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ON HYPERSPIRAL

$$r = a\,\mathrm{e}^{\frac{b}{\theta}}$$

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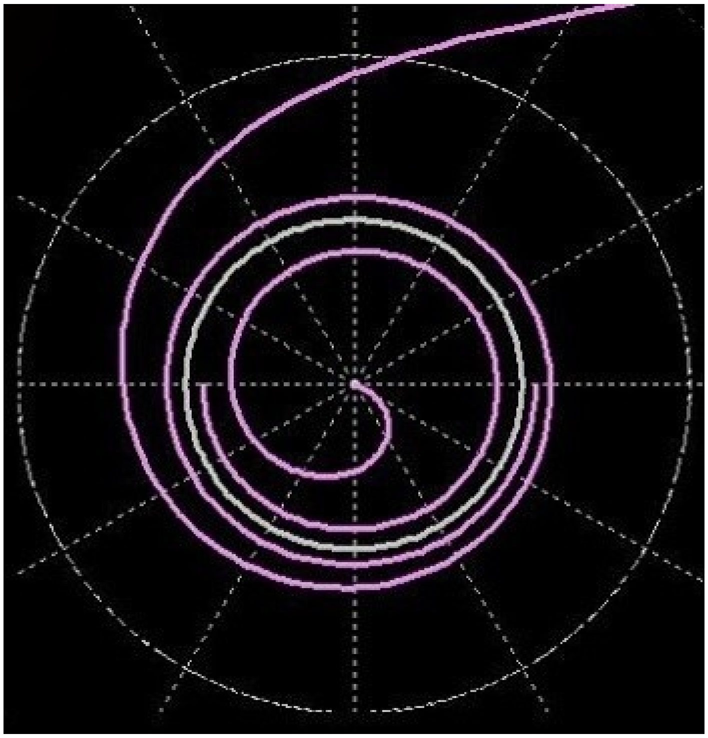

Hyperspiral by Maxima & gnuplot

This spiral (given in polar coordinates r, θ) can be seen as a missing member of the set of known spirals. Namely, if logarithmic spiral would be generalized in a way

$$r = a\,\mathrm{e}^{b\theta^q} \quad , \quad q \in Q$$

(e.g., *hyperlog-spirals*), then in case $q = -1$ follows the above proposed *hyperspiral* $r = a\,\mathrm{e}^{\frac{b}{\theta}}$. The next simplification $a, b = 1$ gives

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$$r = e^{\frac{1}{\theta}} \quad \text{or} \quad \ln r = \frac{1}{\theta} \quad .$$

The spiral has two very distinct parts: the inner part for $\theta < 0$ and the outer part for $\theta > 0$. The circle $r = a$ is the asymptotic one. Polar point is the asymptotic point of the spirals' inner part.

Rate of change of $r(\theta)$ reads

$$\dot{r} = -\frac{b}{\theta^2} r \quad .$$

Because $\psi = \arctan(\frac{r}{\dot{r}})$ defines the angle between radius and tangent in a given point (r, θ) of a polar curve, follows

$$\psi = \operatorname{arccot}(-\frac{b}{\theta^2}) \quad .$$

Second derivative of $r(\theta)$ reads $\ddot{r} = \frac{b(b+2\theta)}{\theta^4} r$. Curvature *k* of polar curves is defined as

$$k = \frac{r^2 + 2\dot{r}^2 + r\ddot{r}}{(r^2 + \dot{r}^2)^{\frac{3}{2}}}$$, hence

$$k = r^{-1} \frac{1 + b^2 \theta^{-4} - 2b\theta^{-3}}{(1 + b^2 \theta^{-4})^{\frac{3}{2}}} \quad .$$

The arc length *s* of polar curves is defined as $s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \dot{r}^2}\, d\theta$, thus follows

$$s = a \int_{\theta_1}^{\theta_2} e^{\frac{b}{\theta}} \sqrt{1 + b^2 \theta^{-4}}\, d\theta \quad .$$

Unlike logarithmic spiral this spiral does not possess simple natural, intrinsic equation because there is no exact solution of the above integral. In fact, *hyperexp* function of the general form *exp*(1/*x*), does not have its exact prime function at all. This very fact must produce deep geometrical consequences onto *hyperspiral* as well.

However, this curve does possess a full polar inversion, i.e. regarding the asymptotic circle

$$r = \frac{a^2}{r(\theta)} = a e^{-\frac{b}{\theta}} \quad .$$

Besides pure geometry, *hyperspiral* may eventually bring new inspiration into areas of science, cosmology, engineering and art.

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