a particle a_i can be associated with the field $A_{\mu\nu}$, the particle b_i can be associated with the field $B_{\mu\nu}$, the particle c_i can be associated with the field $C_{\mu\nu}$, the particle d_i can be associated with the field $D_{\mu\nu}$. In the following, we can define something like $A_{\mu\nu} = a_i \times {}_{F}A_{\mu\nu}$ and $B_{\mu\nu} = b_i \times {}_{F}B_{\mu\nu}$ and $C_{\mu\nu} = c_i \times {}_{F}C_{\mu\nu}$ and $D_{\mu\nu} = d_i \times {}_{F}D_{\mu\nu}$ where the subscript $_F$ can denote an individual particle field. Maxwell's theory unified the electrical and the magnetic field into an *electromagnetic* field. Meanwhile, the electromagnetic and *weak nuclear forces* have been bound together as an *electroweak* force. The electroweak force and the *strong interaction* have been unified into the standard model of particle physics. Such an approach has not enabled a coherent theoretical framework of physics which fully explains and links together the today known physical aspects of objective reality. In contrast to quantum field theory, in this paper, we will not link the electromagnetic and weak nuclear forces together into the *electroweak* force. On the contrary, we link the *strong interaction* and the *weak nuclear force* into an *ordinary force*. In this sense, all but the electromagnetic force is treated or defined as ordinary force. The ordinary force and the electromagnetic force are or can be linked together into the standard model of particle physics. In our above setting, the ordinary force is determined by the tensor $A_{\mu\nu}$ while the electromagnetic force is determined by the tensor $B_{\mu\nu}$. Quantum field theory itself focuses on the three known non-gravitational forces and has been experimentally confirmed with tremendous accuracy under some appropriate domains of applicability while general relativity itself focuses on gravity. Still, quantum field theory and general relativity, as they are currently formulated, are mutually incompatible. Lastly, only one of these two theories can be correct or be both are incorrect.

Definition: The stress energy tensor of the electro-magnetic field ${}_{0}\underline{E}_{\mu\nu}$

We define the second rank *anti tensor* ${}_{0}\underline{E}_{\mu\nu}$ [2] of the tensor $A_{\mu\nu} = {}_{0}E_{\mu\nu}$ [2] as

$$B_{\mu\nu} \equiv {}_{0}\underline{E}_{\mu\nu} \equiv {}_{0}\underline{H}_{\mu\nu} \tag{8}$$

Under conditions of general relativity, where $A_{\mu\nu} = {}_{0}E_{\mu\nu}$ is tensor of ordinary energy/matter, the electromagnetic field is an *anti tensor* [2] of ordinary energy/matter. Under conditions of general relativity, the tensor of the electromagnetic field is determined by an anti-symmetric second-order tensor known as the electromagnetic field (Faraday) tensor F. In general, under conditions of general relativity, the second rank covariant tensor of the electromagnetic field in the absence of 'ordinary' matter, which is different from the electromagnetic field tensor F, can be derived as

$$B_{\mu\nu} \equiv {}_0\underline{E}_{\mu\nu} \equiv {}_0\underline{H}_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu} \right) \right) \right) \tag{9}$$

where F is the electromagnetic field tensor and $g_{\mu\nu}$ is the metric tensor.

Scholium.
The associated probability tensor [2] is determined as

$$p\left(B_{\mu\nu}\right) \equiv p\left({}_0\underline{E}_{\mu\nu}\right) \equiv p\left({}_0\underline{H}_{\mu\nu}\right) \equiv \frac{\left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu} \right) \right) \right)}{R_{\mu\mu}} \tag{10}$$

One possible theoretical geometric formulation of the stress-energy tensor of the electromagnetic field [2] follows as

$$\left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu} \right) \right) \right) \equiv p\left({}_0\underline{E}_{\mu\nu}\right) \cap R_{\mu\mu} \equiv p\left({}_0\underline{H}_{\mu\nu}\right) \cap R_{\mu\mu} \equiv p\left(B_{\mu\nu}\right) \cap R_{\mu\mu} \tag{11}$$

Definition: The tensor $n({}_0E_{\mu\nu})$

We define the second rank tensor $n({}_0E_{\mu\nu})$ as

$$n\left({}_0E_{\mu\nu}\right) \equiv n\left(A_{\mu\nu}\right) \equiv \frac{A_{\mu\nu}}{R} \equiv \frac{{}_0E_{\mu\nu}}{R} \equiv \frac{{}_0H_{\mu\nu}}{R} \equiv \frac{\left(\frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{\mu\nu} \right) - \left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu} \right) \right) \right)}{R} \tag{12}$$

where R denotes the Ricci scalar.

Definition: The tensor $n({}_0\underline{E}_{\mu\nu})$

We define the second rank tensor $n({}_0\underline{E}_{\mu\nu})$ as

$$n\left(B_{\mu\nu}\right) \equiv \frac{B_{\mu\nu}}{R} \equiv \frac{{}_0\underline{E}_{\mu\nu}}{R} \equiv \frac{{}_0\underline{H}_{\mu\nu}}{R} \equiv \frac{\left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu} \right) \right) \right)}{R} \tag{13}$$

where R denotes the Ricci scalar.

Definition: The tensor $n({}_{R}E_{\mu\nu})$

We define the second rank tensor $n({}_{R}E_{\mu\nu})$ as

$$n\left({}_{R}E_{\mu\nu}\right) \equiv n\left(A_{\mu\nu}\right) + n\left(B_{\mu\nu}\right) \equiv \frac{A_{\mu\nu} + B_{\mu\nu}}{R} \equiv \frac{\left(\frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{\mu\nu}\right)}{R} \tag{14}$$

where R denotes the Ricci scalar.

Definition: The tensor $n(C_{\mu\nu})$

We define the second rank tensor $n(C_{\mu\nu})$ as

$$n\left(C_{\mu\nu}\right) \equiv n\left(A_{\mu\nu}\right) \equiv \frac{C_{\mu\nu}}{R} \equiv \frac{\left(\left(\frac{1}{4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu}\right)\right)\right) - \left(\Lambda \times g_{\mu\nu}\right)}{R} \tag{15}$$

where R denotes the Ricci scalar.

Definition: The tensor $n(D_{\mu\nu})$

We define the second rank tensor $n(D_{\mu\nu})$ as

$$n\left(D_{\mu\nu}\right) \equiv \frac{\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\left(\frac{1}{4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu}\right)\right)\right)}{R} \tag{16}$$

where R denotes the Ricci scalar.

Definition: The tensor $n({}_{R}\underline{E}_{\mu\nu})$

We define the second rank tensor $n({}_{R}\underline{E}_{\mu\nu})$ as

$$n\left({}_{R}\underline{E}_{\mu\nu}\right) \equiv n\left(C_{\mu\nu}\right) + n\left(D_{\mu\nu}\right) \equiv \frac{C_{\mu\nu} + D_{\mu\nu}}{R} \equiv \frac{R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c \times c \times c \times c} \times T_{\mu\nu}\right)}{R} \equiv \frac{\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)}{R} \tag{17}$$

where R denotes the Ricci scalar.

Definition: The tensor n($G_{\mu\nu}$)

We define the second rank tensor n($G_{\mu\nu}$) as

$$n\left({}_{R}G_{\mu\nu}\right) \equiv n\left(A_{\mu\nu}\right) + n\left(C_{\mu\nu}\right) \equiv \frac{A_{\mu\nu} + C_{\mu\nu}}{R} \equiv \frac{R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right)}{R} \tag{18}$$

where R denotes the Ricci scalar and $G_{\mu\nu}$ is the Einsteinian tensor.

Definition: The tensor n($\underline{G}_{\mu\nu}$)

We define the second rank tensor n($\underline{G}_{\mu\nu}$) as

$$n\left({}_{R}\underline{G}_{\mu\nu}\right) \equiv n\left(B_{\mu\nu}\right) + n\left(D_{\mu\nu}\right) \equiv \frac{B_{\mu\nu} + D_{\mu\nu}}{R} \equiv \frac{\left(\frac{R}{2} \times g_{\mu\nu}\right)}{R} \tag{19}$$

where R denotes the Ricci scalar and $\underline{G}_{\mu\nu}$ is the anti Einsteinian tensor.

Definition: The tensor n($\underline{E}_{\mu\nu}$)

We define the second rank tensor n($\underline{E}_{\mu\nu}$) as

$$n\left({}_{R}\underline{E}_{\mu\nu}\right) \equiv n\left(C_{\mu\nu}\right) + n\left(D_{\mu\nu}\right) \equiv \frac{C_{\mu\nu} + D_{\mu\nu}}{R} \equiv \frac{\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right)}{R} \tag{20}$$

where R denotes the Ricci scalar and $G_{\mu\nu}$ is the Einsteinian tensor.

Definition: The tensor n($R_{\mu\nu}$)

We define the second rank tensor n($R_{\mu\nu}$) as

$$n\left(R_{\mu\nu}\right) \equiv n\left(A_{\mu\nu}\right) + n\left(B_{\mu\nu}\right) + n\left(C_{\mu\nu}\right) + n\left(D_{\mu\nu}\right) \equiv \frac{R_{\mu\nu}}{R} \equiv \frac{1}{R} \times R_{\mu\nu} \tag{21}$$

where R denotes the Ricci scalar and $R_{\mu\nu}$ denotes the Ricci tensor.

Scholium.
The following 2x2 table 3 (**Table 3**) may illustrate the basic relationships above

Table 3. The decomposition of the Ricci tensor $\mathbf{R}_{\mu\nu}$.

		Curvature		
		yes	**no**	
Energy / momentum	**yes**	$n(A_{\mu\nu})$	$n(B_{\mu\nu})$	$n({}_R E_{\mu\nu})$
	no	$n(C_{\mu\nu})$	$n(D_{\mu\nu})$	$n({}_R \underline{E}_{\mu\nu})$
		$n(G_{\mu\nu})$	$n(\underline{G}_{\mu\nu})$	$n(R_{\mu\nu})$

The unified field under conditions of the general theory of relativity.

Definition: The tensor $g_{\mu\nu}$

The mathematics of general relativity are more or less complex. As a result, the curvature of space (represented by the Einstein tensor $G_{\mu\nu}$) is caused by the presence of matter and energy (represented by the stress–energy tensor $T_{\mu\nu}$) and vice versa. The curvature of space is the cause or determines how matter/energy has to move. The Riemannian metric tensor for a curved space-time of general relativity theory, a kind of generalization of the gravitational potential of Newtonian gravitation, is denoted as

$$g_{\mu\nu} \tag{22}$$

Scholium.
The metric tensor $g_{\mu\nu}$ is a central object in general relativity and describes more or less the local geometry of space-time while representing the gravitational potential. The metric tensor determines the invariant square of an infinitesimal line element, denoted as *ds* and often referred to as an interval. In general, the generalization of the standard measure of distance *ds* between two points in Euclidian space is defined as

$$ds^2 \equiv (d_1x \times d_1x) + (d_2x \times d_2x + \ldots + d_nx \times d_nx) \equiv g_{\mu\nu}dx^{\mu}dx^{\nu} \tag{23}$$

Under conditions, where

$$c^2 \times_0 t^2 \equiv (d_1x \times d_1x) \tag{24}$$

it is

$$ds^2 \equiv (c^2 \times_0 t^2) + (d_2x \times d_2x + \ldots + d_nx \times d_nx) \equiv g_{\mu\nu}dx^{\mu}dx^{\nu} \tag{25}$$

Dividing the equation before by the speed of the light squared, c^2, it is

$$\frac{ds^2}{c^2} \equiv \frac{(c^2 \times_0 t^2)}{c^2} + \frac{(d_2x \times d_2x)}{c^2} + \ldots + \frac{(d_nx \times d_nx)}{c^2} \equiv \left(\frac{1}{c^2}\right) \times g_{\mu\nu}dx^{\mu}dx^{\nu} \tag{26}$$

The term ds^2/c^2 yields the time squared or $ds^2/c^2 = d_R t^2$ as do the other terms. We define *the (metric) tensor of time* as ${}_R t_{\mu\nu} = (1/c^2) \times g_{\mu\nu}$. The (metric) tensor of time ${}_R t_{\mu\nu}$ follows as

$$d_R t^2 \equiv \left({}_0 t^2\right) + \left(d_2 t \times d_2 t\right) + \ldots + \left(d_n t \times d_n t\right) \equiv \left(\frac{1}{c^2}\right) \times g_{\mu\nu} dx^\mu dx^\nu \equiv_R t_{\mu\nu} dt^\mu dt^\nu \tag{27}$$

The relationship between the tensor of the gravitational field ${}_R g_{\mu\nu}$ and the tensor of time ${}_R t_{\mu\nu}$ is determined [2] as ${}_R t_{\mu\nu} = c^2 \times {}_R g_{\mu\nu}$. Due to the relationship above, it is ${}_R t_{\mu\nu} = (1/c^2) \times g_{\mu\nu}$. Under these assumptions, we obtain the equation ${}_R t_{\mu\nu} = c^2 \times {}_R g_{\mu\nu} = (1/c^2) \times g_{\mu\nu}$. At the end, it is $g_{\mu\nu} = c^2 \times c^2 \times {}_R g_{\mu\nu}$ where $g_{\mu\nu}$ is the metric tensor of general relativity and ${}_R g_{\mu\nu}$ is the tensor of the gravitational field. Under conditions, where ${}_R t_{\mu\nu} = (R/2) \times g_{\mu\nu} - \Lambda \times g_{\mu\nu}$ and where $((R/2) - \Lambda) = (1/c^2)$, the metric tensor $g_{\mu\nu}$ is equivalent with the tensor of time ${}_R t_{\mu\nu}$. Finally, the metric tensor $g_{\mu\nu}$ is not identical with the tensor of the gravitational field ${}_R g_{\mu\nu}$ but represents more or less the gravitational potential.

Definition: The metric tensor of the electromagnetic field ${}_{EM} g_{\mu\nu}$

We define the second rank metric tensor of the electro-magnetic field ${}_{EM} g_{\mu\nu}$ of preliminary unknown structure as

$$ {}_{EM} g_{\mu\nu} \equiv \frac{B_{\mu\nu}}{Y} \tag{28}$$

where Y denotes an unknown (scalar) parameter. Due to this definition, it is $B_{\mu\nu} = Y \times {}_{EM} g_{\mu\nu}$.

Definition: The anti metric tensor of the electromagnetic field ${}_0 g_{\mu\nu}$ or ${}_{GW} g_{\mu\nu}$

We define the second rank *anti metric* tensor of the electro-magnetic field ${}_{GW} g_{\mu\nu}$ of preliminary unknown structure as

$$ {}_{GW} g_{\mu\nu} \equiv {}_{EM} \underline{g}_{\mu\nu} \equiv \frac{D_{\mu\nu}}{Y} \tag{29}$$

where Y denotes an (scalar) parameter. Due to this definition, it is $D_{\mu\nu} = Y \times {}_{GW} g_{\mu\nu}$.

Definition: The relationships between the metric tensors

In general, the metric tensor for a curved space-time of general relativity theory is equally determined as

$$g_{\mu\nu} \equiv g_{\mu\nu} + 0 \equiv g_{\mu\nu} - {}_{GW} g_{\mu\nu} + {}_{GW} g_{\mu\nu} \equiv {}_{EM} g_{\mu\nu} + {}_{EM} \underline{g}_{\mu\nu} \equiv {}_{EM} g_{\mu\nu} + {}_{GW} g_{\mu\nu} \tag{30}$$

where and ${}_{EM} g_{\mu\nu} = g_{\mu\nu} - {}_{GW} g_{\mu\nu}$ and ${}_{EM;} g_{\mu\nu}$ denotes the second rank metric tensor of the electro-magnetic field while ${}_{\cdot GW} g_{\mu\nu}$ is the second rank *anti metric* tensor of the electro-magnetic field. Both tensors are still of unknown structure. From this definition, it follows that

$$\frac{R}{2} \times g_{\mu\nu} \equiv \frac{R}{2} \times \left({}_{EM} g_{\mu\nu} + {}_{GW} g_{\mu\nu}\right) \equiv \frac{R}{2} \times {}_{EM} g_{\mu\nu} + \frac{R}{2} \times {}_{GW} g_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} \tag{31}$$

Scholium.
The true meaning of the metric tensor ${}_{GW}g_{\mu\nu}$ is not clear at this moment. One is for sure, the same tensor is an *anti-tensor* of the metric tensor of the electromagnetic field ${}_{EM}g_{\mu\nu}$. There is some theoretical possibility that the tensor ${}_{GW}g_{\mu\nu}$ is related to something like the metric tensor of the gravitational waves, therefore the abbreviation ${}_{GW}g_{\mu\nu}$.

2.2. Axioms.

2.2.1. Axiom I. (Lex identitatis. Principium identitatis. The identity law)

The foundation of all what may follow is the following axiom:

$$+1 \equiv +1. \tag{32}$$

2.2.2. Axiom II.

$$\frac{+1}{+0} \equiv +\infty. \tag{33}$$

2.2.3. Axiom III.

$$\frac{+0}{+0} \equiv +1. \tag{34}$$

3. Results

3.1. Theorem. The unification of gravity and electromagnetism

Claim. (Theorem. Proposition. Statement.)
In general, the gravitational and the electromagnetic field can be joined into one single hyperfield which itself is completely determined by the geometrical structure of the space-time. We obtain

$$2 \times C_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) = +B_{\mu\nu} + C_{\mu\nu} \tag{35}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{36}$$

Multiplying this equation by the stress-energy tensor of general relativity $((4 \times 2 \times \pi \times \gamma)/(c^4)) \times T_{\mu\nu}$, it is

$$+1 \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \times T_{\mu\nu}\right) = +1 \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \times T_{\mu\nu}\right) \tag{37}$$

where γ is Newton's gravitational 'constant' [17], [18] , c is the speed of light in vacuum and π , sometimes referred to as 'Archimedes' constant', is the ratio of a circle's circumference to its diameter. Due to Einstein's general relativity, the equation before is equivalent with

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \times T_{\mu\nu}\right) \tag{38}$$

$R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant and $T_{\mu\nu}$ is the stress–energy tensor. By defining the Einstein tensor as $G_{\mu\nu} = R_{\mu\nu} - (R/2) \times g_{\mu\nu}$, it is possible to write the Einstein field equations in a more compact as

$$G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \times T_{\mu\nu}\right) \tag{39}$$

Due to our definition above it is $G_{\mu\nu} = A_{\mu\nu} + C_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$A_{\mu\nu} + C_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \times T_{\mu\nu}\right) \tag{40}$$

Under these conditions, we recall our definition above where $((8\times\pi\times\gamma)/(c^4))\times T_{\mu\nu} = A_{\mu\nu} + B_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$A_{\mu\nu} + C_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = A_{\mu\nu} + B_{\mu\nu} \tag{41}$$

where $A_{\mu\nu}$ denotes the stress energy tensor of ordinary matter and $B_{\mu\nu}$ denotes the stress energy tensor of the electromagnetic field. Simplifying equation, it is

$$+C_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = +B_{\mu\nu} \tag{42}$$

where $C_{\mu\nu}$ denotes the gravitational field due to the stress energy tensor of ordinary matter $A_{\mu\nu}$. We add the tensor $C_{\mu\nu}$ on both sides of the equation before. The unification of gravity and electromagnetisms under conditions of general relativity theory follows as

$$+C_{\mu\nu} + C_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = +B_{\mu\nu} + C_{\mu\nu} \tag{43}$$

or in general as

$$2 \times C_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = +B_{\mu\nu} + C_{\mu\nu} \tag{44}$$

Quod erat demonstrandum.

3.2. Theorem. The anti Einsteinian tensor $\underline{G}_{\mu\nu}$

Claim. (Theorem. Proposition. Statement.)
The anti Einsteinian tensor $G_{\mu\nu}$ is determined as

$$\underline{G}_{\mu\nu} = B_{\mu\nu} + D_{\mu\nu} = +\left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{45}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{46}$$

Multiplying this equation by the Ricci Tensor $R_{\mu\nu}$, it is

$$+1 \times \left(R_{\mu\nu}\right) = +1 \times \left(R_{\mu\nu}\right) \tag{47}$$

Due to our definition above, it is $A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} \tag{48}$$

The sum of the tensor $\underline{G}_{\mu\nu} = B_{\mu\nu} + D_{\mu\nu}$ can be obtained as

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} - A_{\mu\nu} - C_{\mu\nu} \tag{49}$$

which can be simplified as

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} - \left(A_{\mu\nu} + C_{\mu\nu}\right) \tag{50}$$

Due to our definition, Einsteinian tensor $G_{\mu\nu}$ is defined as $G_{\mu\nu} = A_{\mu\nu} + C_{\mu\nu}$. Rearranging equation above, it is

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} - \left(G_{\mu\nu}\right) \tag{51}$$

Einstein's tensor $G_{\mu\nu}$ is defined as $G_{\mu\nu} = R_{\mu\nu} - ((R/2) \times g_{\mu\nu})$. Substituting this relationship into the equation before, we obtain

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)\right) \tag{52}$$

Rearranging equation, we obtain

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} - R_{\mu\nu} + \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{53}$$

At the end, we obtain

$$\underline{G}_{\mu\nu} \equiv B_{\mu\nu} + D_{\mu\nu} = +\left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{54}$$

Quod erat demonstrandum.

3.3. Theorem. The determination of the unknown parameter Y

Claim. (Theorem. Proposition. Statement.)
The unknown parameter Y is determined as

$$Y = \left(\frac{R}{2}\right) \tag{55}$$

where R denotes the Ricci scalar.

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{56}$$

Multiplying this equation by the *anti Einsteinian tensor* $\underline{G}_{\mu\nu}$, it is

$$+1 \times \left(\underline{G}_{\mu\nu}\right) = +1 \times \left(\underline{G}_{\mu\nu}\right) \tag{57}$$

The same tensor was determined by the theorem before as $B_{\mu\nu} + D_{\mu\nu} = \underline{G}_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$B_{\mu\nu} + D_{\mu\nu} \equiv \underline{G}_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{58}$$

Due to our definition above, it is as $B_{\mu\nu} = Y \times {}_{EM}g_{\mu\nu}$ and $D_{\mu\nu} = Y \times {}_{GW}g_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$B_{\mu\nu} + D_{\mu\nu} \equiv \left(Y \times {}_{EM}g_{\mu\nu}\right) + \left(Y \times {}_{GW}g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{59}$$

Due to our definition $g_{\mu\nu} = {}_{EM}g_{\mu\nu} + {}_{GW}g_{\mu\nu}$, the equation before can be rearranged as

$$B_{\mu\nu} + D_{\mu\nu} \equiv Y \times \left({}_{EM}g_{\mu\nu} + {}_{GW}g_{\mu\nu}\right) \equiv Y \times \left(g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{60}$$

In other words, it is

$$Y \times \left(g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{61}$$

A further manipulation of the equation before yields the result that

$$Y = \left(\frac{R}{2}\right) \tag{62}$$

Quod erat demonstrandum.

3.4. Theorem. The geometrization of the electromagnetic field

Claim. (Theorem. Proposition. Statement.)
The geometrizsation of the electromagnetic field under conditions of general relativity follows as

$$B_{\mu v} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu v} \tag{63}$$

where R denotes the Ricci scalar and ${}_{EM}g_{\mu v}$ denotes the metric tensor of the electromagnetic field.

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{64}$$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, denoted as $B_{\mu v}$, it is

$$+1 \times (B_{\mu v}) = +1 \times (B_{\mu v}) \tag{65}$$

Due to our definition above, the stress-energy tensor of the electromagnetic field was determined by the relationship $B_{\mu v} = Y \times {}_{EM}g_{\mu v}$. Rearranging the equation before we obtain

$$B_{\mu v} = Y \times {}_{EM}g_{\mu v} \tag{66}$$

According to the theorem before, the unknown parameter Y is determined as Y = R/2. The geometrizsation of the electromagnetic field under conditions of general relativity follows as

$$B_{\mu v} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu v} \tag{67}$$

Quod erat demonstrandum.

Scholium.
In a more far reaching development, at least since general relativity theory brought the geometry to the scenario of physics, many attempts were made to extend general relativity's geometrization of gravitation to non-gravitational fields. In particular, *the geometrization of the electromagnetic field* became a principal focus and a cornerstone of physical interest and inquiry. The many geometric theories of the electromagnetism as published meanwhile are still not consistent with the framework of the quantum theory or self-contradictory, despite the fact that the electromagnetic theory as consolidated on the 19th century. The present theorem before describes the stress-energy tensor of the electro-magnetic field as directly related or determined by to the space-time geometry or the metric tensor ${}_{EM}g_{\mu v}$. A unified field theory, in the sense of a completely geometrical field theory of all fundamental interactions, is no longer only a theoretical desire.

3.5. Theorem. The determination of the metric tensor of the electromagnetic field ${}_{EM}g_{\mu\nu}$

Claim. (Theorem. Proposition. Statement.)
The metric tensor of the electromagnetic field ${}_{EM}g_{\mu\nu}$ under conditions of general relativity is determined as

$$ {}_{EM}g_{\mu\nu} = \left(\frac{2}{R}\right) \times \left(\left(\frac{1}{4\times\pi}\right) \times \left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right) - \left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right) \right) \right) = \left(\frac{2}{R}\right)\times B_{\mu\nu} \tag{68} $$

where R denotes the Ricci scalar and ${}_{EM}g_{\mu\nu}$ denotes the metric tensor of the electromagnetic field.

Direct proof.
In general, using axiom I is it

$$ +1 = +1 \tag{69} $$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, abbrivated as $B_{\mu\nu}$, it is

$$ +1\times\left(B_{\mu\nu}\right) = +1\times\left(B_{\mu\nu}\right) \tag{70} $$

Due to our theorem before, the geometrizsation of the electromagnetic field under conditions of general relativity is determined as

$$ \left(\frac{R}{2}\right)\times {}_{EM}g_{\mu\nu} = B_{\mu\nu} \tag{71} $$

The stress-energy tensor of the electromagnetic field $B_{\mu\nu}$ is determined in detail i. e. by the relationship

$$ B_{\mu\nu} \equiv {}_{0}\underline{E}_{\mu\nu} \equiv {}_{0}\underline{H}_{\mu\nu} \equiv \left(\left(\frac{1}{4\times\pi}\right) \times \left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right) - \left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right) \right) \right) \tag{72} $$

where F is the electromagnetic field tensor and $g_{\mu\nu}$ is the metric tensor. The equation before changes to

$$ \left(\frac{R}{2}\right)\times {}_{EM}g_{\mu\nu} = \left(\left(\frac{1}{4\times\pi}\right) \times \left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right) - \left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right) \right) \right) = B_{\mu\nu} \tag{73} $$

The metric tensor of the electromagnetic field ${}_{EM}g_{\mu\nu}$ under conditions of general relativity is determined as

$$ {}_{EM}g_{\mu\nu} = \left(\frac{2}{R}\right) \times \left(\left(\frac{1}{4\times\pi}\right) \times \left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right) - \left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right) \right) \right) = \left(\frac{2}{R}\right)\times B_{\mu\nu} \tag{74} $$

Quod erat demonstrandum.

3.6. Theorem. The tensor of the 'ordinary' gravitational field

Claim. (Theorem. Proposition. Statement.)
The tensor of the 'ordinary' gravitational field denoted as $C_{\mu\nu}$ follows as

$$C_{\mu\nu} = B_{\mu\nu} - (\Lambda \times g_{\mu\nu}) = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{75}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{76}$$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, denoted as $B_{\mu\nu}$, it is

$$+1 \times (B_{\mu\nu}) = +1 \times (B_{\mu\nu}) \tag{77}$$

Due to our theorem before, the metric tensor of the electromagnetic field under conditions of general relativity is determined as

$$B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} \tag{78}$$

where R denotes the Ricci scalar and ${}_{EM}g_{\mu\nu}$ denotes the metric tensor of the electromagnetic field. Due to the another theorem above, it is

$$+C_{\mu\nu} + (\Lambda \times g_{\mu\nu}) = +B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} \tag{79}$$

The tensor of the 'ordinary' gravitational field as $C_{\mu\nu}$ is determined by the stress energy tensor of the electromagnetic field $B_{\mu\nu}$ as

$$C_{\mu\nu} = B_{\mu\nu} - (\Lambda \times g_{\mu\nu}) = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{80}$$

Quod erat demonstrandum.

3.7. Theorem. The tensor of the hyperfield of gravitation and electromagnetism

Claim. (Theorem. Proposition. Statement.)
The geometrized form of the hyper-tensor of unification of gravitation and electromagnetism is determined as

$$C_{\mu\nu} + B_{\mu\nu} = R \times {}_{EM}g_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right)$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{81}$$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, denoted as $B_{\mu\nu}$, it is

$$+1 \times \left(B_{\mu\nu}\right) = +1 \times \left(B_{\mu\nu}\right) \tag{82}$$

Due to our theorem before, the stress energy tensor of the electromagnetic field under conditions of general relativity is determined as

$$B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} \tag{83}$$

where R denotes the Ricci scalar and ${}_{EM}g_{\mu\nu}$ denotes the metric tensor of the electromagnetic field. Due to a theorem before, it is

$$+C_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) = +B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} \tag{84}$$

The tensor of the 'ordinary' gravitational field as $C_{\mu\nu}$ is determined by the stress energy tensor of the electromagnetic field $B_{\mu\nu}$ as

$$C_{\mu\nu} = B_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) \tag{85}$$

Adding the stress-energy tensor of the electromagnetic field $B_{\mu\nu} = (R/2) \times {}_{EM}g_{\mu\nu}$, we obtain the geometrized form of the hyper-tensor of gravitation and electromagnetism as

$$C_{\mu\nu} + B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} + \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) = R \times {}_{EM}g_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) \tag{86}$$

Quod erat demonstrandum.

3.8. Theorem. The tensor $D_{\mu\nu}$

Claim. (Theorem. Proposition. Statement.)
The tensor $D_{\mu\nu}$ is determined as

$$D_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} \tag{87}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{88}$$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, denoted as $B_{\mu\nu}$, it is

$$+1 \times \left(B_{\mu\nu}\right) = +1 \times \left(B_{\mu\nu}\right) \tag{89}$$

Due to our theorem before, the stress energy tensor of the electromagnetic field under conditions of general relativity is determined as

$$B_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} \tag{90}$$

where R denotes the Ricci scalar and ${}_{EM}g_{\mu\nu}$ denotes the metric tensor of the electromagnetic field. Adding the tensor $D_{\mu\nu}$, we obtain

$$B_{\mu\nu} + D_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} + D_{\mu\nu} \tag{91}$$

According to a theorem before, this relationship is equivalent with

$$B_{\mu\nu} + D_{\mu\nu} = \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} + D_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{92}$$

Rearranging equation, the tensor $D_{\mu\nu}$ is determined as

$$D_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} - B_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} = \left(\frac{R}{2}\right) \times \left(g_{\mu\nu} - {}_{EM}g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} \tag{93}$$

Quod erat demonstrandum.

3.9. Theorem. The geometrization of 'ordinary' matter

Claim. (Theorem. Proposition. Statement.)

The geometrization of 'ordinary' matter follows as

$$A_{\mu\nu} = R_{\mu\nu} - R \times {}_{EM}g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \tag{94}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{95}$$

Multiplying this equation by the Ricci Tensor $R_{\mu\nu}$, it is

$$+1 \times \left(R_{\mu\nu}\right) = +1 \times \left(R_{\mu\nu}\right) \tag{96}$$

Due to our definition above, it is $A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu}$. Substituting this relationship into the equation before, we obtain

$$A_{\mu\nu} + B_{\mu\nu} + C_{\mu\nu} + D_{\mu\nu} = R_{\mu\nu} \tag{97}$$

The tensor of ordinary matter $A_{\mu\nu}$ follows as

$$A_{\mu\nu} = R_{\mu\nu} - B_{\mu\nu} - \left(C_{\mu\nu} + D_{\mu\nu}\right) \tag{98}$$

The tensor $B_{\mu\nu}$ itself was determined as $B_{\mu\nu} = (R/2)\times_{EM}g_{\mu\nu}$. The addition of the tensors $D_{\mu\nu}$ plus $C_{\mu\nu}$ is determined as $D_{\mu\nu} + C_{\mu\nu} = (R/2)\times g_{\mu\nu} - \Lambda\times g_{\mu\nu}$ .The equation before changes to

$$A_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right)\right) \tag{99}$$

Rearranging equation before, we obtain

$$A_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \tag{100}$$

The tensor $(R/2)\times g_{\mu\nu}$ is detrmined as $(R/2)\times g_{\mu\nu} = (R/2)\times_{EM}g_{\mu\nu} + (R/2)\times_{GW}g_{\mu\nu}$. The equation can be simplified as

$$A_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \tag{101}$$

The geometrization of 'ordinary' matter follows in general as

$$A_{\mu\nu} = R_{\mu\nu} - R \times {}_{EM}g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \tag{102}$$

Quod erat demonstrandum.

3.10. Theorem. The probability tensor of the electromagnetic field $p(B_{\mu\nu})$

Claim. (Theorem. Proposition. Statement.)
The probability tensor $p(B_{\mu\nu})$ of the electromagnetic field is determined as

$$p\left(B_{\mu\nu}\right) \equiv \left(\frac{1}{2}\right) \times g^{\mu\nu} \times {}_{EM}g_{\mu\nu} = \frac{\left(\left(\frac{1}{4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu}\right)\right)\right)}{R_{\mu\nu}} = \frac{B_{\mu\nu}}{R_{\mu\nu}} \tag{103}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{104}$$

Multiplying this equation by the stress-energy tensor of the electromagnetic field, denoted as $B_{\mu\nu}$, it is

$$+1 \times \left(B_{\mu\nu}\right) = +1 \times \left(B_{\mu\nu}\right) \tag{105}$$

Due to our theorem before, the geometrizsation of the electromagnetic field under conditions of general relativity is determined as

$$\left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} = B_{\mu\nu} \tag{106}$$

The stress-energy tensor of the electromagnetic field was determined i.e. by the relationship

$$B_{\mu\nu} \equiv {}_{0}\underline{E}_{\mu\nu} \equiv {}_{0}\underline{H}_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{d\nu} \times F^{d\nu}\right)\right)\right) \tag{107}$$

where F is the electromagnetic field tensor and $g_{\mu\nu}$ is the metric tensor. The equation before changes to

$$\left(\frac{R}{2}\right)\times {}_{EM}g_{\mu\nu}=\left(\left(\frac{1}{4\times\pi}\right)\times\left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right)\right)\right)=B_{\mu\nu} \tag{108}$$

The Ricci scalar R is defined as the contraction of the Ricci tensor $R_{\mu\nu}$ or it is $R = g^{\mu\nu}R_{\mu\nu}$. The equation before changes to

$$\left(\frac{1}{2}\right)\times g^{\mu\nu}\times R_{\mu\nu}\times {}_{EM}g_{\mu\nu}=\left(\left(\frac{1}{4\times\pi}\right)\times\left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right)\right)\right)=B_{\mu\nu} \tag{109}$$

A *commutative division* [2] by the Ricci tensor $R_{\mu\nu}$ leads to the relationship

$$\left(\frac{1}{2}\right)\times g^{\mu\nu}\times {}_{EM}g_{\mu\nu}=\frac{\left(\left(\frac{1}{4\times\pi}\right)\times\left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right)\right)\right)}{R_{\mu\nu}}=\frac{B_{\mu\nu}}{R_{\mu\nu}} \tag{110}$$

This equation is identical with the probability tensor $p(B_{\mu\nu})$ of the electromagnetic field. In general it is

$$p\left(B_{\mu\nu}\right)\equiv\left(\frac{1}{2}\right)\times g^{\mu\nu}\times {}_{EM}g_{\mu\nu}=\frac{\left(\left(\frac{1}{4\times\pi}\right)\times\left(\left(F_{\mu c}\times F_{\nu}{}^{c}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{d\nu}\times F^{d\nu}\right)\right)\right)}{R_{\mu\nu}}=\frac{B_{\mu\nu}}{R_{\mu\nu}} \tag{111}$$

Quod erat demonstrandum.

Scholium.
The probability tensor is determined by the metric. This is of principle importance.

3.11. Theorem. The probability tensor is determined by the metric tensor

Claim. (Theorem. Proposition. Statement.)
We defined $n(X_{\mu v})=X_{\mu v}/R$ where $X_{\mu v}$ denotes a second rank co-variant tensor and R denotes the Ricci scalar, the contraction of the Ricci tensor as $R=g^{\mu v}R_{\mu v}$. Further, $p(X_{\mu v})=X_{\mu v}/R_{\mu v}$ denotes the probability tensor [2] of the tensor $X_{\mu v}$. In general, the probability tensor $p(X_{\mu v})$ of a tensor $X_{\mu v}$ is determined by the metric tensor as

$$p\left(X_{\mu v}\right) \equiv g^{\mu v} \times n\left(X_{\mu v}\right) \tag{112}$$

Direct proof.
In general, using axiom I is it

$$+1 = +1 \tag{113}$$

Multiplying this equation by the tensor $X_{\mu v}$, it is

$$+1 \times \left(X_{\mu v}\right) = +1 \times \left(X_{\mu v}\right) \tag{114}$$

A commutative [2] division of the tensor $X_{\mu v}$ by the Ricci tensor $R_{\mu v}$ yields

$$\frac{X_{\mu v}}{R_{\mu v}} = \frac{X_{\mu v}}{R_{\mu v}} \tag{115}$$

which is equivalent with

$$p\left(X_{\mu v}\right) \equiv \frac{X_{\mu v}}{R_{\mu v}} = \frac{X_{\mu v}}{R_{\mu v}} \tag{116}$$

or with

$$p\left(X_{\mu v}\right) \equiv \frac{X_{\mu v}}{R_{\mu v}} \tag{117}$$

Rearranging equation, we obtain

$$p\left(X_{\mu v}\right) \equiv \frac{g^{\mu v}}{g^{\mu v}} \times \frac{X_{\mu v}}{R_{\mu v}} \equiv \frac{g^{\mu v} \times X_{\mu v}}{g^{\mu v} \times R_{\mu v}} \tag{118}$$

Due to the relationship $R=g^{\mu v}R_{\mu v}$, the equation before simplifies as

$$p\left(X_{\mu v}\right) \equiv \frac{g^{\mu v}}{g^{\mu v}} \times \frac{X_{\mu v}}{R_{\mu v}} \equiv \frac{g^{\mu v} \times X_{\mu v}}{g^{\mu v} \times R_{\mu v}} \equiv \frac{g^{\mu v} \times X_{\mu v}}{R} \tag{119}$$

and due to our definition $n(X_{\mu\nu})=X_{\mu\nu}/R$ it is

$$p\left(X_{\mu\nu}\right) \equiv \frac{g^{\mu\nu}}{g^{\mu\nu}} \times \frac{X_{\mu\nu}}{R_{\mu\nu}} \equiv \frac{g^{\mu\nu} \times X_{\mu\nu}}{g^{\mu\nu} \times R_{\mu\nu}} \equiv \frac{g^{\mu\nu} \times X_{\mu\nu}}{R} \equiv g^{\mu\nu} \times n\left(X_{\mu\nu}\right) \tag{120}$$

or in general. The probability tensor of a tensor $X_{\mu\nu}$ is determined by the (*inverse*) metric tensor as

$$p\left(X_{\mu\nu}\right) \equiv g^{\mu\nu} \times n\left(X_{\mu\nu}\right) \tag{121}$$

Quod erat demonstrandum.

Scholium.

Quantum physics (quantization) focuses on the probability (amplitudes) while general relativity theory relies on geometry (tempo-spatial points). The definition of a probability tensor $p(X_{\mu\nu})$ of a tensor $X_{\mu\nu}$ marks a remarkable degree of interaction between probability theory and high-dimensional theory of general relativity and a key step to the unification of quantum physics and general relativity by probabilitizing general relativity's geometric background. The contradiction free transformation of a geometrical mathematical framework into a probabilistic mathematical framework is possible in principle and vice versa. A geometrization of probability theory appears to be possible too.

4. Discussion

A new approach to quantum gravity and the unified field theory developed by the author is already published [2]. Besides of the misprint in this paper [2] in Eq. (76)

$${}_{0}\omega_{\mu\nu} \equiv 2_{\mu\nu} \cap_{R} \pi_{\mu\nu} \cap {}_{R}f_{\mu\nu} \equiv \frac{1_{\mu\nu}}{{}_{R}\hbar} \frown \left(C \;{}_{\nu} - \Lambda \times g_{\mu\nu} \right) \tag{76}$$

which should be changed to

$${}_{0}\omega_{\mu\nu} \equiv 2_{\mu\nu} \cap_{R} \pi_{\mu\nu} \cap {}_{R}f_{\mu\nu} \equiv \frac{1_{\mu\nu}}{{}_{R}\hbar} \frown \left(C \;{}_{\nu} + \Lambda \times g_{\mu\nu} \right) \tag{76}$$

one way how to geometrize the electromagnetic field is already provided. In this paper the geometrization of the electromagnetic field under conditions of general relativity theory is developed in more (technical) detail. This paper has answered the question about the geometrization of the electromagnetic field under conditions of general theory of relativity. Still, this publication has not answered the question whether does *geometrization* excludes *quantization* and vice versa. In other words, is there a dualism between geometrization and quantization in the sense *either* geometrization *or* quantization. In this context, the geometric entity 'line' is determined by points. But what is a point, something quantized? Thus far, can we find any geometry within a quantized 'objects' and vice versa? Is the geometry of space-time at the end determined by quantized 'objects'? The answer to such questions may be considered for future work. In this paper, the tensor $D_{\mu\nu}$ was derived in Eq. (78) as

$$D_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} - B_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} - \left(\frac{R}{2}\right) \times {}_{EM}g_{\mu\nu} = \left(\frac{R}{2}\right) \times \left(g_{\mu\nu} - {}_{EM}g_{\mu\nu}\right) = \left(\frac{R}{2}\right) \times {}_{GW}g_{\mu\nu} \qquad (78)$$

The concrete meaning of the tensor $D_{\mu\nu}$ is not clear at this moment. The information carried by the tensor $D_{\mu\nu}$ is very different from the information as carried by the electromagnetic waves. The interaction between electromagnetic and gravitational waves and the transformation of one wave into another is already discussed [21] in literature. In this context, the tensor $D_{\mu\nu}$ is an anti-tensor of the stress-energy tensor of the electro-magentic field. Thus far, it is possible and highly desirable that the metric tensor ${}_{GW}g_{\mu\nu}$ is determined by fuctuations of gravitational fields and that the same tensor represents something like "ripples" in spacetime. Under these assumptions, the tensor $D_{\mu\nu}$ could represent the metric tensor of the gravitational waves. The assumption $D_{\mu\nu}$ represents a fourth until known unknown field, does not make any sense today.

5. Conclusions

In the 1940s, the theoretical framework of quantum electrodynamics consolidated electromagnetism with quantum physics. It has also to be noted that the trial to geometrize the electromagnetic field within the theoretical framework of general relativity has still not met with much success. In this publication, the electromagnetic field has been geometrized under conditions of general relativity.

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Appendix

None.

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