

Elegant and Accurate Quark Mass Equations

–Evidence for Quantum Gravity

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The masses of the electron and the up-type quarks may be written elegantly in terms of the Planck Mass, the natural scale of quantum gravity. Since the quarks of each generation are evidently arranged symmetrically about mass levels that descend in geometric sequence from the Planck Mass, the masses of the down-type quarks may also be written in terms of the Planck Mass. The new (2016) central values of Particle Data Group quark mass evaluation lie very close to the values, divulged in 2013, of the Planck Model.

The quark mass pattern

All particles occupy the mass levels and sublevels of three geometric sequences that descend from the Planck Mass, $1.220910(29) \times 10^{19}$ GeV [1]: Sequence 1 is of common ratio $1/\pi$, Sequence 2 is of common ratio $2/\pi$ and Sequence 3 is of common ratio $1/e$ [2]. The principal levels¹ of each sequence number up from $n = 0$ (Planck scale).

The quarks of each generation are arranged symmetrically about mass levels in Sequences 2 and 3 [3], as shown in Figure 1. Particle Data Group evaluations (2016) of quark mass have been used [4]. The occupied mass levels, having level-numbers that are multiples of 5, are superlevels of type 2 [5]. The charged leptons and their partners occupy superlevels of type 1, which have level-numbers that are multiples of 3 [5].

The up quark and the electron, themselves, lie close to superlevels, as shown in Figure 2.

The masses of the electron and the up-type quarks

The Bohr radius $a_0 = (\pi/2)^{125} \cdot l_{\text{Planck}}$ ($= 0.529 \times 10^{-10}$ m) [6], where the Planck length $l_{\text{Planck}} = 1.616229(38) \times 10^{-35}$ m [1]. As a result of this critical observation, one may write $m_{\text{electron}} = \alpha^{-1}(\pi/2)^{-125} \cdot m_{\text{Planck}}$ ($= 0.511$ MeV), where $\alpha = 1/137.036$ is the fine structure constant.

Evidently, the up and top quarks lie in symmetrical arrangement with the electron in Sequence 2, as shown in Figure 3. Their masses may be written as $m_{\text{up}} = \alpha(\pi/2)^{-100} \cdot m_{\text{Planck}}$ ($= 2.177$ MeV) and $m_{\text{top}} = \alpha(\pi/2)^{-75} \cdot m_{\text{Planck}}$ ($= 174.1$ GeV). From inspection, $m_{\text{charm}} = \alpha^2(\pi/2)^{-75} \cdot m_{\text{Planck}}$ ($= 1.271$ GeV). These are the up-type quark masses of the Planck Model.

Furthermore, $m_{\text{GUT}} = \alpha^{-1}(\pi/2)^{-25} \cdot m_{\text{Planck}} = (\pi/2)^{100} \cdot m_{\text{electron}}$ ($= 2.09 \times 10^{16}$ GeV).

The masses of the down-type quarks

From the symmetrical arrangement of the quarks of each generation, as shown in Figure 1, it follows that $m_{\text{down}} = \alpha^{-1}(\pi/2)^{-120} \cdot m_{\text{Planck}}$ ($= 4.888$ MeV), $m_{\text{strange}} = \alpha^{-2}e^{-90}(\pi/2)^{75} \cdot m_{\text{Planck}}$ ($= 96.13$ MeV) and $m_{\text{bottom}} = \alpha^{-1}(\pi/2)^{-105} \cdot m_{\text{Planck}}$ ($= 4.275$ GeV). These are the down-type quark masses of the Planck Model.

¹ Principal levels are of integer level-number n

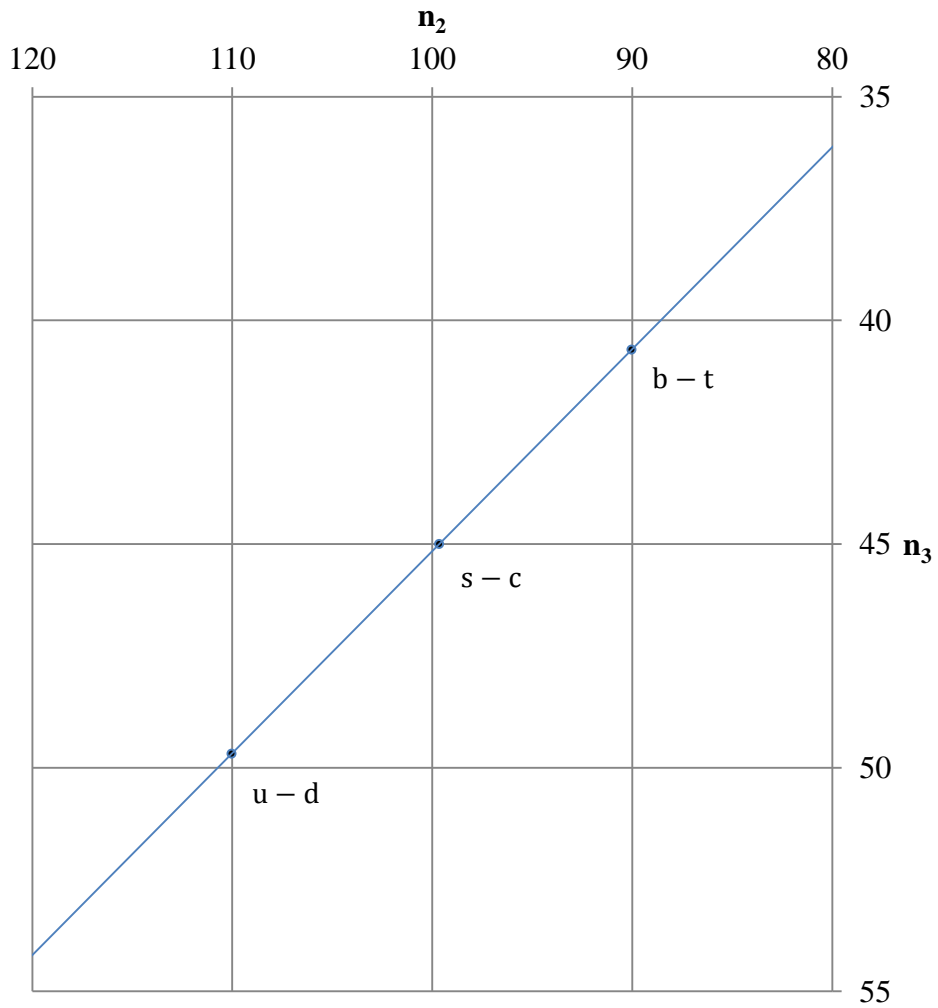


Figure 1: The quark doublets on the mass levels of Sequences 2 and 3. Each doublet is represented by the geometric mean of the two quark masses. Particle Data Group evaluations (2016) have been used. The markers lie on a straight line since n_2 and n_3 are in constant ratio. The doublets occupy type 2 superlevels, whose level-numbers are multiples of 5.

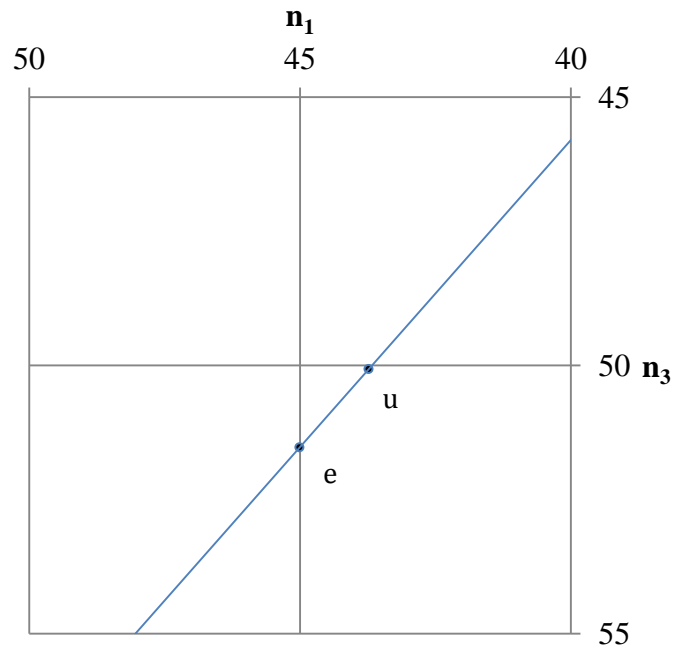


Figure 2: The up quark and the electron in Sequences 1 and 3. Particle Data Group evaluations of particle mass (2016) have been used. The markers lie on a straight line since n_1 and n_3 are in constant ratio. The up quark lies close to a superlevel of type 2, while the electron lies close to a superlevel of type 1 and type 2.

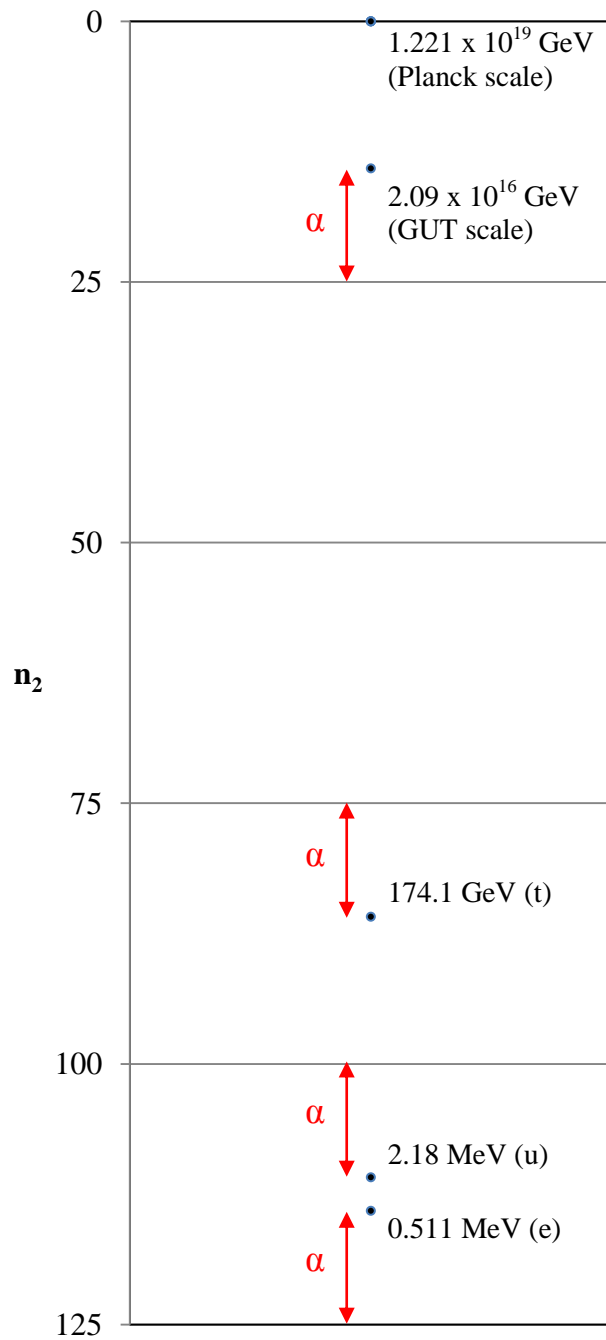


Figure 3: Mass scales of the Planck Model, in Sequence 2. Scales differing by a factor $\alpha = 1/137.036$ are indicated. Note the symmetries in the pattern of scales.

A comparison of the quark masses of the Planck Model with the evaluations (2016) of the Particle Data Group

The two sets of quark mass values are shown in Table 1. There is a high degree of concurrence between the values derived in the Planck Model and the evaluations of the Particle Data Group. This is evidence for quantum gravity.

Quark	Mass (Planck Model)	Mass (Particle Data Group)
up	2.2 MeV	$2.2^{+0.6}_{-0.4}$
down	4.9 MeV	$4.7^{+0.5}_{-0.4}$
strange	96 MeV	96^{+8}_{-4}
charm	1.27 GeV	1.27 ± 0.03
bottom	4.28 GeV	$4.18^{+0.04}_{-0.03}$
top	174.1 GeV	$173.21 \pm 0.51 \pm 0.71$

Table 1: The quark masses of the Planck Model and the evaluations (2016) of the Particle Data Group

References

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4. C. Patrignani, (Particle Data Group), *Chin. Phys. C*, **40**, 100001 (2016)
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