

# The origin of the coupling constant ( $e$ ) and some other important dimensionless physical constants within General Relativity and Quantum Mechanics

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**Abstract:** This paper will answer the mystery of the coupling constant ( $e$ ), a puzzle of its origin that was made popular by Richard Feynman, by using what will be defined as “temporal kinematics”. Temporal kinematics studies the motion of time, we will name this “temporal motion” and provide a detailed explanation and kinematics to why this concept is far more accurate than the current concept of “repulsive gravity” that dominates in the cosmic inflation studies. Temporal motion should not be confused with cosmic inflation, it can only act as an initiator of it. Temporal kinematics functions as a bridge between General Relativity and Quantum Mechanics.

## Introduction

Some of the unexplained problems in physics can be explained and proven in a relatively simple way if we apply the logic of General Relativity on other fields of physics. The simplest way is to use “temporal motion” instead of “repulsive gravity” [1] to explain the inflation of space from the initial inflation, often called “cosmic inflation”, to the present time.

We use a  $(-, +, +, +)$  metric, where  $(-)$  marks the dimension of time  $(t)$  as usual [2]. Even in the simplest form of a  $(R^4)$  flat spacetime with  $(t, x, y, z)$  we have a metric:

$$(1) ds^2 = -c^2 dt^2 + x^2 + y^2 + z^2$$

We will proclaim that temporal motion inflates space; the inflation is its equivalent of what a trajectory is for spatial motion. Temporal motion has a velocity  $(c)$  which is the speed of light and can be thought of as a speed limit of the Universe. This limit exists due to temporal motion since nothing can move in space faster than time due to the entanglement of space and time known as the spacetime continuum.

## Cosmological model

The Universe will be represented as homogenous and isotropic. Isotropy means that the metric must be diagonal since it will be show that space is allowed to be curved. Therefore we will use spherical coordinates to describe the metric.

The metric is given by the following line element:

$$(2) ds^2 = dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

where we measure  $(\theta)$  from the north pole and at the south pole it will equal  $(\pi)$ .

In order to simplify the calculations, we abbreviate the term between the brackets as:

$$(3) d\omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

because it is a measure of angle, which can be thought of as “on the sky” from the observers point of view [4]. It is important to mention that the observers are at the center of the spherical coordinate system.

Due to the isotropy of the Universe the angle between two galaxies, for the observers, is the true angle from the observers’ vantage point and the expansion of the Universe does not change this angle.

Finally, we represent flat space as:

$$(4) ds^2 = dr^2 + r^2 d\omega^2$$

Robertson and Walker proved that the only alternative metric that obeys both isotropy and homogeneity is:

$$(5) ds^2 = dr^2 + f_K(r)^2 d\omega^2$$

where  $(f_K(r))$  is the curvature function given by:

$$(6) f_K(r) = \begin{cases} K^{-1/2} \text{ for } K > 0 \\ r \text{ for } K = 0 \\ K^{-1/2} \sin h(K^{1/2}r) \text{ for } K < 0 \end{cases}$$

which means that the circumference of a sphere around the observers with a radius  $(r)$  is, for  $(K \neq 0)$ , not anymore equal to  $(C = 2\pi r)$  but smaller for  $(K > 0)$  and larger for  $(K < 0)$ .

The surface area of that sphere would no longer be  $(S = (4\pi/3)r^3)$  but smaller for  $(K > 0)$  and larger for  $(K < 0)$ . If  $(r)$  is  $(r \ll |K|^{-1/2})$  the deviation from  $(C = 2\pi r)$  and  $(S = (4\pi/3)r^3)$  is very small, but as  $(r)$  approaches  $(|K|^{-1/2})$  the deviation can become rather large.

The metric in the equation (1) can also be written as:

$$(7) ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\omega^2$$

If we determine an alternative radius  $(r)$  as:

$$(8) r \equiv f_K(r)$$

This metric is different only in the way we chose our coordinate  $(r)$ .

We can now build our model by taking for each point in time a RW space. We allow the scale factor and the curvature of the RW space to vary with time [3]. This gives the generic metric:

$$(9) ds^2 = -dt^2 + a(t)^2 [dx^2 + f_K(x)^2 x^2 d\omega^2]$$

the function  $(a(t))$  is the spatial scale factor that depends on time and it will describe the spatial expansion of the Universe. We use  $(x)$  instead of  $(r)$  because the radial coordinate, in this form, no longer has meaning as a true distance.

## Temporal Motion

Temporal motion needs three spatial equations for a trajectory, specifically it needs  $(2 + 1)$  spatial dimensions.

Therefore  $(D = 2 + 1)$  is the number of spatial dimensions.

The equation for temporal motion has to be on a quantum level to satisfy the observational evidence that suggests the opinion that the Universe comes from a singularity. We do so by establishing  $(d\mathcal{D})$  where  $(\mathcal{D})$  represents the number of temporal dimensions which is (1), however having in mind that time has a negative value in the metric  $(-, +, +, +)$  we give it a value of  $(-1)$ . We will represent this as  $(-p)$  where  $(p = 1)$  and it is a very low constant pressure. We use it in a dimensionless form:

$$(10) p = \int_1^e \frac{1}{x} dx = \int_1^e \frac{1}{y} dy = 1$$

We name this constant the Ellis dimensionless constant, in honor of George Ellis.

We write a simple equation of motion:

$$(11) \delta \rightarrow = \delta \int d\mathcal{D} L(a(t), \dot{a}(t))$$

Where  $(\rightarrow)$  is the symbol for temporal motion,  $(a(t))$  is the three-dimensional trajectory/inflation and  $(\dot{a}(t))$  is the velocity that equals  $(c)$  the speed of light. However:

$$(12) \dot{a}^{-1}(t) = -pc$$

Where the pressure  $(p)$  equals 1. This allows us to form the equations, three of them, for the temporal course of inflation.

And the trajectory describing inflation  $(a(t))$  becomes  $(a^{-1}(t))$  and functions as:

$$(13) a^{-1}(t) \begin{cases} \rightarrow (x) = \log \lim_{x \rightarrow \infty} \left(\frac{p}{x+p}\right)^x (x) \\ \rightarrow (y) = \log \lim_{y \rightarrow \infty} \left(\frac{p}{y+p}\right)^y (y) \\ \rightarrow (z) = \pi y(\dot{t}) + \delta x(\dot{t}) \end{cases}$$

Where  $(\dot{t})$  is the positive first derivative of time. We name it Feynman time in honor of Richard Feynman.

$$(14) \dot{t} = z + E_v$$

Where  $(E_v)$  is the energy state of the vacuum and it is zero by definition. Therefore the first derivation of time, Feynman time, functions as the third dimension  $(z)$  and it has the energy value of the vacuum, which is zero.

The equations might seem too complicated to comprehend but they are practical when we apply that  $(p = 1)$  we get the solutions  $(\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 1\right)^x)$  equals  $(e)$  and  $(\lim_{y \rightarrow \infty} \left(\frac{1}{y} + 1\right)^y)$  equals  $(e)$  as well, where the first two equations become  $(\rightarrow (x) = \log_e(x))$  and  $(\rightarrow (y) = \log_e(y))$  and the

number ( $e$ ) is the Euler's number, not the coupling constant we are looking for. The third equation ( $\rightarrow (z)$ ) is the "wave function", representing "temporal waves". Every temporal wave can be thought of as a spatial layer, or a frame. We will define them as "z-frames" and state that each value of ( $z$ ) represents every individual z-frame from (1) to ( $n$ ).

The mathematical core of the equations is:

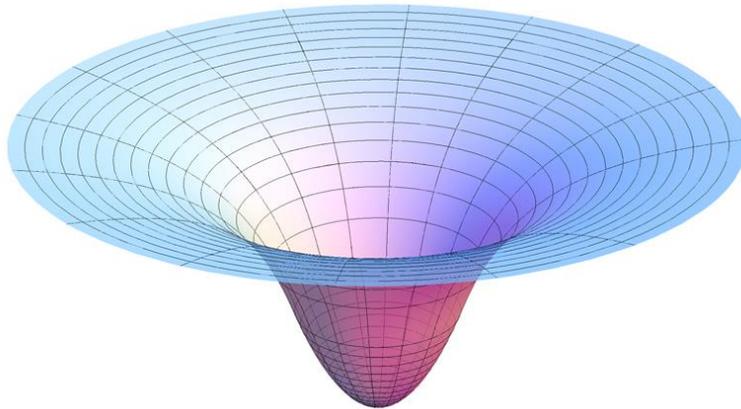
$$(15) e = \pi - \delta$$

This equation is the symmetry of temporal motion and therefore it is the mathematical logic behind the temporal kinematics, simply put the mathematical foundation of temporal motion. We shall name this equation the "logos equation". Now we conclude from the equation that ( $\delta = 0.423310825130748$ ) and define that:

$$(16) \delta = \frac{e}{D} + \Omega + (\delta_{CKM} - \delta_{PMNS})$$

Where ( $D = 2 + 1$ ) is the number of spatial dimensions, ( $\Omega \approx 0.3$ ) is the ratio of the actual density of the Universe to the critical minimal density necessary for the Big Crunch scenario to occur in the distant future, ( $e$ ) is the coupling constant and it is measured to be ( $e = 0.08542455$ ), ( $\delta_{CKM} \approx 0.995$ ) is the CKM cp-violating phase and ( $\delta_{PMNS}$ ) is the PMNS cp-violating phase and its value is currently unknown. After doing the calculus we conclude that ( $\delta_{PMNS} = 0.900164024869252$ ) however we approximate it to be ( $\delta_{PMNS} \approx 0.900164 \pm 0.0000001$ ) where we are making a prediction that can be tested experimentally in order to confirm the claims of this paper. The first objective of the paper was to explain how the coupling constant ( $e$ ) arises in physics.

When we draw the functions, we get an image:



*Figure 1: Trajectory/cone of temporal motion where every ellipse is a temporal wave.*

## Dimensions of the Universe

The Universe has four dimensions, a  $(-1)$  temporal dimension and  $(2 + 1)$  spatial dimensions where the  $(+1)$  is the  $(z)$  dimension that is described in temporal kinematics as the positive first derivative of time. The best way to explain this is to say that the dimensions  $(t)$  and  $(z)$  are entangled, which forms the relationship of spacetime that General Relativity studies. In simpler terms we could say that temporal waves are spatial layers or frames. We name them  $z$ -frames.

This explains the lack of curved space in the Universe (it is observed to be near flat), and its near homogeneous and near isometric nature.

The constant pressure  $(p)$  prevents anything in space to travel faster than time  $(c)$  due to the equation (12).

### Z-frames

Every individual  $z$ -frame is represented by a value of  $(z)$  from eq. (13), for example the current period is  $(z = 1)$ , to represent different eras of the Universe. [5]

For:

$$(17) (z \simeq 1000)$$

we have a value:

$$(18) a(t) \simeq \left( \frac{3}{2} H_0 \sqrt{\Omega_{m,0}} t \right)^{2/3}$$

which is a  $z$ -frame known as “matter dominated era”. Earlier than that, in a  $z$ -frame known as the “radiation dominated era”, a period when the Universe was dominated by radiation, around  $(z \gtrsim 3200)$  we have a value:

$$(19) a(t) \simeq \left( 2H_0 \sqrt{\Omega_{r,0}} t \right)^{1/2}$$

The early, radiation dominated Universe expanded as:

$$(20) a \propto \sqrt{t}$$

Every frame has slightly more temporal-kinetic energy, or “dark energy”, than the previous one but since the differences in the trillions of frames is complicated to determine it is therefore simpler and more productive to only use some frames.

Due to the low negative pressure of temporal motion, its kinetic energy which is “dark energy”, also has a low negative pressure  $(-p = -1)$  [6]. Having such a pressure, dark energy accelerates the inflation of space conducted by temporal motion.

## Fundamental interactions

Due to the relationship of space and time fundamental interactions also have their temporal equations.

### Electromagnetic interaction

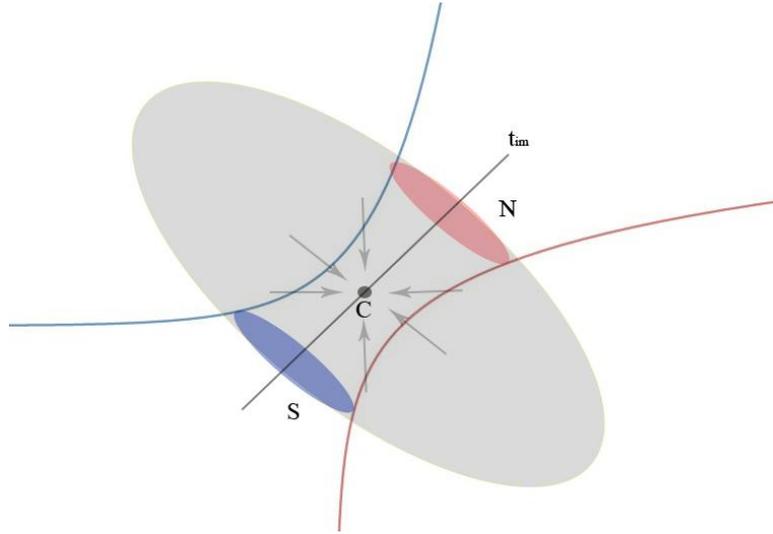


Figure 2: Geomagnetic field of Earth.

$$(21) a^{-2}(t) \begin{cases} \rightarrow (x) = \log \lim_{\alpha_x \rightarrow 0} (p + \alpha_x)^{p/\alpha_x} (x) \\ \rightarrow (y) = \log \lim_{\alpha_y \rightarrow 0} (p + \alpha_y)^{p/\alpha_y} (y) \\ \rightarrow (z) = \pi y(\dot{t}) + \delta x(\dot{t}) \end{cases}$$

Same as with the first temporal equations, ( $p = 1$ ) providing the solutions ( $e$ ) meaning that ( $\rightarrow (x) = \log_e(x)$ ) and ( $\rightarrow (y) = \log_e(y)$ ) where ( $e$ ) is the Euler's number. The third equation also functions as a wave equation due to the positive second derivative of time ( $\dot{t}$ ) and the ( $e = \pi - \delta$ ) also applies explaining how waves spread through space and time, that is the spacetime continuum. Since ( $\alpha_x \rightarrow 0$ ) and ( $\alpha_y \rightarrow 0$ ) we get two timelike curves that form an electromagnetic potential well. Here ( $\dot{t}$ ) functions as a wave field for the fundamental interaction. We will name it Mileva time, in honor of Mileva Maric/Marjity Einstein, and it equals:

$$(22) \dot{t} = z + \alpha_z$$

Where:

$$(23) \alpha_{zx} = \alpha_z \cdot e^{\alpha_x}$$

and:

$$(24) \alpha_{zy} = \alpha_z \cdot e^{\alpha_y}$$

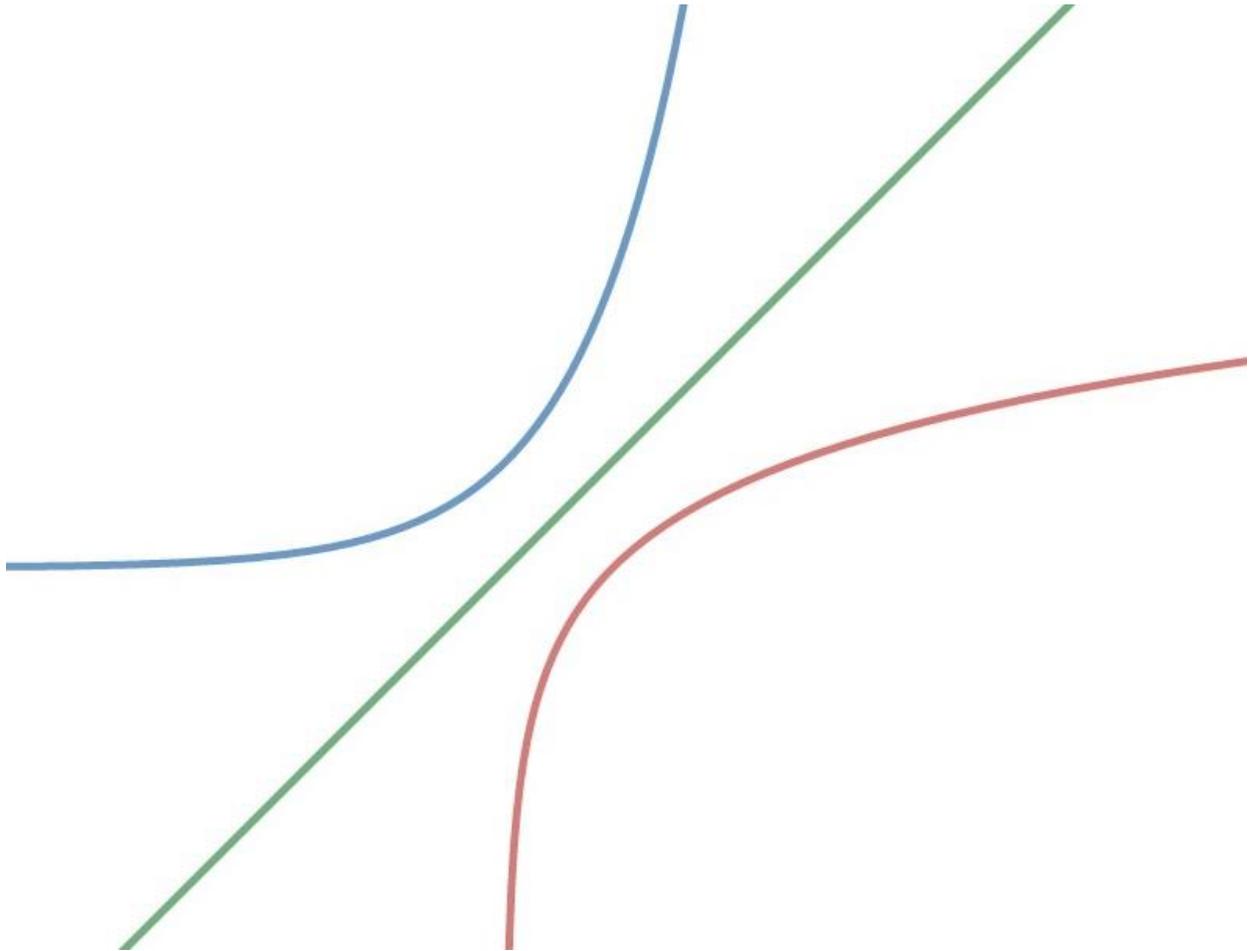
Therefore:

$$(25) \frac{d\alpha_{zx}}{d\alpha_x} = \frac{d(\alpha_z \cdot e^{\alpha_x})}{d\alpha_x} = \alpha_z \frac{d(e^{\alpha_x})}{d\alpha_x} = \alpha_{zx}$$

and:

$$(26) \frac{d\alpha_{zy}}{d\alpha_y} = \frac{d(\alpha_z \cdot e^{\alpha_y})}{d\alpha_y} = \alpha_z \frac{d(e^{\alpha_y})}{d\alpha_y} = \alpha_{zy}$$

We define that  $(\alpha_z = \alpha)$  where  $(\alpha)$  is the fine-structure dimensionless constant. This describes the electromagnetic interaction between two elementary charged particles.



*Figure 3: The electromagnetic field where the red and the blue lines are timelike curves that form a potential well and the green line is the axis.*

We can also explain the "mass gap" using Temporal Kinematics.

### The mass gap

We will describe the mass gap as a difference of Feynman time ( $\dot{t}$ ) and Mileva time ( $\ddot{t}$ ).

$$(27) \dot{t} \xrightarrow{\alpha} \ddot{t}$$

where ( $\dot{t} = z + E_\nu$ ) and ( $E_\nu = 0$ ) while ( $\ddot{t} = z + \alpha_z$ ) where, as the lightest elementary particle, we take the photon, thus ( $\alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} = 0.0072973525664$ ). Other elementary particles can be used as well.

### Gravitational interaction

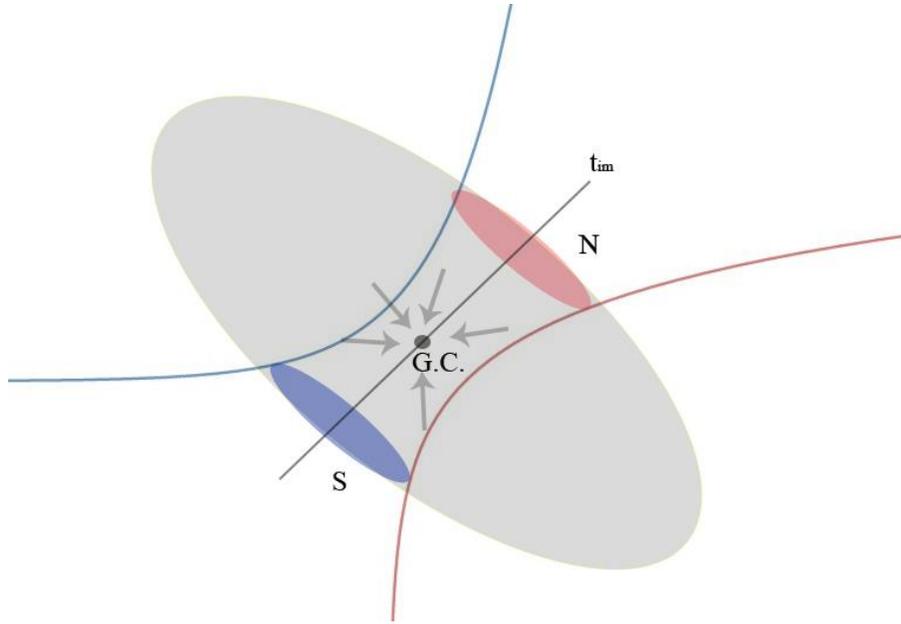


Figure 4: The Gravitational field of Earth.

The temporal equations are:

$$(28) \alpha^{-2}(\dot{t}) \begin{cases} \rightarrow (x) = \log \lim_{\alpha_{G_x} \rightarrow 0} (p + \alpha_{G_x})^{p/\alpha_{G_x}} (x) \\ \rightarrow (y) = \log \lim_{\alpha_{G_y} \rightarrow 0} (p + \alpha_{G_y})^{p/\alpha_{G_y}} (y) \\ \rightarrow (z) = \pi y(\ddot{t}) + \delta x(\ddot{t}) \end{cases}$$

Where ( $\alpha_{G_x} \rightarrow 0$ ) and ( $\alpha_{G_y} \rightarrow 0$ ) form timelike curves and therefore a gravitational well and the solutions for ( $\rightarrow (x), \rightarrow (y)$ ) are again ( $e$ ), same as before. For gravity, Mileva time ( $\ddot{t}$ ) is:

$$(29) \ddot{t} = z + \alpha_{G_z}$$

Same as with the electromagnetic interaction, we have:

$$(30) \alpha_{G_{zx}} = \alpha_{G_z} \cdot e^{\alpha_{G_x}}$$

and:

$$(31) \alpha_{G_{zy}} = \alpha_{G_z} \cdot e^{\alpha_{G_y}}$$

Therefore:

$$(32) \frac{d\alpha_{G_{zx}}}{d\alpha_{G_x}} = \frac{d(\alpha_{G_z} \cdot e^{\alpha_{G_x}})}{d\alpha_{G_x}} = \alpha_{G_z} \frac{d(e^{\alpha_{G_x}})}{d\alpha_{G_x}} = \alpha_{G_{zx}}$$

and:

$$(33) \frac{d\alpha_{G_{zy}}}{d\alpha_{G_y}} = \frac{d(\alpha_{G_z} \cdot e^{\alpha_{G_y}})}{d\alpha_{G_y}} = \alpha_{G_z} \frac{d(e^{\alpha_{G_y}})}{d\alpha_{G_y}} = \alpha_{G_{zy}}$$

We define that  $(\alpha_{G_z} = \alpha_G)$  where  $(\alpha_G)$  is the gravitational coupling dimensionless constant.

Where we have described the gravitational interaction between a pair of elementary particles of our choice.

### **Gravitational polar inequality**

Due to the asymmetry of the poles (specifically their center) the gravitational poles are unequal, which influences solar instability.

The more massive a star is the less stable it will be. With medium size stars, gravitational polar inequality leads to a gradual shrinking of the core since it is the most massive part of the star, until the core shrinks to the size sufficient for the medium sized star such as the Sun, to become a red giant. The core seeks to attain a balance of equal poles, which it can never achieve as shown on the image bellow.

As we can see on the figure five, no matter how much we shrink the circles the poles will never be equal until point (0) is reached which represents the point of singularity. This is best noticeable with super massive stars where the core does not shrink but implodes into point (0) while the other layers of the star explode. This makes black holes the only gravitationally symmetrical celestial bodies.

Gravitational polar inequality is the second prediction this paper makes.

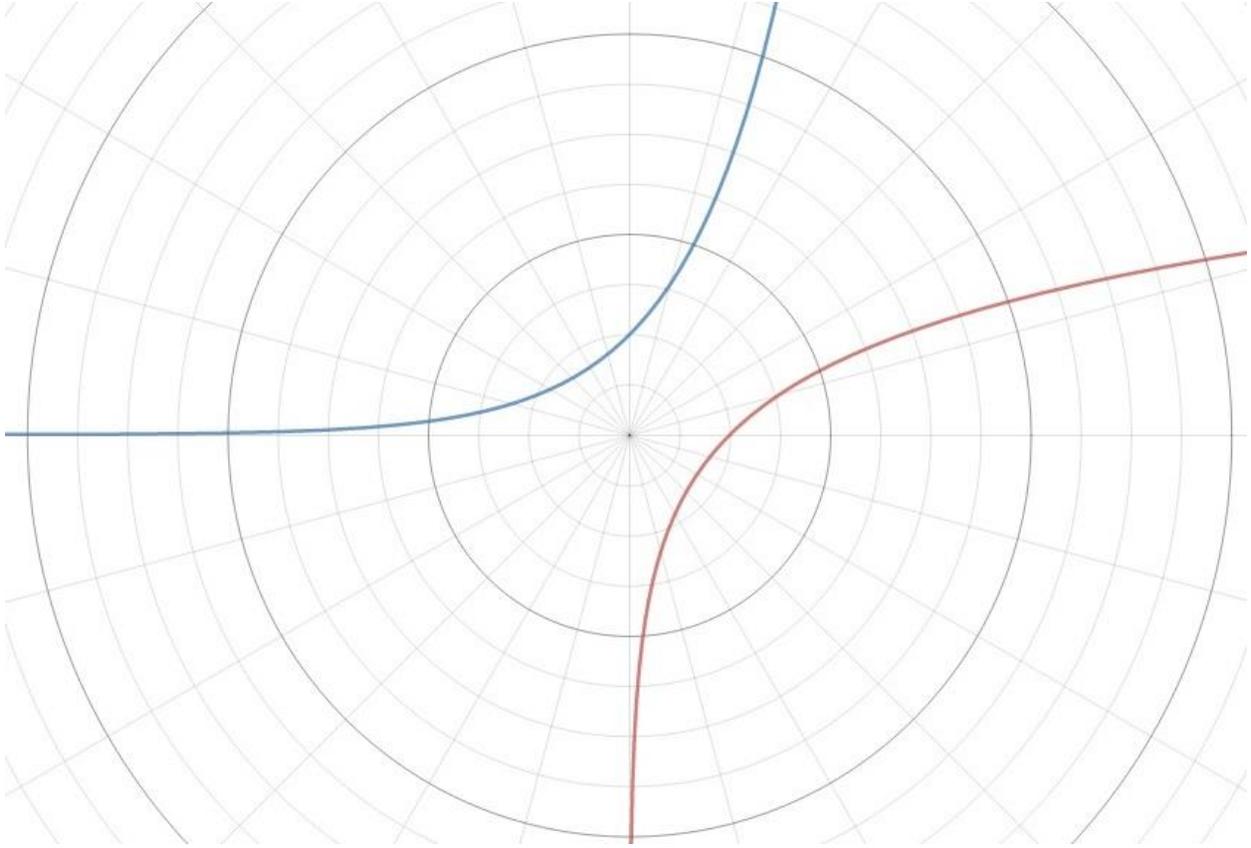


Figure 5: Gravitational polar inequality in 2D.

Gravity and electromagnetism have the  $(x \rightarrow \infty, y \rightarrow \infty)$  range and  $(a^{-2}(t) = pc)$  velocity.

### The Strong and the Weak interaction

Unlike the gravitational and electromagnetic interaction, strong and weak interactions have nowhere near infinite range  $(x \rightarrow \infty, y \rightarrow \infty)$ , which is why instead of  $(p)$  we will use  $(\alpha_s)$  which is the strong coupling constant  $(\alpha_s \approx 1)$ .

$$(34) a^{-2}(t) \begin{cases} \rightarrow (x) = \log \lim_{g_x \rightarrow 0} (\alpha_s + g_x)^{\alpha_s / g_x} (x) \\ \rightarrow (y) = \log \lim_{g_y \rightarrow 0} (\alpha_s + g_y)^{\alpha_s / g_y} (y) \\ \rightarrow (z) = \pi y(\ddot{t}) + \delta x(\ddot{t}) \end{cases}$$

Where, as before,  $(g_x \rightarrow 0, g_y \rightarrow 0)$  make timelike curves and  $(\ddot{t})$  is:

$$(35) \ddot{t} = z + g_z$$

As before:

$$(36) g_{zx} = g_z \cdot e^{g_x}$$

and:

$$(37) g_{zy} = g_z \cdot e^{g_y}$$

Therefore:

$$(38) \frac{dg_{zx}}{dg_x} = \frac{d(g_z \cdot e^{g_x})}{dg_x} = g_z \frac{d(e^{g_x})}{dg_x} = g_{zx}$$

and:

$$(39) \frac{dg_{zy}}{dg_y} = \frac{d(g_z \cdot e^{g_y})}{dg_y} = g_z \frac{d(e^{g_y})}{dg_y} = g_{zy}$$

We define that ( $g_z = g$ ) where ( $g$ ) is the coupling constant (gauge coupling parameter).

### Common symmetry

If the dimensionless coupling constant ( $g$ ) is much lesser than one ( $g \ll 1$ ) then the second derivative temporal waves are weakly coupled. If ( $g$ ) is of order one ( $g = 1$ ) or higher ( $g > 1$ ), then the second derivative temporal waves are strongly coupled. Second derivative temporal waves are fields of fundamental interactions.

We have there by explained why all elementary particles have dual nature by applying temporal equations, and we found a common symmetry for all four fundamental interactions by using dimensionless physical constants, hence explaining their origin and importance.

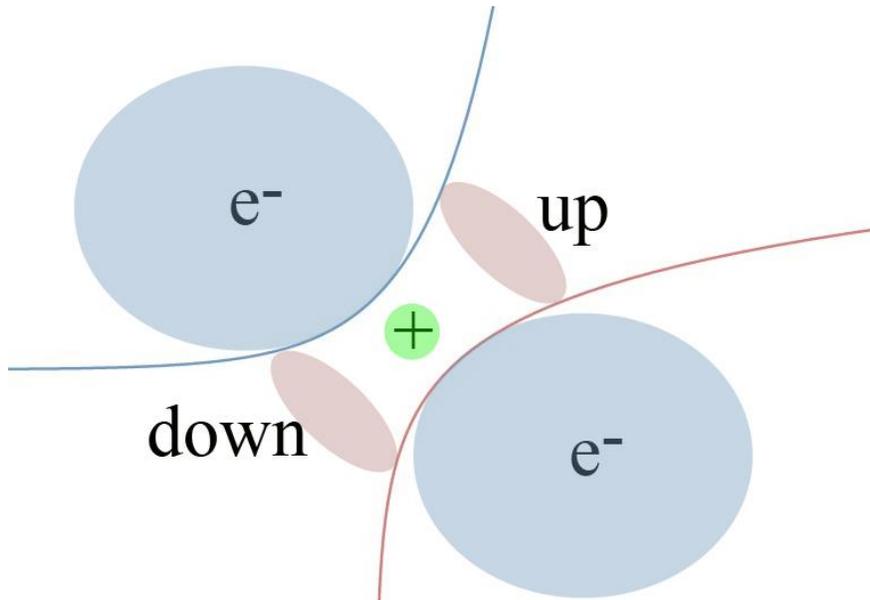


Figure 6: The Atom, where + and the green center is the nucleus and the blue cloud and  $e^-$  represents the electron cloud.

## Conclusion

If we apply the new values from the temporal equations assuming no distinction between the spatial directions, we can change some of the equations in the cosmic inflation theory in order to make them more logical. We write the FRLW metric as:

$$(40) ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{p - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

With a time-invariant Hubble constant, we have a de Sitter metric where:

$$(41) ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$

We also define the density parameter:

$$(42) \Omega_p \equiv \frac{\rho}{\rho_c} = \frac{3/8\pi G (H^2 + k/a^2)}{3H^2/8\pi G} = p + \frac{k}{a(t)^2 H^2}$$

When ( $a(t) = e^{Ht}$ ) and ( $H = \text{const.}$ ) we have:

$$(43) \Omega_{inflation} = p + \frac{k}{a(t)^2 H^2} = p + kH^{-2}e^{-2Ht}$$

Where, as before ( $p = 1$ ) therefore the pressure drives ( $\Omega_p$ ) very rapidly to the value of (1). With enough influence by temporal motion, the initial value of ( $\Omega_p$ ) that may have differed from (1) could have been driven close enough to (1) that it would be approximately equal to it in the present period of the Universe.

Using temporal equations allows for a much simpler and more accurate theory of inflation. Temporal kinematics creates the cone (seen on Figure 1), setting the course of inflation from the beginning and to an end as well.

Possible quantum fluctuations:

$$(44) \phi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

Resulting in:

$$(45) \delta\phi = \left[ (L a(t))^3 \right]^{-1/2} \sum_{\vec{k}} \left[ a_{\vec{k}} g_k(t) e^{i\vec{k}\vec{x}\vec{y}} + H.C. \right]$$

The time dependent part of the fluctuation is:

$$(46) \Psi_k \equiv a(t)^{-3/2} g_k$$

Therefore:

$$(47) |\delta\phi|^2 = L^{-3} |\delta\Psi_k|^2$$

where ( $L \rightarrow \infty$ ).

We consider an inflation field composed of a spatially homogenous term plus a first order:

$$(48) \phi(\vec{x}, t) = \phi^{(0)}(t) + \delta\phi(\vec{x}, t)$$

In units of:

$$(49) \hbar = c = p = 1$$

we get the evolution equation:

$$(50) \partial_t^2 \delta\phi + 3H\partial_t \delta\phi - a^{-2}(t) \sum_{i=1}^D \partial_i^2 \delta\phi + m(\phi^{(0)})^2 \delta\phi = 0$$

Here the ( $a^{-2}(t)$ ) represents the evolution of the fundamental forces. This proves that there is only one form of evolution and that is the evolution of the Universe (celestial/cosmic evolution). In other words, laws of physics, including evolution, dictate everything in the Universe including life, natural selection and even death. This is explained by temporal laws.

The first temporal law:

Time does not move forward nor can it move backward, time simply moves on. Temporal waves move on from the beginning, the Big Bang, to the end, the Big Crunch with possibility of some other similar scenarios. This is dictated by the ( $\Omega$ ) factor in the basis of ( $\delta$ ), due to its value ( $\Omega \approx 0.3$ ) which is the value necessary for an event such as the Big Crunch to occur in the distant future. In other words, everything that has a beginning must have an end. Time has a deterministic nature while space, due to the t-z entanglement, has a near-deterministic nature.

The second temporal law:

The second temporal law is a derivation of the first, specifically regarding life. Everything that is born (is alive) must eventually die. This is also due to the ( $\Omega$ ) factor in the basis of ( $\delta$ ), meaning that both life and death are dictated by the basic laws of physics.

### **Temporal Kinematics**

Temporal kinematics successfully unites General Relativity and Quantum Mechanics by providing a common temporal symmetry for all four fundamental interactions. Temporal Kinematics is a falsifiable theory since it makes a prediction regarding ( $\delta_{PMNS}$ ) and claiming the existence of gravitational polar inequality, both of which are observable.

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