On the Secular Recession of the Earth-Moon System as an Azimuthal Gravitational Phenomenon

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Abstract We here apply the ASTG-model to the observed secular trend in the mean Sun-(Earth-Moon) and Earth-Moon distances thereby providing an alternative explanation as to what the cause of this secular trend may be. Within the margins of observational error; for the semi-major axis rate of the Earth-Moon system, in agreement with observations (of Standish 2005), we obtain a value of about $+(5.10 \pm 0.10)\,\mathrm{cm/yr}$. The ASTG-model predicts orbital drift as being a result of the orbital inclination and the Solar mass loss rate. The Newtonian gravitational constant G is assumed to be an absolute time constant. Krasinsky and Brumberg (2004); Standish (2005) reported for the Earth-Moon system, an orbital recession from the Sun of about $+(15.00 \pm 4.00)\,\mathrm{cm/yr}$ and $+(7.00 \pm 2.00)\,\mathrm{cm/yr}$ respectively; while Williams et al. (2004); Williams and Boggs (2009); Williams et al. (2014) report for the Moon, a semi-major axis rate of about $+(38.08 \pm 0.04)\,\mathrm{mm/yr}$ from the Earth. The predictions of the ASTG-model for the Earth-Moon system agrees very well with those the findings of Standish (2005); Krasinsky and Brumberg (2004). The lost orbital angular momentum for the Earth-Moon system – which we here hypothesize to be gained as spin by the two body Earth-Moon system; this lost angular momentum accounts very well for the observed Lunar drift, therefore, one can safely safely say that the ASTG-model does to a reasonable degree of accuracy predict the observed Lunar semi-major axis rate of about $+(38.08 \pm 0.04)\,\mathrm{mm/yr}$ from the Earth.

Keywords astrometry, celestial mechanics, ephemerides, planetary recession

1 Introduction

Gravitational "*anomalies*" have puzzled the scientific community for quite sometime now. First, was the discovery of the so-called darkmatter by the eccentric Swiss astronomer Fritz Zwicky (1933*a*,*b*) – this discovery was latter confirmed latter by Rubin and Ford (1970); Rubin et al. (1970, 1985); this was followed by the Pioneer anomaly in the late 1980's by the United States of America's National Aeronautic Space Administration (*NASA*) scientists Anderson et al. (1998, 2002), then came the Earth-flyby anomalies in the early 1990's again by *NASA* scientists (*cf.* Antreasian and Guinn 1998; Anderson et al. 2007, 2008; Iorio 2009*a*; Turyshev and Toth 2009; Iorio 2014*a*, 2015) and more recently, there has emerged the phenomenon of the secular recession of about $7 - 15\,\mathrm{cm/yr}$ in the mean Earth-Moon distance (*cf.* Acedo 2013*b*) and this measurement was conducted by independent group of American and Russian astronomers Standish (2005); Krasinsky and Brumberg (2004); Pitjeva and Pitjev (2012) respectively. Even the Moon has been found by Williams et al. (2004); Williams and Boggs (2009); Williams et al. (2014) to be receding from the Earth (*cf.* Iorio 2011*a*,*b*) at a rate of about $38\,\mathrm{mm/yr}$.

What really is going on with gravitation? We ask?! What is the matter? Do we really understand gravitation? Why suddenly an upsurge of these gravitational anomalies? For a conscience review of Solar gravitational anomalies, see *e.g.* Anderson and Nieto (2009); Iorio (2015). That said – here at the outset, let it be known that the present endeavour does not claim nor purport to answer these questions but merely makes a modest contribution to that end. Our prime focus is the observed secular recession of the Earth-Moon system from the Sun.

Our working philosophy in seeking a solution to these problems that are manifesting as gravitational anomalies is that one must first look deep into the anatomy and labyrinth of existing theories before setting sail to seek more exotic ideas. The widely accepted gravitational model is Einstein (1916)'s General Theory of Relativity (GTR). On the Solar scale, excluding minute corrections, the GTR and Newtonian gravitation are in good agreement. In the present reading, we shall be applying the recently proposed Azimuthally Symmetric Theory of Gravitation (hereafter ASTG-model,

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see Nyambuya 2010, 2015*b*) to this problem of the observed secular recession of the Earth-Moon system.

The ASTG-model – which we present only as a plausible alternative model of gravitation; this model is nothing more than the azimuthal solutions of the well known Poisson-Laplace equation applied to the scenario of gravitation. That is to say, this theory (ASTG-model) is (in our modest view) a natural extension and next logical step in the development of Newtonian gravitation (and perhaps an alternative to the GTR). Actually, this theory is part of a much larger model of gravitomagnetism (see Nyambuya 2014*e*,*c*,*g*). At first glance, this theory (ASTG-model) appears as nothing more than the mundane azimuthally symmetric solutions of the well known Poisson-Laplace equation, namely:

$$\boldsymbol{\nabla}^2\Phi = 4\pi G\varrho, \qquad (1)$$

where G is Newton's universal constant of gravitation, Φ is the gravitational potential, ϱ is the density of matter and $\boldsymbol{\nabla}^2$ is the usual Laplacian differential operator.

At its inception, it was assumed that the ASTG-model is but a banal theory of gravitation only extending the gravitational theory of Sir Isaac Newton from just being a central field phenomenon to an azimuthal and polar field phenomenon, but overtime that view has been changed (Nyambuya 2015*b*). The ASTG-model is a "seemingly non-relativistic classical theory" where spin is not only taken into account but takes center stage in the theory, especially when the spin is relatively and significantly high. This is not the case with classical theories of gravity hence this very development makes it a new theory of gravitation.

As argued in Nyambuya (2015*b*), the ASTG-model is surely a new classical theory of gravitation which makes the *seemingly ambitious hypothesis* that the spin of a gravitating mass has a significant and decisive role to play in the emergent gravitational field of a spinning mass. The ASTG-model is based[1] on the solutions $[\Phi = \Phi(r,\theta)]$ of equation (1), *i.e.*:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}}{r}\left[1 + \sum_{\ell=1}^{\infty}\lambda_\ell\left(\frac{G\mathcal{M}}{rc^2}\right)^{\ell}\mathcal{P}_\ell(\sin\theta)\right], \qquad (2)$$

where:

$$\mathcal{P}_\ell(\sin\theta) = \begin{cases} P_\ell(\sin\theta), & \text{for} \quad \ell = 2,4,6,\dots\ etc \\ \left|P_\ell(\sin\theta)\right|, & \text{for} \quad \ell = 1,3,5,7\dots\ etc \end{cases}; \qquad (3)$$

[1]This theory can be extended to include the polar solutions $\Phi(r,\theta,\varphi)$. Exploration of these solutions is a task we hope to look into in future readings.

and $P_\ell(\sin\theta) = P_\ell[\cos(\pi/2-\theta)]$ are Legendre polynomials, $\mathcal{M}$ is the mass of the central gravitating body, c is the speed of light in vacuum, r is the radial distance from this gravitating body, and $(\lambda_\ell : \ell = 1,2,3,\dots\ etc)$ are some dynamic parameters which in the ASTG-model are assumed to be related to the gravitating body in question and the explicit dependence of these λ-parameters on the gravitating body's spin have been explored and made clear in the reading Nyambuya (2015*b*).

About these λ-parameters, it should be mentioned that this property that the λ's are dynamic parameters assumed to be related to the gravitating body in question is the novelty of the ASTG-model. Putting weight to what we already have said; in a way, the dynamism of the λ-parameters makes the ASTG-model a new classical theory of gravitation where the spin of the gravitating mass enters the gravitational podium. Further, of the λ-parameters, for all conditions of existence, it is assumed that $(\lambda_\ell \equiv 0)$ whenever spin is dropped (switched-off). What this all means is that with the spin switched off, the ASTG-model reduces to the traditional Newtonian gravitational theory that we are used to know.

Furthermore, it should be mentioned [as was done in the reading Nyambuya (2010)], that, the λ-parameters are free parameters whose dependence on spin and the resulting numerical coefficients are all to be determined from empirical data, intuition and imagination. This is clearly a weak point of the theory. We can only hope that the λ-parameters that have been proposed in the reading Nyambuya (2015*b*) will prove to be universal in that they will apply to other gravitational systems without the need for further adjustments.

As already said, the novel feature of the ASTG-model is that it brings the spin of a gravitating object into the fold of the classical gravitation (*i.e.*, non-relativistic gravitation). The spin now plays an important and decisive role in generating the gravitational field that has a bearing on test bodies in the vicinity of this gravitating object. But the ASTG-model is not the only theory that does this. For example, we have the gravitomagnetic effects such as the *Lense-Thirring Effect* (Lense and Thirring 1918; Iorio 2012*b*), the *Gyroscope Precession Effect* (Pugh 1959; Schiff 1960*a*,*b*) and the *Gravitomagnetic Clock Effect* (Zeldovich 1965; Vladimirov et al. 1987; Cohen and Mashhoon 1993; Lichtenegger et al. 2006; Iorio and Lichtenegger 2005; Iorio et al. 2002; Iorio 2001*a*,*b*; Mashhoon et al. 2001). The ASTG-model is yet to be applied to these three important gravitomagnetic effects so as to see what it has to say about them.

Of these three important effects, the *Pugh-Schiff Gyroscope Effect* (Pugh 1959; Schiff 1960*a*,*b*) has been measured to a convincing accuracy using University of Stanford's[2]

[2]See Gravity Probe B. Websites: `http://einstein.stanford.edu` and `http://www.gravityprobeb.com`

Gravity Probe B Experiment (Everitt et al. 2011) while the Lense-Thirring orbital precession were tentatively measured with artificial satellites orbiting some Solar system major bodies (Ginzburg 1959; Cugusi and Proverbio 1978; Ciufolini and Pavils 2004; Iorio 2006, 2009*b*, 2010, 2012*a*; Iorio et al. 2011, 2013; Renzetti 2013*a*,*b*, 2014). This Gravity Probe B measurement is one of the latest in a series of measurements that have confirmed the accuracy of the GTR. It places the GTR ahead of most of the competing models of gravitation. However, it should be noted that in this regime of measurements, the GTR is being tested in the low energy regime and not in the regime of very high spacetime curvature where its predictions are clearly at variance with most of the competing models (cf. Will 2006, 2014).

As consistently pointed out by Will (2006, 2009, 2014), there is need to test the GTR in regimes of high spacetime curvature if the short-comings of the GTR are ever to surface or any cracks in it are to emerge and these shortcomings may pave the way for alternative models of gravitation to demonstrate their supremacy (if any) in those regimes. Therefore, despite the accuracy with which the GTR is confirmed by experiments in the low energy regime, motivation to compare the GTR, experiment and alternative models remains not only high, but necessary in-order to find better models of gravitation that might explain current gravitational anomalies.

In-closing this introductory section, we shall give a synopsis of the present reading. In §(2), we present the working hypothesis and thereafter, in §(3) we present some computations that are necessary for latter purposes. In §(4), we present a summary of the work presented in Nyambuya (2014*c*). This work is presented in context which gives its relevance in the present reading and in §(5) we present the relevant equations of motion corresponding to the gravitational potential presented in §(4). In §(6) we show how the ASTG-model lead to the loss of planetary orbital orbital angular momentum. In §(7) we tackle the problem of the present paper where we apply the resulting equations to the recession of the Earth-Moon system. In §(8) we tackle the problem of Lunar recession and finally in §(9) and (10), we give a general discussion and the conclusion drawn thereof.

2 Hypothesis

The observed recession of the Earth-Moon system from the Sun (and possibly other planets as-well) is possibly due to the loss of orbital angular momentum which is induced by the Sun's luminosity and *via* its azimuthal gravitational field $\Phi = \Phi(r, \theta)$ where $\Phi = \Phi(r, \theta)$ is a solution to (1) and is given in (2).

3 Computations

We need to establish a single value for the semi-major axis rate of the Earth-Moon system from the Sun. Standish (2005)'s (7.00 ± 2.00) cm/yr measurement is taken by some as an improvement on Krasinsky and Brumberg (2004)'s earlier measurement of (15.00 ± 4.00) cm/yr. Our approach in the present is to take these two measurements as being equally good measurements, therefore we need to apply statistical methods and establish a single working value out of these two measurements.

As is common knowledge, the mean distance from the center of mass of the Sun and the common center of mass of the Earth-Moon system has traditionally been referred to as the Astronomical Unit and is denoted by the symbol AU and in some cases au (cf. Iorio 2015). The AU is used as a fundamental unit of measurement in astronomy and astrophysics. This unit determines and defines the Solar system scale. The term "Astronomical Unit" appears at the beginning of the 20^{th} century (cf. Pitjeva 2012) and it is not until 1976 that the International Astronomical Union (IAU) adopted a formal definition of the AU. As a measure of the mean Sun-(Earth-Moon) distance, the AU is no longer an appropriate way of referring to the mean Sun-(Earth-Moon) distance, as this has since been fixed by the 2012 IAU General Assembly[3] (Capitaine 2012). This makes perfect sense as units are not supposed to be dynamic but sacrosanct and eternally static. Therefore, the distance between the Sun and the Earth-Moon system, we shall refer to as the mean Sun-(Earth-Moon) distance – not the AU.

Assuming that these two measurements (of Krasinsky and Brumberg 2004; Standish 2005) are governed by Gaussian statistics and that the errors in the measurements are random and independent, then, the best estimate of these two measurements can be obtained by taking the weighted mean of the two values. For example if $(x_i + \delta x_i : i = 1, 2, \ldots n)$ is a set of n measurements of a constant quantity x, where x_i is the best value of i^{th} measurement and δx_i is its accompanying error margin, then, the best estimate (x_{best}) of x from this set is $x_{\text{best}} = \sum w_i x_i / \sum w_i$ where w_i are the weights such that $w_i = 1/(\delta x_i)^2$ and the best estimate in the error margin δx_{best} is $\delta x_{\text{best}} = (\sum w_i)^{-1/2}$ (*cf.* Taylor 1982, p.150). Applying this prescription to the two measurements of Standish (2005); Krasinsky and Brumberg (2004), we obtain:

$$\dot{a}_{\text{em}} = +9.50 \pm 0.20 \, \text{cm/yr}, \tag{4}$$

[3] Resolution B2 of the XXVIII IAU General Assembly, available on the Internet at `http://syrte.obspm.fr/IAU_resolutions/Res_IAU2012_B2.pdf`

where $a_{\rm em}$ is the semi-major axis of the Earth-Moon system's orbit. We shall herein adopt this value (4) as currently the best representative of the accuracy with which the change in the mean distance between the Sun and Earth-Moon system can be determined from observations.

The maximum distance of the Earth from the Sun $\mathcal{R}_{\rm orb}^{\rm max} = 1.52098232 \times 10^{11}\,{\rm m}$ and minimum distance is $\mathcal{R}_{\rm orb}^{\rm min} = 1.47098290 \times 10^{11}\,{\rm m}$ (Standish and Williams 2010). In our calculation, we need one single value for the mean distance between the Sun and the Earth-Moon system. From $\mathcal{R}_{\rm orb}^{\rm min}$ and $\mathcal{R}_{\rm orb}^{\rm max}$, the best estimate would the average of these two values, that is, $\mathcal{R}_{\rm orb}^{\oplus} = (\mathcal{R}_{\rm orb}^{\rm max} + \mathcal{R}_{\rm orb}^{\rm min})/2$ and the best estimate in the error $\Delta\mathcal{R}_{\rm orb}^{\oplus}$ to this value is $\Delta\mathcal{R}_{\rm orb}^{\oplus} = (\mathcal{R}_{\rm orb}^{\rm max} - \mathcal{R}_{\rm orb}^{\rm min})/2$, so that the best value for the mean distance between the Sun and the Earth-Moon system $\mathcal{R}_{\rm mean}^{\oplus}$ is:

$$\mathcal{R}_{\rm mean}^{\oplus} = (1.50 \pm 0.03) \times 10^{11}\,{\rm m} = (1.00 \pm 0.02)\,{\rm AU}. \quad (5)$$

In column three of Table (1), this same approach is used to compute the mean distances of Solar planets from the Sun. These distances are required in order to calculate the predicted recession of these planets from the Sun.

For the Earth-Moon system, the Earth is the central massive body and the Moon is the orbiting test body. At the apogee of the Moon, the centres of mass of the two systems are $4.055 \times 10^8\,{\rm km}$ while at perigee, they are $3.633 \times 10^8\,{\rm km}$ apart. This means the mean distance of the Earth-Moon system is $(3.80 \pm 0.20) \times 10^8\,{\rm km}$. The "error" $\pm 0.20 \times 10^8\,{\rm km}$ is not an error bar but a "margin" expressing the range between the maximum and minimum distance. From this, it follows that $(\dot{a}/a)_{\rm em} = (10.00 \pm 0.50) \times 10^{-11}\,{\rm yr}^{-1}$.

4 The Five Components of the Gravitational Force

In the reading Nyambuya (2014*c*), the Four Poisson-Laplace equation:

$$\boldsymbol{\nabla}^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\varrho, \quad (6)$$

is solved in the context to the gravitomagnetic theory given in Nyambuya (2014*e*). It must be mentioned here that the gravitomagnetic theory proposed in Nyambuya (2014*c*) is not championed in the same spirit as the gravitatomagnetic theory that emerges from the GTR in the weak field approximation. No! It is championed from the vantage point of the new proposed *Unified Field Theory* of all the forces of *Nature* (see Nyambuya 2014*g*) *albeit*, in the original quest and spirit of Maxwell-Heaviside (Maxwell 1865; Heaviside 1893, 1894) gravitomagnetic theory.

It is shown in the reading Nyambuya (2014*c*) that equation (6) admits five solutions which in the radial case *i.e.* $\Phi_j = \Phi_j(r) : j = 1, 2, 3, 4, 5$; these solutions are for the case $(j = 1)$:

$$\Phi_{(1)} = -\frac{G\mathcal{M}_{\rm star}}{r}, \quad (7)$$

which is the usual Newtonian gravitational potential. In the present reading and therein the reading Nyambuya (2014*c*), despite the existing provision for a dynamic Newtonian gravitational constant, it is assumed that the Newtonian gravitational constant G is not a time variable.

Insofar as measurements by Pitjeva and Pitjev (2012, 2013); Pitjev and Pitjeva (2013); Pitjeva (2013); Pitjeva and Pitjev (2014) are concerned the heliocentric gravitational parameter $(\mu_\odot = G\mathcal{M}_\odot)$ has been found to vary, *i.e.* $(\dot{\mu}_\odot/\mu_\odot \simeq -5 \times 10^{-14}\,{\rm yr}^{-1})$. This variation of $\mu_\odot$ can either be due to a variation of G or $\mathcal{M}_\odot$. Since the Sun is a luminous object, it is certain that $\mathcal{M}_\odot$ is significant player in the time variation of $\mu_\odot$. According to Pitjeva and Pitjev (2012, 2013); Pitjev and Pitjeva (2013); Pitjeva (2013); Pitjeva and Pitjev (2014), the issue is whether or not G varies with time. Using data of at least thirty seven years of Lunar Laser Ranging (LLR), Williams et al. (2004); Müller and Biskupek (2007) have made independent determinations of $\dot{G}/G$. They (Williams et al. 2004; Müller and Biskupek 2007) found no variation at the level of accuracy of their measurements. If these measurements (Williams et al. 2004; Müller and Biskupek 2007) are indicative of a no variation in $\dot{G}/G$, this leaves the variation of $\mu_\odot$ as being caused by the Sun's mass loss rate. As stated, our working hypothesis is that $(\dot{G}/G = 0)$; this assumption is not unreasonable given *e.g.* Williams et al. (2004); Müller and Biskupek (2007)'s measurements.

Now, for the case $(j = 2)$, we have:

$$\Phi_{(2)} = -\frac{G_2(t)\mathcal{M}_{\rm star}e^{-\mu_2 r}}{r} \quad \text{where} \quad G_2 = G_2(0)e^{-\mu_2 ct}, \quad (8)$$

where $G_2(t)$ is the corresponding time variable gravitational constant associated with this potential and this gravitational constant has the same dimensions as the Newtonian gravitational constant G and μ_2 is a constant parameter with the dimensions of inverse length. The potential equation (8) is the usual Yukawa potential and this potential has been slated for investigation of the Pioneer anomaly.

The Yukawa potential has been used to try and explain the Pioneer anomaly (*cf.* Brownstein and Moffat 2006; Iorio 2007*a*), the rotation curves of spiral galaxies (*cf.* Moffat 1995, 2005) and the extra-anomalous apsidal precession of Solar planetary orbits (Iorio 2007*b*,*c*, 2008). The fact that in

Nyambuya (2014*c*), this potential is derived from within an acceptable framework of gravitomagnetism, this justifies not only its application in gravitational physics but its existence in the gravitation physics.

Despite the fact that current thinking holds that the Pioneer anomaly may not be a gravitational phenomenon but a result on-board problems with the heating systems of the spacecrafts (Iorio 2007*a*; Turyshev and Toth 2010; Turyshev et al. 2011, 2012), in a future reading, the Yukawa potential equation (8) together with the potential $\Phi_{(4)}$, these are going to be applied to the problem of the Pioneer anomaly. We are of the strong view that the resolution of the Pioneer anomaly may very well be far from resolved, all plausible causes must be considered and only a dedicated mission is going to decide which of the proposed mechanisms corresponds with physical and natural reality.

For the case $(j=3)$, we have:

$$\Phi_{(3)} = -\frac{G_3(t)\mathcal{M}_{\rm star}\cos(\mu_3 r)}{r}, \qquad (9)$$

where $G_3(t) = G_3(0)\cos(\mu_2 ct)$ is the time variable gravitational constant associated with this potential and this constant has the same dimensions as the Newtonian gravitational constant G and μ_3 is a constant parameter with the dimensions of inverse length. The potential equation (9) is a new gravitational potential and this potential has been slated for investigation of the existence of rings system around planets.

For the case $(j=4)$, we have:

$$\Phi_{(4)} = -\frac{G_4(r,t)\mathcal{M}_{\rm star}}{\mathcal{R}_{\rm star}}\left[\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{\alpha_4} + \kappa_4\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{1-\alpha_4}\right], \qquad (10)$$

where as before $G_4(r,t)$ is the time and space variable gravitational constant associated with this potential and $\mathcal{R}_{\rm star}$ is the radius of the gravitating object in-question and α_4 is constant while κ_4 is a dimensionless parameter which is such that $(|\kappa_4| \geq 0)$. The potential equation (10) is a new gravitational potential and this potential has been slated for investigation of the flat rotation curves of spiral galaxies.

Lastly, for the case $(j=5)$, we have:

$$\Phi_{(5)} = -\frac{G_5(r,t)\mathcal{M}_{\rm star}}{\mathcal{R}_{\rm star}}\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{\frac{1}{2}}\cos\left[\ln\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{\alpha_5}\right], \qquad (11)$$

and again $G_5(r,t)$ is a space and time variable gravitational constant associated with this potential. Like the other potentials, this potential equation (11) is a new gravitational potential; it has been slated for investigation of the origins of the Titius-Bode Law (see *e.g.* Huang and Bakos 2014; Lara et al. 2012; Neito 1972, for Titius-Bode Law).

The potentials equation (7), (8), (9), (10) and (11) all have a radial dependence. Except for equation (7), the azimuthal gravitational components of equation (8), (9), (10) and (11) have up to now not been worked out. However, these potentials will suffice for the work we intent to carry out here. All the potentials equation (7), (8), (9), (10) and (11) are assumed to act simultaneously on any gravitating body so that the total or resultant gravitational potential Φ is such that $\Phi = \sum_{j=1}^{5}\Phi_{(j)}$.

In the next section, we will derive the equations of motion for a test body in the vicinity of a spinning gravitating body under the influence of the Newtonian potential (7). We will here not consider the potentials (8), (9), (10) and (11) and the reason for this is because this is unnecessary as the Newtonian potential with the spin included in accordance with the ASTG-model, this gives us good results.

5 Equations of Motion

In spherical coordinates, for a three dimensional space, the acceleration $(\boldsymbol{a} = \ddot{\boldsymbol{r}})$ is given by:

$$\begin{aligned}\boldsymbol{a} = \ & (\ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta)\hat{\boldsymbol{r}} \\ & +(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} \\ & +(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta)\hat{\boldsymbol{\varphi}}\end{aligned} \quad . \qquad (12)$$

The acceleration due to gravity $(\boldsymbol{g} = -\gamma\boldsymbol{\nabla}\Phi)$, where γ is the ratio of the gravitational mass $(\mathrm{m_g})$ to the inertial mass (m_i) *i.e.* $(2\gamma = \mathrm{m_g}/\mathrm{m}_i)$. Though there may be reasons for $(\gamma \neq 1/2)$ (*cf.* Nyambuya and Simango 2014), for the present, we shall – as is usually assumed, take the gravitational and inertial mass of a test particle to be identical physical quantities (*i.e.*, $\mathrm{m}_i \equiv \mathrm{m_g} \Longrightarrow \gamma \equiv 1/2$); however, in our derivation, we shall keep alive this γ-term because we would like to use the resulting equations for latter investigations where this term's variability plays center stage – we do not want to re-derive these equations but conduct this task here once and for all. Once we have derived our equations with the γ-factor included, we shall at the appropriate moment drop this term. Hence, comparing the different components (*i.e.*, the radial, $\hat{\theta}$ and $\hat{\varphi}$-components) of this equation of motion *i.e.* $\boldsymbol{a} \equiv 2\gamma \boldsymbol{g}$, one obtains the following equations:

$$\frac{\partial^2 r}{\partial t^2} - r\left(\frac{\partial \theta}{\partial t}\right)^2 - r\left(\frac{\partial \varphi}{\partial t}\right)^2 \sin^2\theta = -\frac{\partial(2\gamma\Phi)}{\partial r}, \quad (13)$$

$$\frac{\partial J_\theta}{\partial t} - \frac{J_\varphi^2 \sin\theta\cos\theta}{r^2} = -\frac{\partial(2\gamma\Phi)}{\partial\theta}, \quad (14)$$

$$\frac{\partial J_\varphi}{\partial t} + \frac{2J_\theta J_\varphi \cos\theta}{r^2} + \frac{2(\sin\theta - 1)\dot{r}J_\varphi}{r} = -\frac{\partial(2\gamma\Phi)}{\partial\varphi}, \quad (15)$$

for the $\hat{\boldsymbol{r}}$, $\hat{\theta}$, and the $\hat{\varphi}$-component respectively. In the above equations, J_φ and J_θ are the specific orbital angular momentum in the $\hat{\varphi}$ and $\hat{\theta}$-directions.

Now, making the substitution $u = 1/r$, the equations (13), (14) and (15) transform to:

$$\frac{\partial^2 u}{\partial\varphi^2} + \frac{\dot{J}_\varphi}{u^2 J_\varphi^2}\frac{\partial u}{\partial\varphi} + \left(\sin^2\theta + \zeta^2\right)u = -\frac{1}{J_\varphi^2}\frac{\partial(2\gamma\Phi)}{\partial u}, \quad (16)$$

$$\frac{\partial J_\theta}{\partial t} - u^2 J_\varphi^2 \sin\theta\cos\theta = -\frac{\partial(2\gamma\Phi)}{\partial\theta}, \quad (17)$$

$$\frac{\partial J_\varphi}{\partial t} + 2u^2 J_\varphi J_\theta \cos\theta - 2(\sin\theta - 1)u^2 J_\varphi^2 \frac{\partial u}{\partial\varphi} = -\frac{\partial(2\gamma\Phi)}{\partial\varphi}, \quad (18)$$

respectively.

In equation (16) ($\zeta = J_\theta/J_\varphi = \mathcal{T}_\varphi/\mathcal{T}_\theta$) where $\mathcal{T}_\varphi$ and $\mathcal{T}_\varphi$ are the orbital periods of revolution in the $\hat{\theta}$ and $\hat{\varphi}$ directions respectively. In the present reading, equation (18), is the equation of interest. We shall investigate equation (17) in a future reading. For example, if ($J_\theta \neq 0$), it follows that the planets will experience a drift away from the Solar equator. It is this (and other possible phenomenon) that we will investigate in the future to see if the ASTG-model does explain the tilt of Solar planetary orbits *etc*.

Before closing this section, it must be pointed out that, at a *prima facie* level, it is easy to set ($\partial(2\gamma\Phi)/\partial\theta = \partial(2\gamma\Phi)/\partial\varphi = 0$), so that the right hand-side of equations (17) and (18) will be zero. If – however – one where to realise that ($\theta = \omega_\theta t$) and ($\varphi = \omega_\varphi t$), then, they will soon realise that the right hand-side of equation (17) and (18) are not exactly zero as might be deduced at face value but these will only be zero in the case of a static potential. Certainly, a luminous body such as the Sun is going to have a non-static gravitational potential since its mass is a time variable, hence the right hand-side of these equations (17) and (18) will not equal zero. These equations (17) and (18) will, accordingly be such that:

$$\frac{\dot{J}_\theta}{J_\theta} - \frac{u^2 J_\varphi^2 \sin\theta\cos\theta}{J_\theta} = -\frac{1}{\omega_\theta J_\theta}\frac{\partial(2\gamma\Phi)}{\partial t}, \quad (19)$$

$$\frac{\dot{J}_\varphi}{J_\varphi} + \frac{2J_\theta\cos\theta}{r^2} + 2(\sin\theta - 1)\frac{\dot{r}}{r} = -\frac{1}{\omega_\varphi J_\varphi}\frac{\partial(2\gamma\Phi)}{\partial t}, \quad (20)$$

respectively.

6 Loss of Planetary Orbital Angular Momentum

As stated in §(2), the working hypothesis here is that the observed recession of the Earth-Moon system from the Sun (and possibly other planets as-well) is possibly due to the loss of orbital angular momentum which is induced by the Sun's luminosity and azimuthal gravitational field. For our purpose here, we shall make the following assumptions:

1. The colatitude orbital angular moment J_θ is small enough that we can neglect it. This implies that in equation (20) we can set ($J_\theta = 0$).
2. For the relationship between ($\dot{\mathcal{R}}_{\rm orb}, \mathcal{R}_{\rm orb}, \dot{\mathcal{T}}_{\rm orb}$ & $\mathcal{T}_{\rm orb}$) and ($\dot{J}_\varphi, J_\varphi, \dot{\mathcal{R}}_{\rm orb}$ & $\mathcal{R}_{\rm orb}$), we place the following constraints:

$$\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}} = \frac{1}{1+\varepsilon}\frac{\dot{\mathcal{T}}_{\rm orb}}{\mathcal{T}_{\rm orb}}, \quad \Rightarrow \quad \frac{\dot{J}_\varphi}{J_\varphi} = (1+\varepsilon)\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}, \quad (21)$$

where ε is a constant. From our on-goinging work on the problem of Earth Flyby Anomalies (*cf.* Antreasian and Guinn 1998; Anderson et al. 2007, 2008; Iorio 2009*a*; Turyshev and Toth 2009; Iorio 2014*a*, 2015), we find that ($\varepsilon = 12.00 \pm 1.00$). In the present work, this value (parameter) does not in any way affect anything connected with what we want to achieve herein. In addition to the constraints given in equation (21), we shall impose the additional constraint:

$$\frac{d}{dt}\left(\frac{\dot{J}_\varphi/J_\varphi}{u^2 J_\varphi}\right) = \frac{d}{dt}\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}\omega_\varphi}\right) \equiv 0. \quad (22)$$

Consequently, the constraint (22) implies that:

$$\frac{\ddot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}} = -\varepsilon\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)^2. \quad (23)$$

3. While the new gravitational potentials equation (8), (9), (10) and (11) may contribute to the recession of the Earth-Moon system, we shall assume as our first approach to the problem, that the Newtonian gravitational component equation (8) contributes the most to the recession of the Earth-Moon system.

4. For nearly circular orbits – as is the case with Solar planetary orbits, to first order approximation, for J_φ, we have $(J_\varphi^2 \simeq G\mathcal{M}_{\rm star}\mathcal{R}_{\rm orb})$. We will use this approximation throughout.

Now, considering only the Newton gravitation potential equation (7) and effecting the afore-stated assumptions into equation (20), we will have:

$$\frac{\dot{J}_\varphi}{J_\varphi} + 2(\sin\theta - 1)\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}} = \frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}} - \frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}} + \frac{\dot{\gamma}}{\gamma}. \quad (24)$$

From equation (21), it follows that the contribution from the first gravitational component equation (7) on the assumptions ($\dot{\gamma} = 0$), is:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(1)} = \frac{1}{2\sin\theta}\frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}}. \quad (25)$$

At this point, in-order to get the correct sign in the drift, we need to take into account the type of coordinate system used as specified in Nyambuya (2015*a*) by making the replacement ($\dot{r} \rightarrow -\dot{r}$) – this tacit adjustment will without notice be made to equations (27), (28), (29) and (30); in the case of equation (25), this will transform to:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(1)} = -\frac{1}{2\sin\theta}\frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}}; \quad (26)$$

and in the same manner as we have derived (26) above, the contribution from the second gravitational component equation (8), is:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(2)} = -\frac{1}{2\sin\theta}\left(\frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}} - \mu_2 c\right)\frac{G_2 e^{-\mu_2 r}}{G}; \quad (27)$$

and further, the contribution from the third gravitational component equation (9), is:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(3)} = -\frac{1}{2\sin\theta}\left(\frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}} - \mu_3 c\right)\frac{G_3\cos(\mu_3 r)}{G}; \quad (28)$$

and the contribution from the fourth gravitational component equation (10), is:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(4)} = -\frac{1}{2\sin\theta}\left(\frac{\dot{G}_{04}}{G_4} + \frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}}\right)\left[\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{\alpha_4} + \kappa_4\left(\frac{\mathcal{R}_{\rm star}}{r}\right)^{1-\alpha_4}\right]; \quad (29)$$

and, lastly, the contribution from the fifth gravitational component equation (11), is:

$$\left(\frac{\dot{\mathcal{R}}_{\rm orb}}{\mathcal{R}_{\rm orb}}\right)_{(5)} = -\frac{1}{2\sin\theta}\left(\frac{\dot{\mathcal{M}}_{\rm star}}{\mathcal{M}_{\rm star}} - \mu_5 c\right)\left(\frac{\mathcal{R}_{\rm star}}{\mathcal{R}_{\rm orb}}\right)^{\frac{1}{2}}\cos\left[\ln\left(\frac{\mathcal{R}_{\rm star}}{\mathcal{R}_{\rm orb}}\right)^{a}\right]. \quad (30)$$

In equation (29) and (30), we have taken into account the case where the radius of the gravitating object in-question undergoes a secular variation. For luminous objects such as the Sun and the stars, certainly, their radii will undergo a secular variation and this must be taken into account in the case where the components equation (10) and (11) contribute significantly to the secular drift of test bodies. The contributions of the terms equation (27), (28), (29) and (30) to the recession of the Earth-Moon system are here considered to negligible. So, we will not say much about these terms and their inclusion here is to simple say that these terms may be necessary for certain gravitating systems.

Table 1 Theoretical Predictions of Secular Solar Planetary Drifts

Planet	Tilt Angle (θ) (1.0°)	Mean Radius ($\mathcal{R}_{\rm orb}^{\rm best}$) (AU)	Theoretical Value ($\dot{a}$) (cm/yr)	Theoretical Value ($\dot{a}/a$) ($10^{-13}{\rm yr}^{-1}$)	Pitjeva and Pitjev (2012)'s Model Values ($\dot{a}/a$) ($10^{-13}{\rm yr}^{-1}$)
Mercury	14.0	0.390 ± 0.080	$+1.10 \pm 0.30$	$+1.50 \pm 0.50$	$+33.0 \pm 59.5$
Venus	10.4	0.726 ± 0.005	$+2.74 \pm 0.02$	$+2.50 \pm 0.03$	$+37.4 \pm 29.0$
Earth	7.0	1.000 ± 0.020	$+5.10 \pm 0.10$	$+3.65 \pm 0.10$	$+0.135 \pm 0.032$
Mars	8.9	1.500 ± 0.100	$+6.50 \pm 0.50$	$+2.50 \pm 0.50$	$+0.235 \pm 0.054$
Jupiter	8.3	5.200 ± 0.300	$+24.50 \pm 1.00$	$+3.00 \pm 0.30$	$+36300 \pm 22400$
Saturn	9.5	9.600 ± 0.500	$+39.50 \pm 2.00$	$+2.60 \pm 0.30$	$+9440 \pm 1380$
Uranus	7.8	19.300 ± 0.900	$+97.500 \pm 5.00$	$+3.20 \pm 0.30$	–
Neptune	8.8	30.200 ± 0.300	$+134.00 \pm 2.00$	$+2.85 \pm 0.05$	–
Pluto	24.2	40.000 ± 10.000	-65.00 ± 20.00	-0.90 ± 0.50	–

Note: The planetary data on the tilt angle θ of planetary orbits relative to the Solar spin equator, the perihelion and apehilion distances of planets used in the present table are adapted from the *NASA* website: `http://nssdc.gsfc.nasa.gov/planetary/factsheet/` on this day 15 Nov. 2014@16h07 GMT+2. The angle θ has been calculated as follows (1) we obtained the tilt of Solar planetary orbits relative to the ecliptic plane and these values are available on the *NASA* website; (2) we then add to this the tilt angle of the Solar spin equator relative to the ecliptic plane and this is known to be 7°. In this way, we obtained the tilt angles of the planes of these orbits relative to the Solar spin equator. This same method has been used in Table (1) of Nyambuya (2010).

7 Recession of Earth-Moon System

According to equation (26), there are two parameters involved in the secular drift of the a planet around and the Sun, these are the its tilt (θ) angle and the Solar mass loss rate ($\dot{\mathcal{M}}_\odot/\mathcal{M}_\odot$). The actual cause is the Solar mass loss rate. According Noerdlinger (2008), the total Solar mass rate is ($\dot{\mathcal{M}}_\odot/\mathcal{M}_\odot = -9.13 \times 10^{-14}\ {\rm yr}^{-1}$). This Solar mass rate includes electromagnetic radiation, the Solar neutrino luminosity and Solar wind. Given that: the tilt of the Earth-Moon's orbit about the Solar equator is ($\theta_{\rm em} \sim 7.0^\circ$), it follows from equation (2) that ($\dot{a}_{\rm em} \sim 5.10 \pm 0.10\,{\rm cm/yr}$). Certainly, within the margins of error, this value ($5.10 \pm 0.10\,{\rm cm/yr}$) is in good agreement with the observations of Standish (2005) so much that it surely gives one some reasonable degree of confidence in the theory producing this value. It is akin to Einstein (1916)'s GTR at its inception when it correctly predicted the $43.01''$ anomalous secular advance of the perihelia of the planet Mercury.

Perhaps, ridding on this confidence, one can proceed to use equation (26) to make predictions about the other Solar planets. Table (1) does exactly that. These are predictions which future measurements can either verify or refute. We have therein Table (1) applied equation (26) to the rest of the Solar planets. If the present ideas prove to have reasonable correspondence with physical and natural reality, then, the planet Neptune must have the largest secular advance and Pluto must instead of receding, it must be advancing toward the Sun and this is because of the fact that it orbits in a direction opposite to other planets.

In the last column of Table (1), we have included the values of $\dot{a}/a$ by Pitjeva and Pitjev (2012) in-order to compare them with our own. As can be read of from this table, these values do not compare with ours. In general, Pitjeva and Pitjev (2012)'s values are significantly large. Pitjeva and Pitjev (2012)'s values are based on the particular model they adopted; *i.e.*, these values are a result of fitting of data to the gravitational model that they have adopted and this model is different from ours. For example, Pitjeva and Pitjev (2012) attribute the change in the mean Sun-Planet distance as being a result of the secular decrease of the Sun's heliocentric gravitational parameter $G\mathcal{M}$ value whereas in our model, we assume $\dot{G}/G \equiv 0$. This is a possible source of variance between our model values and those of Pitjeva and Pitjev (2012). These are issues to be addressed in the future.

8 Lunar Recession

On a rather similar note, we have the recession of the Moon from the Earth. Recent analysis of Lunar Laser Ranging (LLR) data records panning $43\,{\rm yr}$ (*cf.* Williams et al. 2014) revealed – at a 3σ level of statistical significance (*cf.* Iorio 2011b), an increase in the mean Earth-Moon distance of about $38.08 \pm 0.04\,{\rm mm/yr}$ (Williams et al. 2004; Williams and Boggs 2009; Williams et al. 2014). Present-day models of dissipative phenomena occurring in the interior of both the Earth and the Moon are not able to explain it this (*cf.* Iorio 2011*b*). Researchers (Williams et al. 2004; Iorio 2011*a*,*b*; Xin 2011; Riofrio 2012; Acedo 2013*a*,*b*; Ziefle 2013; Iorio 2014*b*; Nyambuya 2014*f*) have proposed various mechanisms to explain this seemingly observation.

In the majority of cases, what is considered to be anomalous in the orb of the Moon is the increase in its eccentricity rate and not the $38\,\mathrm{mm}$ annual drift (*cf.* Iorio 2011*a*; Riofrio 2012; Ziefle 2013; Acedo 2014; Iorio 2014*b*, 2015). The Lunar eccentricity e_{moon} is known to increase at a rate such that $[\dot{e}_{\mathrm{moon}} = (5.00 \pm 2.00) \times 10^{-12}\ \ \mathrm{yr}^{-1}]$ (Williams et al. 2014). As already said, this increase in the eccentricity is assumed by some researchers (Iorio 2011*a*; Riofrio 2012; Ziefle 2013; Acedo 2014; Iorio 2014*b*, 2015) to be divorced from the Lunar drift. Against this line of thinking, it has been argued in reading Nyambuya (2015*a*) that these two phenomenon (*i.e.* Lunar drift and eccentricity rate) are intimately linked. Actually, according to Nyambuya (2015*a*), it has argued that the Lunar drift explains the increase in the eccentricity.

Now – surely – if the same mechanism explained in §(7), *i.e.* the mechanism that causes the Earth-Moon system to recede from the Sun is here the same mechanism that leads to the secular recession of the Moon, then, equation (26) is the appropriate equation to explain this. Alas, according to equation (26), the contribution of the Earth's azimuthal gravitational field to the Lunar recession is essentially zero because the Earth is a non-luminous object. In-order for the Earth to cause a secular recession of about $38\,\mathrm{mm/yr}$ of the Moon as required by equation (26), then, given the Moon's inclinations to the Earth's equator of 5.1° and the mean Earth-Moon distance of $\sim 3.84 \times 10^{8}\,\mathrm{m}$, the Earth would have to have a luminosity of about $14000\mathcal{L}_{\odot}$ in-order to cause the $38\,\mathrm{mm/yr}$ recession; surely this – on any scale of imagination; is absurd! Clearly, equation (26) can not explain the secular Lunar recession.

So – one may wonder – if in the present instance equation (26) fails to explain the Lunar recession, what then is the cause of the observed Lunar recession? Actually, the fact that equation (26) explains very well the secular recession of the Earth-Moon system and this same mechanism fails on the same basis to explain the secular recession of Moon, this may lead one to doubt the validity of the theory because one naturally expects that this same mechanism must in general explain the secular recession of any test body around a massive object. As demonstrated in Nyambuya (2014*f*), a closer look will reveal that this is not the case.

It has been argued in Nyambuya (2014*f*), that the cause of the Lunar secular recession may be a direct consequence of the recession of the Earth-Moon system from Sun. As the Earth-Moon system goes about its secular drift due to the Sun's azimuthal gravitational field, it losses orbital angular momentum ($\boldsymbol{J}_{\mathrm{em}}$). From the law of conservation of angular momentum, it is required that the total angular momentum be conserved, this means that the sum total of orbital angular momentum and the total spin ($\boldsymbol{S}_{\mathrm{em}}$) the Earth-Moon system must be preserved *i.e.* $\delta(\boldsymbol{J}_{\mathrm{em}} + \boldsymbol{S}_{\mathrm{em}}) \equiv 0$, therefore $\delta\boldsymbol{J}_{\mathrm{em}} = -\delta\boldsymbol{S}_{\mathrm{em}}$. That is to say, the lost orbital angular momentum is by the law of conservation of total angular momentum transferred to the spin of the Earth-Moon system. As argued there-in Nyambuya (2014*f*), this transfer leads to three things:

1. The recession of the Moon from the Earth. This effectively leads to a secular change in the spin period of the Earth-Moon system about their common center of mass.

2. The radial expansion of the Earth. This effectively leads to a secular change in the Earth day.

3. The contraction of the Moon. This effectively leads to a secular change in the Lunar day.

This reading takes the explanation given Nyambuya (2014*f*) as the cause of the Lunar secular recession. This clears the present ideas on the explanation of the recession of the Earth-Moon system has not having a fault in that this same explanation fails to explain the Lunar recession.

9 General Discussion

We have applied the ASTG-model to the observed secular drift in the mean Sun-(Earth-Moon) and Earth-Moon distances. For the Earth-Moon system, our findings are in tandem with the measurements of Standish (2005); we obtain an annual recession of about $+(5.10\ \pm\ 0.10)\,\mathrm{cm/yr}$. This prediction from the ASTG-model is seen as being a result of the orbital inclination, θ, and the Solar mass loss rate, $\dot{\mathcal{M}}_{\odot}/\mathcal{M}_{\odot}$. The tilt angle that enters in the formulae equation (19) and (20) is measured relative to the plane of the spin equator of the gravitating system. While the gravitational terms perculiar to the ASTG-model have not been used to making this prediction because these terms are too small to be significant in the present instance, the fact that the angle θ is measured in-accordance with the ASTG-model, this alone qualifies the ASTG-model as the model making this prediction. Therefore, the observed secular recession of the Earth-Moon system may very well be a result of the Solar azimuthal gravitational field. The θ-dependence is also exhibited by other gravitomagnetic models *e.g.* Acedo (2014).

Though other researchers (*cf.* Iorio 2005; Miura et al. 2009; Riofrio 2012; Acedo 2013*a*,*b*; Iorio 2014*a*) have used the same mechanism leading to the secular recession of the Earth-Moon system to explain the recession of the Moon from the Earth, the present ideas suggest that the ASTG-model's mechanism leading to the secular recession of the

Earth-Moon system may not be the same mechanism causing the observed Lunar drift. The cause of the Lunar drift may very well be a result of the transfer of orbital angular momentum of the Earth-Moon system to the spin angular momentum of the Earth-Moon system. This transfer of orbital angular momentum of the Earth-Moon system to the spin angular momentum of the Earth-Moon system is required by the law of conservation of total angular momentum of the Earth-Moon system.

Whether or not the ASTG-model will stand the test of time is something only time will tell. With the exception of Pluto because it rotates in a direction opposite to the rest of the planets, not only is the Earth receding, the ASTG-model predicts that the rest of the planets must be receding too. According to the ASTG-model's predictions, Mercury, with a predicted drift of about $1\,\mathrm{cm/yr}$, it has the least recession while Neptune is predicted to have the largest drift of about $135\,\mathrm{cm/yr}$. Pluto is predicted to be approaching the Sun as a rate of $65\,\mathrm{cm/yr}$. Thus, holding all things constant and given the predicted rates of drift of Pluto and Neptune, they must in the future collide and this will happen in about 185 billion years from today. Of cause, this is a prediction that can not be verified given our short human lifespans. What we can verify is perhaps the predicted secular drifts.

On attempts by other researchers, we have the work by Miura et al. (2009)'. The attempt by Miura et al. (2009) makes use of conventional physics by appealing to the theory of tides and the conservation of the total angular momentum. In their theory, the tides are assumed to transfer angular momentum between the Sun and planets. Effectively, they predict an annual secular change of $21.00\,\mathrm{ms/yr}$ in the Sun's spin period. As pointed in Nyambuya (2010), the ASTG-model also makes similar predictions on the spin angular momentum. With the latest modifications on the ASTG-model, there may be a need to re-visit this calculation.

Further on attempts by other researchers, we have the work by Riofrio (2012); Acedo (2013*a*,*b*). In-order to explain explain the planetary recession, Riofrio (2012); Acedo (2013*a*,*b*) use exotic ideas; ideas to do with a to a *Variable Speed of Light* (VSL) and in the case of Acedo (2013*a*,*b*), he in in-cooperates the idea that the inertial and gravitational mass depend on the age of the Universe. Acedo (2013*a*,*b*)'s model explains both the secular increase of the astronomical unit and the increase of the Moon's orbital eccentricity. Based on the work conducted in Nyambuya (2014*a*,*b*,*d*,*g*), we are very hesitant to consider a VSL theory.

In the similar vein of exotic ideas we have Iorio (2005, 2014*a*). Iorio (2014*a*) considers post-Newtonian effects of the cosmological expansion, and of the slow temporal variation of the relative acceleration rate of the cosmic scale factor and concludes that none of them is successful since their predicted secular rates of the Lunar eccentricity as they are too small by several orders of magnitude. In Iorio (2005), using the exotic Dvali-Gabadadze-Porrati multi-dimensional braneworld scenario, Iorio (2005) calculates variation of about $6.00\,\mathrm{cm/yr}$ for the Earth-Sun distance. All the above cited works are clear indications that this problem of the recession of the Earth-Moon system is an emerging *hot topic* – if not already a *hot topic*.

In-closing, allow us to say that this reading – despite the good agreement of the ASTG-model's predictions with observations; it does not purport to answer the question of what is the cause of the observed recession of the Earth-Moon system and the recession of the Moon from the Earth. The present work must be taken only as work seeking answers to this question. Before the ASTG-model can be taken seriously – and any other model for that matter; there is need for it to explain a wider range of physical phenomenon. Otherwise its explanation of the secular drift of the Earth-Moon system may just very be a chance opportunity coinciding numerically with observational values.

10 Conclusion

Assuming the correctness (*i.e.*, acceptability) of the thesis posited herein, and its consequences thereof as applied herein, we hereby make the following conclusions:

1. The observed secular recession of the Earth-Moon system may very well be a result of the Solar azimuthal gravitational field.
2. The mechanism leading to the secular recession of the Earth-Moon system may not be the mechanism causing the observed Lunar drift. The cause of the Lunar drift may very well be a result transfer of orbital angular momentum of the Earth-Moon system to the spin of the Earth-Moon system. This transfer of orbital angular momentum of the Earth-Moon system to the spin of the Earth-Moon system is required by the law of conservation of total angular momentum of the Earth-Moon system.

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