

The Ordinalcy & Residuality Foundations: And Now for Greater Simplicity

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Abstract

The dominant functional or “cardinal” paradigm, building on the natural yet arbitrary dichotomy of causality and probabilism or time versus space, can be relaxed in a manner suggesting a straightforward integration for a host of the otherwise intricate and seemingly unrelated areas. Proposed is an alternative dual paradigm of Ordinalcy and Residuality building on higher-order relationships.

A Case against “Cardinalcy”—or Azimuthality?

The computing power has exploded, processing speed and memory only competing with the manufacturers’ avarice—and thirst after “wide moat,” as per further updates stumbling into lower expected earnings amidst a “fast cycle” that has increasingly marked the sector. It does not, however, appear that it is the capacity limitations, or lack of a functional quantum computing architecture, that has prevented this competitive turbulence from making any sense of weather forecasting or climate turbulence—and nor does it seem that it is all about a sheer lack of closed-form solutions for Navier--Stokes fluids. Or can there be any?

For the same token, the desperate financial analyst may be forced to draw upon meaningless levels-targeting for the GDP vector autoregression models or nearly as helpless an attempt at separating the supply and demand in the structural-form regressions as previously done by the social planners and think tanks. For one thing, the selfsame proverbial professional might do a better job with a handful of simpler styled computations on hand, e.g. with the ratios and multiples she is used to—as long as another level of ratios is considered. The key idea would be that, while *quotients* tend to be far better behaved (stopping little short of stationarity prerequisites obtaining, notably cointegration), this is all the more characteristic of *higher-order* quotients.

The above-mentioned policy analyst could end up better off contemplating an inherently interactive nature of their explanatory variables—which is not do deny merit to questioning the *hierarchy* of the latter category versus those maintained as dependent. In fact, this can not be reduced to conventional endogeneity issues, and nor can it be fixed by either embarking on *interactive* coefficients (which are at times naïve and hard to pin down) or even *implicit* functions.

That said, even these early guesses at or analogies for the domain in question suggest enough leeway for straightforward generalization. A particular set of variables, unless one is willing to go with correlations or Granger causality or linear dependence, may not fit just squarely into isomorphism of any meaningful sort. Two or more variables clearly enter a *relationship*, which goes two or multiple ways, albeit without allowing for any conventional reduction—whether it be a correspondence or a function or an arcane operator acting on a single object without capturing all of them as a fitting aggregate.

Ordinalcy at Long Last

Functions and their extensions may well seem a very resourceful mode of getting rich and meaningful results—regrettably, at times, without getting the discipline anywhere unless a cross-disciplinary aide arrives early. A “functional representation,” or indeed a worldview, is what we have grown so accustomed to since a Cartesian account on clarity, with “the method” spawning a host of related notions such as space, its dimensions, and a representation of any process as axial projections. Algebraically, a function (and perhaps a spacetime) would appear as a holistic, global “body” seen all at once—a straightforward implication of causality or hard determinism working as a kind of *perpetuum mobile*: Indeed, it might appear, if one were to zoom in and meditate on the notion, that an abstract function maps an argument into a correspondent one—as if to create both these *ab nihilo* (yet in a well-defined and consistent manner), or maintaining sufficiency and necessity for the *two*—and not for a *larger* [unknown] set of related variables yet to be tapped into.

Some of the transgressions—not least a homogeneous space and a straightlined time—have been redeemed beyond Riemannian accounts or strong causality. However, some of the flipside solutions—notably probabilism and stochasticity as well as two-way time travel—might again appear about as much accustomed-to as they are excessive. The plausible yet unnecessary schizophrenia, incidentally leading to splits and schisms (as between a corpuscular versus wave or general relativistic versus quantum mechanic representations), ironically has been posited as an attainment and abundance of results in and of itself. This fact has also been accepted without objection over time.

So be it: There is the space whose every dimension can be extended indefinitely—and the same infinity may easily carry over to dimensionality. At least this sounds plausible and useful, and after all can be *calibrated* with respect to the exact number of dimensions (hidden not least) to fit the experimental data. And, time may too have to an extent been endogenized beyond linear isotropic yet not necessarily ergodic premises (and anyway that’s not what Lorenzian relativism maintain) or “relaxed” at a sub-Planck level (as would quantum-mechanic and string-theoretic accounts suggest, as per sheer lack of observability “down low”).

Needless to say, those very accounts or their philosophical underpinnings cannot be made subject to direct falsifiability (remember the azimuthality chain)—for reasons beyond the measurable threshold. Again, the dual extreme would be about the ad-hoc categories (in

excess of calibrating dimensions) that arise from empirical checks as fudge. Oh, and those fermions and Higgs bosons—they do indeed feature some extra, interactive bridges on top of entangled states.

These may be dubbed particles, strings, m-branes, or matrix spectra; may or may not “exist” literally or as metaphors; but do they really exist as *standalone* entities? Can one be defined without *referring to the others*? Or is this maintained to be “the best of the possible worlds” as a matter of each regularity being held as optimal *with respect* to the other laws, or as a special (zero) value for the Ricci tensor?

And now for something different: How about more of a parsimonious enterprise to be espoused as one way of rethinking the intuitive, plausible, and reasonable (or “clear”)?

An axiomatization could build on a set of inter-related assumptions or possibilities without necessarily being complete or ultimate:

(A1) Objects do “exist” as long as they can enter a relationship

(A2) Objects do “not exist” as neutral entities in the full-fledged sense of the term

(A3) They *may* so exist or be defined in some narrowed-down, *special* sense

(A4) It can be shown that, among other things, a conventional functional or causal representation is one such special mode of existence

(A5) This partial modality embarks on *strong* symmetry

(A6) Symmetry is generalized to build naturally on *duality*

(A7) A *relationship* is one parsimonious way of defining duality between or across objects as diverse as, dimensions, elements, categories, classes (sets, groups, topoi), properties, and relationships to name a few

(A8) It readily follows that select properties as standalone objects are not defined unless shown to be part of a relationship

(A9) It readily follows that *higher-order relationships*, or “relationships of relationships,” constitutes the core domain of interest that cannot be reduced to functionality at a low enough cost

(A10) Although there is no one-for-one reduction, a plethora or bridges can be proposed between Ordinalcy and what can be dubbed as Cardinal (functional, azimuthal) representations

(A11) Dimensions need not exist and are ill-defined *outside* specific [sets of] objects

(A12) *Standalone* dimensions are unlikely to exist, whereas their [higher-order] relationships (e.g. spacetime) lend more existential validity to them as a matter of clarity or simplicity as secured by *completeness*.

For instance, what is known as *Lame functions*—also commonly referred to as the constant elasticity of substitution (CES) in economics microfoundations—would qualify as one optimal balance between what amounts to regular implicit functions versus a relationship aggregate as defined by a particular parameter (elasticity) to lend meaning as well as fit to the otherwise set-like category that cannot even be generalized in group-theoretic terms per se.

A host of alternative illustrations can be suggested to showcase the analogies, with their rethinking to fare as a reduction bridge. To begin with, modulo congruence of the form $A \equiv B \pmod n$, syllogistic implications $A \xrightarrow{R} B$, or even regular Cartesian mappings $f: A^m \rightarrow B^n$ as well as regular choice-theoretic ranking ARB or $R(A, B)$ could be invoked to hint at how a relationship is to be generalized in a reconciliatory manner. In passing, note that the latter may refer to the convention that has long pervaded the economic theory whereby the otherwise unmeasurable utility has been defined in terms of *ordinal* or weak comparative preferences.

It is with reference to this early analogy yet with a far broader perspective that I came up with the Ordinalcy paradigm back in the late 2000 through early 2001. Indeed, from day one, Ordinalcy has aimed to extend the notion of *comparison* so far as to capture distant yet related notions such as ratios (as well as meromorphic analogies), ordering (in a qualitative manner rather than as in set theory), or relationship at large. A relationship can be seen at the end of the day as a *grand comparison*:

$$R(A, B) \neq R(B, A)$$

B is related to A, in that the two (or more objects) are R-compared in a non-commutative way. The generalized setup could look as follows:

$$\exists R \neq R': R\{A_i\} \neq R\{A_{k_i}\} \equiv R^k\{A_i\}$$

The very asymmetry between any two mutually inverse sub-relationships suggests duality or indeed *m-ality* as per the complete power of such transitions or basis reshufflings. One should bear in mind that it has been maintained that no select element A_i may carry any independent or individual meaning or information to it.

To account for this inherent duality-based asymmetry, I have deployed the refined *Orduality* notion from 2001 (with the entire enterprise starting with a need to model contagious co-movement across related as well as distant markets for financial assets, initially depicted as implicit trade in information or knowledge inputs in line with the Heckscher--Ohlin--Samuelson theorem).

Residuality

This added domain dates back to that very same quest, even though the Residuality name emerged as late as 2008 to 2009, despite the formal apparatus (yet to be unveiled in future research) having been largely outlined by 2005. The initial rationale held it that, unlike

in mainstream microfoundations, the otherwise rational economic agent will not pick just any *marginal* slacks, as the former basis for equimarginal optimizing conditions. Arguing otherwise would sound too normative, with excesses like strong-form market efficiency or hyper-rationality plaguing the “rational expectations” premises (followed by unnecessary discovery of it all being futile—and winning big both times).

Specifically, I have posited that it is *large* value differentials that matter, which should *inherently* contain the duality (cost or otherwise flipside metrics) to them, for optimization to make every sense without the need to embark on either exogenous constraints or additivity or transferability pertaining to second-stage cost subtraction.

On the other hand, duality has been imputed in how large differentials border on *weak* symmetry (non-commutativity, non-transitivity, etc.) as pointed out in Shevenyonov (2016a). Unlike in conventional calculus faring on strong local symmetry (and two-way algebraic manipulability enabling the otherwise pointless time travel such as non-recurrent aggregate demand equations), a *Residuale* (to be read akin to “rationale”) has a dual flip-side embedded in it beyond the differential versus integral inversion as in linear, Mikusinski (1960) style operators.

One might want to consider the following proposition along the morphism lines: *Homomorphic* correspondences (injective or surjective mappings) do not exhaust duality, unless rethought in ways that posit the “functional values” as none other than the residual subset of laws spawning each remaining regularity as part of a complete set. This being one other “best of the worlds” representation, the *isomorphic* counterpart could be treated as a generally unattainable and meaningless bijective correspondence of laws (or otherwise functional elements), as if to overlook their complete set. As one extension, this allusion (resembling the (B, F) *azimuthality* trajectory as part of the (A, G) *gradiency* path) can hardly be reduced to a functional space of any kind—unless a special rule or relationship akin to a group action or manifold algebra can be proposed marking a transfer between any two (or indeed a $2 < m - k < m$ subset) elements, functional stretchings, dimensions, or categories. But this is what a relational paradigm seeks to maintain and build upon, *inter alia*.

Prior & Further Research

The Residuality topic can now be seen as having arisen from my 1993-2001 interest in the dynamic and endogenous nature of comparative advantage, or indeed the scarcity of augmentable production opportunities as opposed to resources—which related framework I explicitly dubbed as “strategic protectionism” (a term coined in one of the IMF reports that, as I learned from private conversations with Dr. Anne Krueger, “was criticized more than once.”) Whereas no tariff or cross-subsidization was meant, it was apparent early on that any such pursuit would be subjected to zealous scrutiny by the advocates of skewed terms of trade as an instance of “the best of the possible worlds” (or “whatever is, is good” attitude).

In hindsight, the grand agenda appears to fit in with what would later be phased in as “blue ocean strategy” (Kim & Mauborgne, 2005) referring to frontier shifting or demand nudging on a micro-level, and “shared value” beyond externalities or team play (Porter & Kramer, 2011) in the corporate literature, by which time my Residual approach had evolved without embarking on these apparently conflated areas a tiny bit. The silver lining is that the applications as well as overlapping implications are there.

A dichotomy of *X*- versus *XX-transition* was furthermore conceived of early in the fall 1999, which in hindsight may have acted to differentiate between Gradiency versus Residuality domains—somewhat interchangeably so and possibly as a slight resemblance of the *X*-[in]efficiency issue in neoclassical or institutional literature. Whereas the former referred chiefly to [major] differentials under whatever frontier on hand (e.g. major arbitrage), the latter would pertain to frontier shifts—or, alternatively, to the lower-bound utility that is binding as a constraint or threshold not subject to explicit tradeoffs.

The Ordinalcy paradigm has not actually changed in major ways as outlined by the summer 2002. Among other things, I maintained that risk and probabilism, be it in economics domains or elsewhere, was a redundant fiction as long as a reasonably complete setup could be considered. In other words, *completeness means simplicity as well as duality*. By contrast, risk taking could be invoked as a satisficing means trading off deliberation costs (which act to detract from rationality) for some other cost—with the matching type chosen as a mode of risk mitigation, securitization, or “shared value” on the downside.

It was back then that I expanded on one of my original metaphors for Gradiency, namely a distant if tenuous relation to the optimal directional derivative as envisaged by the $(\mathcal{A}, \mathcal{Q})$ framework referring respectively to an “expansion *radius*” (the type of *hyle* elements or dimensionality being fulfilled in) as per the *structure* applied. The two, filler and structure, would generalize the equivalent of polar coordinates expansion of the static “Big Bang” type—indeed suggesting a “continuum of being or creation.” Arcane yet trite as it might seem, the framework will come in handy when rethinking the Heisenberg tradeoff along the applicable calculus lines.

At this point, the careful reader may have gained the [generally correct] impression of the initial Gradiency topic being closely intertwined with the present set of related notions being proposed. This inter-linkage will be elaborated on in future research, with a plethora of discipline-specific and [new] *foundations*-general as well as reduction-facilitating (to serve as bridges or reconciliatory references) to be offered with an eye on *novel calculi* as well as modeling applications for the more conventional means rethought. For now, the Twelve Axioms could usher in a dual implication. First, completeness befits duality, yet not necessarily [symmetrically] the other way around (e.g. referring to the gradiency-azimuthality conjugation). Second, only full-fledged objects (e.g. entire systems or paradigms yet not their select elements) can most meaningfully and simply be compared or related. This is to suggest that higher-order relationships are most existentially valid, as completeness borders on simplicity as well as duality.

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