DARK MATTER AND THE DYNAMICS OF GALAXIES: A NEWTONIAN APPROACH

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Abstract

In this paper I propose a correction to the well-known Newtonian gravitational potential, a correction which explains the form of the radial velocities as a function of the discoid galaxies radii. The main scope of this work is to find a correction to the Newtonian gravitational potential which has to fulfil two major conditions: a) to take into account the entire amount of the experimental data; b) the resulting potential to be a consequence of condition a) from a physical perspective. As a result, the corrected form of the Newtonian gravitational potential was found to belong to a physical cause and this cause can be the existence of the dark matter, evenly distributed within galaxies. This distribution makes dark matter to act as a binder for ordinary matter, so that the discoid galaxies not rotate as a fluid (as standard Newtonian theory states), but as some rigid frames (as the observational data state).

Key words: *modified Newtonian dynamics; dark matter; MOND theory; radial velocities curve.*

1. INTRODUCTION

Dynamics of galaxies is currently one of the major problems of the theory of gravity, until its discovery, the first half of 20-th century, [1]. Most of the galaxies execute around their centers of mass a rotating movement that has some features that distinguish them. If the matter from which the galaxies are consisting would be subject only the law of Newtonian gravity, then galaxies would rotate like some ideal fluids, increasingly faster towards the center and decreasingly slower towards the edges. But, in reality, the rotation of galaxies takes place as if they are some rigid bodies. At a distance from their centers, called critical radius, the radial velocity becomes practically independent of the radius.

Over the time, there have been several attempts to explain the behavior of the rotating galaxies. Firstly, we talk about the existence of dark matter, [2]. This exotic matter acts like a bend for ordinary matter and the resulting dynamics is the observed one.

From an experimentally point of view, the empirical Tully-Fisher relation must be valid. Consequently, $v \propto M^{\beta}$, where β=1/3-1/4, v is the radial velocity and M the galaxy mass. The exponent β represents an interval, not the extreme values of an interval. This exponent takes different values when the observations are made in different wavelengths, [3].

The dark matter theory explains, in principle, the dynamics of galaxies, but not from Tully-Fisher relation perspective.

Another theory which try to explain this dynamics is based on general relativity, [4]. The dark matter is no more considered, but this theory is very general and it has no experimental particularities.

The Weyl-Dirac based on theory, [5], explains the dynamics of galaxies but, once again, generally and with poor references to experimental data.

From all these theories only the Modified Newtonian Dynamics, MOND, takes into account, with some degree of accuracy, the experimental data. MOND considers only β=1/4 as representative value for wavelengths which characterize a large variety of galaxy masses, [6].

Excepting this fact, MOND exhibits some major disadvantages. First of all it is an effective theory; it brings no causal justifications from physical order to elucidate the behavior of rotating galaxies. MOND only postulates the modified dynamics laws and these postulates have not a physical basis.

The aim of this paper is to correct, somehow, these disadvantages. First of all, we intend to consider the entire interval, β= [1/3;1/4], in our theoretical evaluations. Then, we intend to give our theory a physical signification.

2. ATTRACTIVE FORCES AND THE DYNAMICS OF THE GALAXIES

In the following we suppose that one can obtain a theory to explain the rotation curves of discoid galaxies, a theory based on dark matter.

Therefore we have a gravitational potential of the form:

$$\Phi = \frac{GM}{r} - \frac{A}{\alpha+1} r^{\alpha+1} \tag{1}$$

which is the result of solving Poisson's equation:

$$\Delta\Phi = 4\pi G(\rho + \rho_{ue}) \tag{2}$$

The second term on the right side of equation (1) is an empirical term attached to the Newtonian gravitational potential, a term that describes the action of an attractive gravitational potential, due to an unknown form of energy. We have therefore A> 0.

With this potential (1) we try to show that this unknown energy may cause the radial movement of the discoid galaxies observed in reality. From (1) we find the expression of the first derivative of this equation, the acceleration:

$$g = -\frac{GM}{r^2} - Ar^{\alpha} \tag{3}$$

which have a great importance in the development of reasoning in the following. The gravitational acceleration (3) is entirely attractive and it balances with the centrifugal acceleration of galaxy, which is repulsive. Suppose that neither of them tip the balance one way or another, so at equilibrium we must have:

$$\frac{v^2}{r} = \frac{GM}{r^2} + Ar^{\alpha} \tag{4}$$

which leads after a obvious a multiplication with r to a simpler expression:

$$v^2 = \frac{GM}{r} + Ar^{\alpha+1} \tag{5}$$

From observations made on the motion of galaxies and the Tully-Fisher empirical expression it results $v \propto L^{\beta} \propto M^{\beta}$, where L represent the luminosity and $\beta = \frac{1}{3} - \frac{1}{4}$.

So, now we can empirically determine the proper α in two cases corresponding to the two expressions of the potential (1) which provide the shape of the rotation curves of discoid galaxies according to observations. Therefore we have:

$$v \propto M^{\frac{1}{3}} \tag{6}$$

and the case in which we have a particular interest:

$$v \propto M^{\frac{1}{4}} \tag{7}$$

The critical radius from which the velocity $v(r) = const.$ come from the condition:

$$\frac{\partial v(r)}{\partial r} = 0$$

Thus, expression (5) becomes:

$$0 = -\frac{GM}{r_c^2} + (\alpha + 1) A r_c^{\alpha}$$

and the critical radius is:

$$r_c = \left(\frac{GM}{(\alpha + 1) A} \right)^{1/\alpha + 2} \tag{8}$$

Then this is the radius from which the velocity is independent from it:

$$v = A^{\frac{1}{\alpha+4}} (GM)^{\frac{\alpha+1}{\alpha+4}} [(\alpha+1)^{\frac{1}{\alpha+2}} + (\alpha+1)^{-\frac{\alpha+1}{\alpha+2}}]^{1/2} \tag{9}$$

To be in accordance with the observations, conditions (6) and (7), we must have:

$$\frac{\alpha + 1}{\alpha + 4} = \gamma$$

with γ=1/3 and γ=1/4. Under these restrictions we find two values for α, consequently ½ and 0. The case corresponding to α=0 leads to:

$$v = \sqrt[4]{4GMA} \tag{10}$$

a value independent from radius. Under these conditions equation (10) is valid without the need to consider MOND theory. If we admit that a_0 have not the same meaning as in the MOND theory but totally due to other causes, having no connection with the expansion of the universe but only with internal dynamics of galaxies, is a constant specific to each galaxy in part, then all we have talked so far is valid. Otherwise the place of a_0 may be taken by, the general value A which can be determined from experimental curves. Amazingly, if we do this we get to the result $4A = a_0$ (which was also determined from experimental curves, as a mean value, for discoid galaxies). In this case a_0 cannot be conceived as in MOND theory, [6], but as a galactic characteristic without any connection with the expansion of the universe.

The difference from the MOND theory appears to be the double value of the critical radius:

$$r_c = 2\left(\frac{GM}{a_0} \right)^{\frac{1}{2}} \tag{11}$$

But the form (11) can be avoided if we take $A = a_0$. It leads us unexpectedly closer to the MOND theory, from critical radius perspective, but radial velocity is a little bit bigger than the MOND-like velocity. Indeed, if we consider $A = a_0$ we obtain from (10) exactly:

$$v^4 = 4GMa_0 \tag{12}$$

which is the well-known expression obtained in the MOND theory for the radial velocities independent from the galaxies radius, multiplied with four, actually less important. What is important here is that we got these results in the approximation:

$$g = g_N + a_0$$

and not in the approximation:

$$g = (g_N \cdot a_0)^{1/2}$$

like in MOND theory, [6]. The difference lies in the fact that our theory has not worked out with an expression of a modified inertia like MOND theory does, Newton's Second Law remaining unchanged. So, with (1) and (7), some MOND theory results can be obtained without the need to change the law of inertia. We just need to change the definition of constant a_0 as an artifact of galaxies, supposed due to an unknown energy and its distribution into each galaxy in part. If this acceleration is directly connected with dark matter, as a property of dark matter, than a_0 should be a universal constant. It is independent of quantity of dark matter existent in a galaxy. We could think, the small deviations from this value can be assigned to the form of each galaxy in part. How dark matter is distributed in galaxy differ from another galaxy, so the constant A (or a_0) could be different.

This acceleration could explain the anomaly of Pioneer 10 spacecraft, also. There is an attractive constant force in our galaxy, supplementary to the Newtonian one, which is directly due to compression trend of the galaxy caused by the amount of dark matter uniformly spread in it. Equation (4) is valid because dark matter is opposing to the trend of the dispersion caused by the rotation of the galaxy. But all these comments are valid only if dark matter is the origin of the potential (1) for α=0.

3. POSSON'S EQUATION VERIFICATION

The second case we discuss now is corresponding to α=1/2. Following the same steps as in previous case, from (8) we find:

$$r_c = \left(\frac{2GM}{3A}\right)^{2/5} \quad (13)$$

for critical radius, and from (9):

$$v = A^{\frac{2}{9}}(GM)^{\frac{1}{3}}[(3/2)^{2/5} + (2/3)^{1/3}]^{1/2} \quad (14)$$

for radial velocity, results that should fit the observational data. The constant A is, this time, no more acceleration. The acceleration induced by the unknown energy is now increasing with radius. For the same magnitude of constant A as in previous case, it results increased values for critical radius and radial velocity.

The question now is which one of the tow cases is correct from Poisson's equation point of view. The potential (1) for α=0 has no source, except the normal matter. Even if it verifies the Poisson's equation (2), so it is a valid gravitational potential, it can be created from normal matter only. Ironically we find approximately the same results in MOND theory and we have to conclude that the Modified Newtonian Dynamics is correct. The absence of a physical cause and validity of case (7) make this possible. Consequently, there is an equivalence between MOND theory and the theory of modified Newtonian potential (1) for α=0 we previously present. That's why the MOND theory is an effective theory and not a physical theory, the potential (1) for α=0 has source normal matter only.

Therefore, if we want to take into account the effects of an unknown energy to galactic normal matter we must consider the second case, the potential (1) for α=1/2. It could have dark matter as source; otherwise the Poisson's equation (2) would have no sense. This is the reason why a Newtonian theory based on a modified Newtonian potential that can describe the motion of galaxies due to dark matter is valid only in this case, for which:

$$A = -4\pi G\rho_{dm} r^{1/2} \quad (15)$$

The constant A is decreasing/increasing proportionally to the density of dark matter, hence is a feature of each galaxy in part. And we have finally a Newtonian theory which describes close to reality the effects of dark matter in terms of dynamics of galaxies. But the sign minus from equation (15) seems to contradict this affirmation. The only physical justification of it is that the dark matter density comes from a negative pressure:

$$p = -\rho_{dm} \cdot v^2 \quad (16)$$

as a result of self interaction between dark matter's particles in motion into a homogenous compressible fluid, with negative compressibility. The expression (16) is an intuitive one, because the gravity normal force should be a result of ordinary matter motion through the dark matter fluid:

$$p = -\rho \cdot v^2 \quad (17)$$

This is the reason why the velocity in equations (16) and (17) could not be the same.

4. REPULSIVE FORCES AND THE DYNAMICS OF THE GALAXIES

If the nature is more surprising than we expect and ordinary matter somehow succeeds to generate a gravitational potential in form (1) for α=0, then it will produce the observational data which are wrongly interpreted as dark matter effects. Fact is, the observational data we have, in both forms (6) and (7), can so easily give rise to misinterpretations. According to this model only the observational data in form (6) can be attributed to dark matter. The rest of it is due to other causes. Which are these causes, we don't know. But in the following considerations we will show that the cause for all observational data could be the dark matter.

Assume this time that ordinary matter succeeds to describe the dynamics of discoid galaxies in absence of dark matter, through a repulsive gravitational potential. This potential is equivalent with:

$$\Phi = \frac{GM}{r} + \frac{B}{\alpha+1} r^{\alpha+1} \tag{18}$$

for some values α which will be calculated to accomplish the observational requirements. The equation (18) is the result of solving Poisson's equation:

$$\Delta\Phi = 4\pi G(\rho + \rho_{ue})$$

The term ρ_{ue} is generally called it unknown energy. Concerning the potential (18) the balance of accelerations is:

$$\frac{v^2}{r} = \frac{GM}{r^2} - Br^{\alpha}$$

which is different from (3). In order to write correctly the balance of all accelerations involved in dynamics of a galaxy in this case, we must consider, again, the contribution of dark matter. If we conceive a galaxy full of some sort of matter that react with an opposite acceleration to the expansion trend of the galaxy, than this matter could be the dark matter. Therefore we have:

$$a = -\frac{GM}{r^2} + Br^{\alpha} - Br^{\alpha} + \frac{v^2}{r} \tag{19}$$

At a=0, after we reached a critical radius value:

$$0 = -\frac{GM}{r_c^2} + Br_c^{\alpha} - Br_c^{\alpha} + \frac{v^2}{r_c} \tag{20}$$

we can find the equilibrium conditions from which we can determine the critical radius expression and the radial velocity expression. Hence, from:

$$\frac{GM}{r_c^2} = br_c^{\alpha} \tag{21}$$

the expression of this critical radius will be:

$$r_c = \left(\frac{GM}{b}\right)^{\frac{1}{\alpha+2}} \tag{22}$$

On reaching the critical radius, the equilibrium of the galaxy will not be complete unless we consider:

$$\frac{v^2}{r_c} = br_c^{\alpha}$$

result which leads, by taking into account (22), to:

$$v = b^{\frac{1}{\alpha+4}} (GM)^{\frac{\alpha+1}{\alpha+4}} \tag{23}$$

The experimental restrictions (6) and (7), in equation (23), lead to the same cases, α=0 and α=1/2. If $b \neq B$, the equations (22) and (23) have no sense. The general constant $b \in R^+$ indicates an additional repulsive force except the assumed repulsive force Br^{α}. Hence the case b=B is the only valid.

Whence, after we replace the constant B with a_0 and α=0, we can determine the radial velocity of the galaxy as:

$$v^4 = GMa_0$$

which is the well-known expression obtained in the MOND theory for the radial velocities independent from the galaxies radii, and the critical radius from which the small accelerations approximation occurs in the MOND theory:

$$r_c = \left(\frac{GM}{a_0}\right)^{\frac{1}{2}}$$

The case corresponding to α=1/2 is not a valid one because it is expressing, through (18), a repulsive action which cannot be attributed to dark matter.

5. DISCUSSIONS

The results obtained in the previous section are specific to MOND theory. So we must conclude that our goal, to find a Newtonian theory for describe the dynamics of the galaxies due to dark matter, is reached if we consider both potentials, (1) and (18) simultaneously, (1) for α=1/2 and (18) for α=0 More than that, the observational data we have are in form of an interval, β=1/3-1/4, i.e. $\alpha \in [0,1/2]$, $\alpha \in R^+$. The above discussed cases refer to the limits of this interval, β=1/3 and β=1/4. In order to provide an accurate description of the dark matter action we must to establish what are the limits for which the cases corresponding to (1) for α=1/2 and (18) for α=0 are valid. To do this we must solve the Poisson's equation (2) with the potential (1). It results:

$$\alpha A r^{\alpha-1} = -2\pi G \rho_{dm}$$

From this equation we observe that the potential (1) has sources for $\alpha \in (0,1/2]$. Only for α=0 the potential (1) has no sources and the potential (18) is more proper to describe the dark matter action in this case. Therefore, the dark matter effects are described completely by the potential:

$$\Phi = \frac{GM}{r} - \frac{A}{\alpha+1} r^{\alpha+1}, \text{ for } \alpha \in (0,1/2], \quad \Phi = \frac{GM}{r} + Br, \text{ for } \alpha=0 \qquad (24)$$

It is an empirical potential, because there is theory created only for observational data fitting. It is obvious that the gravitational potential which describes properly the action of dark matter is the form (24), without the value corresponding to α=0. A gravitational potential due to some physical causes cannot generate simultaneously an action and a reaction.

The theory presented in [5] deduces only a potential like (24) for α=0. It is produced by ordinary matter, like in our theory. The effects could not be those presented above, because its effects are much smaller than a potential $\propto r^{-2}$, generated by the same ordinary matter.

If we approximate the Yukawa gravitational potential, [7]:

$$\Phi = \frac{Ce^{-r/r_0}}{r} \qquad (25)$$

in terms of power series with respect to the galaxy radius r_0 than we find a gravitational potential like (24), for α=0. The equation (25) represents the potential per mass unit, therefore we meet serious obstacles to properly calibrate the constants in order to make (24) looks like (25). If we take the constant C in the form GM than the constant B must be fourteen times bigger than the constant a_0 (for our galaxy).

Taking into account the above considerations we must admit that the form (24) cannot momentary be reproduced by a physically grounded mathematically correct theory. Under these conditions equation (24), without the form corresponding to α=0, is valid without MOND theory's confirmation. In this final case the general constant A can be determined from experimental curves.

Our theory is only an effective theory, not better than MOND theory, and nothing more. But it involves dark matter in galaxies' dynamics. Instead, it has the same disadvantages as MOND theory: it doesn't solve the mass discrepancies problem. The constant A from (24) depends on the galaxies radii, therefore in the case of some small spherical galaxies and some big galactic clusters, an anomalous mass discrepancy will occur: too large for small spherical galaxies and moderate for big galactic clusters, [6]. Perhaps, more accurate measurements of mass/luminosity conversion factor M/L, will clarify this problem.

6. CONCLUSIONS

In this paper we propose an alternative Newtonian theory for MOND, a theory which describes the effects of dark matter in the dynamics of galaxies. Under the hypothesis that for the shape of the radial velocity curves of galaxies are responsible an attractive and a surprising repulsive form of energy, this influence is found to be expressed by a supplementary potential which it must be added to the Newtonian gravitational potential. Somewhat surprising, we found the results specific to MOND theory, but this time with a modified Newtonian potential.

7. REFERENCES

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