

# A DERIVED EXPRESSION OF NEWTON'S LAW OF GRAVITATION AND OF THE NEWTONIAN CONSTANT G

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## Abstract

I propose a theoretical model of Universal Gravitation based upon hypothetical mass/energy resonance waves, the intensities of which I propose to be casually analogous with those of electromagnetic waves. Using said model, I derive the Newtonian gravitational expression, from which the Newtonian gravitational constant G factors as a combination of other physical constants, resulting in an apparent value of  $6.662936 \times 10^{-11} m^3/kg s^2$ . A second resultant of the theory is a demonstration that the quantum energy states of the hydrogen atom appear related to the length of these waves, shown equal to twice the ground state orbital radius in a Bohr hydrogen atom. Additionally, I have determined apparent values for the Planck mass, length, and time independently of any determination of G.

Key words: Universal Gravitation; Newtonian Constant of Gravitation; Mass/Energy Resonance Waves; Quantum Energy States; Bohr Radius; Planck Mass; Planck Length

## I. INTRODUCTION

The value of the Newtonian gravitational constant has interested physicists for over three hundred years and, except for the speed of light, it has the longest history of measurements. It also holds the distinction of being the least accurately known physical constant of all [1]. The relative uncertainty in the precision of its measurement, let alone its absolute accuracy, being thousands of times larger than those of other important constants, such as the Planck constant and the electron charge. Even the determination by Luther and Towler [2], considered by many to be the most internally precise result ever obtained for G, has a reported uncertainty of about 130 ppm. This, in addition to the disparity in the results of G-value measurements reported by different experimenters over the years, calls into question the true value of G. Further doubt has been cast on the true value of G by relatively recent measurements from respected research teams, those measurements disagreeing wildly with the 2014 official CODATA recommended value. I will further review and reference these results in the presentation and discussion text to follow. It seems that inherent extraneous influences might complicate obtainment of a highly accurate experimental value for G. Therefore, because of these complications, a value for G derived from a theoretical gravitational model is desirable.

My major objectives, therefore, are to present a theoretical model of gravitation, based upon basic precepts, from which the expression for Newton's Universal Law of Gravitation derives as a direct consequence, and from which the Newtonian constant of gravitation factors as an expression of other highly accurate and precise physical constants as promulgated in the 2014 "CODATA Recommended Values of the Physical Constants". Others have attempted to resolve  $G$  into basic components, but few have garnered much interest, as most are simply pure numerology. However, it is neither the purpose nor intent of this paper to review these attempts or to analyze their relative merits.

Five proposals offered as the basis for the following gravitational model and theory follow.

1. The fundamental property of matter responsible for gravitation is the duality of mass and energy.
2. Because of duality, matter has a resonance structure with an associated resonance frequency, said frequency being responsible for the propagation of resonance waves and associated fields through space.
3. Resonance waves are propagated as a disturbance through the fabric of space, are transverse, oscillating in a single plane perpendicular to the direction of propagation, and are propagated with velocity  $c$ , identical with that of electromagnetic waves.
4. Resonance waves are monochromatic, having a single wavelength  $\lambda_\phi$ , and frequency  $\nu_\phi$ , identical with that of the mass/energy resonance frequency.
5. Emitters of resonance waves accelerate towards each other because of eventual interaction of their respective wave fronts and associated resonance fields that propagate through space in all directions from the center of the disturbance.

Although a mechanistic explanation of proposal five is not required insofar as the model is concerned, I suggest that the postulated resonance field associated with matter could act against the free energy of space in such a way that the emitter would experience a force directed towards its center of mass from all directions. Resonance wave front interference occurring between emitters could result in an imbalance of forces and the emitters would experience a mutual acceleration toward each other that would appear to be the result of mutual attractive forces. This is only speculation and is not essential to the theory, as number five states all that is necessary insofar as a working model is concerned.

## **II. DERIVATION OF THE NEWTONIAN GRAVITATIONAL EXPRESSION AND UNIVERSAL CONSTANT $G$**

As an introduction to further consideration of mass/energy resonance wave propagation, it is first necessary to review that for electromagnetic waves.

On the basis of classical electrodynamics, an accelerated charge  $q$  radiates energy in the form of electromagnetic waves. The electromagnetic wave consists of an electric field of intensity  $y_e$

and a magnetic field of intensity  $y_m$ , always at right angles to each other and to the direction of propagation and always numerically equal when  $q$  is expressed in esu. Their magnitudes at a distance  $S$ , the radius of a sphere containing the charge at its center, given by:

$$Y_{e,m} = \left( \frac{Aq}{Sc^2} \right) \sin \theta \quad (1)$$

Where  $q$  is the charge,  $A$  is the acceleration of the charge and  $\theta$  is the angle between  $S$  and the direction of propagation. Considering now the special case of a point mass  $m$ , carrying electron charge  $e$ , and experiencing a linear acceleration equal to that of its angular acceleration in a ground state Bohr type atom:

$$Y_{e,m} = \left( \frac{Ae}{Sc^2} \right) \sin \theta \quad (2)$$

whereby in accordance with the restriction placed on  $A$ :

$$A = \frac{V_o^2}{R_o} = \alpha^2 c^2 / R_o$$

where  $\alpha$  is the fine structure constant and  $R_o$  is the orbital radius. This, upon substitution into Eq. (2), results in:

$$Y_{e,m} = \left( \frac{\alpha^2 e}{SR_o} \right) \sin \theta. \quad (3)$$

Given that:

$$\alpha = \hbar / m_e a_o c$$

where  $\hbar$  is the reduced Planck constant,  $m_e$  the electron rest mass,  $a_o$  the electron orbital radius in a ground state Bohr hydrogen atom, and that:

$$R_o = \hbar / m \alpha c$$

Eq. (3) can be restated as:

$$Y_{em} = \left( \frac{\alpha \hbar e}{m_e a_o c} \right) \left( \frac{m \alpha c}{\hbar S} \right) \sin \theta.$$

This simplifies to:

$$Y_{e,m} = \left( \frac{\alpha^2 e m}{\hbar a_o S} \right) \sin \theta$$

that upon substitution of  $(\alpha \hbar c)^{1/2}$  for  $e$  yields:

$$Y_{e,m} = \left( \frac{\sqrt{\alpha^5} m (\hbar c)^{1/2}}{m_e a_o S} \right) \sin \theta = \left( \frac{km (\hbar c)^{1/2}}{m_e a_o S} \right) \sin \theta. \quad (4)$$

Assuming resonance waves do exist and propagate as proposed, it follows that all matter emits these waves and that emitters would be incased in spherical wave shells of increasing radii from the center of the disturbance, each wave shell being separated a distance  $\lambda_\phi$  from an adjacent wave shell. Resonance waves would obviously be absent if space were devoid of matter. However, if into this void were introduced a single emitter, resonance waves

immediately would propagate throughout space in all directions. Consequently, the propagation of resonance waves and their associated fields result in an infinite number of equal but opposing force vectors operating on the emitter, their magnitude along any direction in space at a specified distance from the center of the disturbance being a function of the resonance field intensity as proposed by:

$$Y_\phi = \left[ \frac{KM(\hbar c)^{1/2}}{m_\phi S^2} \right] \sin \theta \quad (5)$$

that structurally and dimensionally is predicated on an analogy with electromagnetic waves, Eq. (4), where  $K$  is apparently a pure number,  $M$  the mass of the emitter,  $S$  the distance from the emitter, i.e., the radius of a sphere containing the emitter at its center,  $m_\phi$  the mass associated with the resonance wave and  $\theta$  the angle between  $S$  and the direction of propagation. It is apparent from Eq. (5) that the intensity is a maximum in a direction at right angles, and zero in a direction parallel to that of the direction of propagation. Hence, the wave is transverse.

Since the resonance wave is postulated to vibrate in a single plane, the energy per unit volume  $W_\phi$  in the wave is just  $Y_\phi^2/8\pi$ , and because the wave is propagated with velocity  $c$ , the intensity  $I_\phi$ , i.e., the energy flowing per unit time through unit area  $a^*$ , perpendicular to the direction of propagation is:

$$I_\phi = cW_\phi = \frac{cY_\phi^2}{8\pi}. \quad (6)$$

Hence,  $I_\phi$  would represent the instantaneous intensity of the resonance field at any point in space. Just as the rate at which energy radiates from an accelerated charge is obtained by integrating the intensity over the surface of a sphere of radius  $S$  containing the accelerated charge at its center, so likewise may that of resonance energy from a sphere of radius  $S$  containing a point mass  $M$  at its center, as follows:

Consider a small element of surface area included between two small circles of radii  $S(\sin\theta)$  and  $S(\sin\theta + d\theta)$ , and of incremental area:

$$d(a^*) = (2\pi S^2)\sin\theta d\theta. \quad (7)$$

The differential amount of resonance energy  $d(E_\phi)$  passing through this element of surface in unit time is  $I_\phi d(a^*)$ , thus from Eqs. (6) and (7):

$$dE_\phi = I_\phi d(a^*) = \left( \frac{2\pi c Y_\phi^2 S^2}{8\pi} \right) \sin\theta d\theta. \quad (8)$$

Substitution of the right hand member of Eq. (5) into Eq. (8) for  $Y_\phi$  and simplifying results in:

$$dE_\phi = \left( \frac{K^2 \hbar c^2 M^2}{4m_\phi^2 S^2} \right) \sin^3 \theta d\theta.$$

This, when integrated between the limits of  $\theta = 0$  and  $\theta = \pi$ , yields:

$$E_\phi = \frac{K^2 \hbar c^2 M^2}{3m_\phi^2 S^2}.$$

Dividing both sides of the above expression by the velocity of propagation  $c$ , and the mass of the emitter  $M$ , results in:

$$\frac{E_\phi}{cM} = \frac{K^2 \hbar c M}{3m_\phi^2 S^2}. \quad (9)$$

This, because of the resonance field, is the acceleration experienced at any point on the radius of curvature of a spherical segment of radius  $S$  containing a point-mass at its center. It follows, therefore, that this acceleration must be identical with the surface gravity  $g$ , of a spherical body of given mass and radius  $R$  equal to  $S$ . Thus, the resonance and gravitational fields must be one.

For the simple case of two mass centers separated by distance  $S$ , upon eventual interaction of their respective resonance fields, results:

$$F = M_1 g_2 = M_2 g_1. \quad (10)$$

Thus, from Eqs. (9) and (10) it follows that:

$$2F = M_1 g_2 + M_2 g_1 = M_1 \frac{K^2 \hbar c M_2}{3m_\phi^2 S^2} + M_2 \frac{K^2 \hbar c M_1}{3m_\phi^2 S^2}.$$

Therefore:

$$F = (K^2 \hbar c / 3m_\phi^2) M_1 M_2 / S^2 = G M_1 M_2 / S^2.$$

Thus completed is the derivation of the expression for Newton's law of gravitation from the proposed model, wherefrom  $G$  factors as:

$$G = \frac{K^2 \hbar c}{3m_\phi^2}. \quad (11)$$

Since:

$$\lambda_\phi = 2\pi \hbar / m_\phi c$$

Eq. (11) can be restated as:

$$G = \frac{K^2 c^3 \lambda_\phi^2}{12\pi^2 \hbar}. \quad (12)$$

Continuing now with the well-known expression:

$$G = \frac{\hbar c}{m_{pl}^2} \quad (13)$$

where  $m_{pl}$  is termed the Planck mass, a hypothetical entity defined by Eq. (13) in terms of the experimental value of the day for  $G$ , it is shown that  $G$  can also be associated with the fine structure constant and the square of the electron esu charge by replacing  $\hbar c$  in Eq. (13) with  $e^2/\alpha$ , resulting in:

$$G = \frac{e^2}{\alpha m_{pl}^2} \quad (14)$$

that upon substitution of  $a_o m_e \alpha^2 c^2$  into the numerator for  $e^2$  and  $\hbar/a_o m_e c$  into the denominator for  $\alpha$  can be restated as:

$$G = 3\pi^2 \left( \frac{\alpha^2 m_e^2}{m_{pl}^2} \right) \times \frac{1}{3\pi^2} \left( \frac{a_o^2 c^3}{\hbar} \right). \quad (15)$$

A precise value for  $m_{pl}$  is indeterminate absent of an absolute G-value. Thus, the above term  $(\pi \alpha m_e \sqrt{3}/m_{pl})^2$  cannot be determined with certainty, but can be replaced in Eq. (15) with the inverse squared of a yet to be evaluated dimensionless quantity Q, resulting in:

$$G = \frac{a_o^2 c^3}{3\hbar(\pi Q)^2}. \quad (16)$$

Equating the right-hand members of Eqs. (12) and (16) and solving for  $\lambda_\phi$ , assuming  $K$  equal to  $1/Q$  yields:

$$\lambda_\phi = 2a_o \quad (17)$$

from which follows:

$$\lambda_\phi = \frac{2\pi\hbar}{m_\phi c} = 2 \left( \frac{\hbar}{\alpha m_e c} \right) \quad (18)$$

leading directly to:

$$m_\phi = \pi \alpha m_e \quad (19)$$

that upon substitutions of the right-hand member squared, along with  $K = 1/Q$ , into Eq. (11) results in:

$$G = \frac{\hbar c}{3\pi^2 \alpha^2 m_e^2 Q^2} = \frac{\hbar c}{(\pi \alpha m_e Q \sqrt{3})^2}. \quad (20)$$

From Eqs. (13) and (20) it therefore follows that:

$$m_{pl} = \pi \alpha m_e Q \sqrt{3}. \quad (21)$$

The Planck length  $l_{pl}$  expressed in terms of the Planck mass as  $\hbar/m_{pl}c$  in conjunction with Eq. (21), (18), and (17) results in:

$$l_{pl} = \frac{a_o}{\pi Q \sqrt{3}}. \quad (22)$$

In addition, an alternate expression found for  $l_{pl}$  that appears completely independent is:

$$l_{pl} = \frac{a_o \alpha^{10}}{32\pi^3 \sqrt{2}} = 1.614878 \times 10^{-35} m. \quad (23)$$

Upon equating the right-hand symbolic members of Eqs. (22) and (23) and solving for  $Q$  results:

$$Q = \sqrt{\frac{8}{3}} \left( \frac{4\pi}{\alpha^5} \right)^2 = 6.02213934 \times 10^{23} = N. \quad (24)$$

Thus results a very large number in terms of two fundamental constants,  $\alpha$  and  $\pi$ . The observation that this result is almost exactly equal to the 2014 recommended value for Avogadro's number  $N_A$  is inescapable, differing by only about 0.25 ppm. This is not to claim

that  $Q$  is Avogadro's number, merely that it appears to be a near numerical identity. Thus, henceforth,  $Q$ , designated simply as  $N$ , is understood to be essentially the numerical equivalent of  $N_A$ , but resulting from Eq. (24).

Now, easily obtained from Eq. (23) is the Planck time:

$$t_{pl} = \frac{l_{pl}}{c} = 5.3866550 \times 10^{-44} s$$

and from result (24) and Eqs. (20) and (21), obtained respectively are:

$$G = \frac{\hbar c}{(\pi \alpha m_e N \sqrt{3})^2} = 6.662936 \times 10^{-11} m^3 kg^{-1} s^{-2}. \quad (25)$$

and:

$$m_{pl} = \pi \alpha m_e N \sqrt{3} = 2.178290 \times 10^{-8} kg$$

As illogical as result (24) appearing in an expression for  $G$  may at first seem, it is not a fatal flaw as some may think. U.V.S. Seshavatharam and S. Lakshminarayana have co-authored papers, *Logic behind the squared Avogadro Number* [3] and *Role of Avogadro number in grand unification* [4] that strongly support numerical  $N_A$  as a fundamental constant of nature.

Considered key in supporting the validity of Eq. (25) and its derivation are Eqs. (22) through (24). Symbolically Solving Eq. (16) for  $\alpha_0$  and substituting the result, along with the symbolic solution for  $Q$  into Eq. (23) to obtain  $\sqrt{\hbar G/c^3}$ , the well-known experimental  $G$ -dependent expression for the Planck length, provides additional verification. As shown above, however, obtained now is the  $G$ -independent values for the Planck length and of  $m_{pl}$  and  $t_{pl}$  as well.

As a first consideration, when compared to mantissas of  $(6.67408 \pm 0.0031)$ ,  $(6.67384 \pm 0.0008)$ ,  $(6.67428 \pm 0.00067)$ ,  $(6.674 \pm 0.001)$ ,  $(6.673 \pm 0.01)$  for the 2014, 2010, 2006, 2002, and 1998 CODATA recommended  $G$  values respectively, it would seem that theoretical result (25) is in serious disagreement, and for that matter, with other experimental results reported in the literature. Of the above, the only exception being the 1998 recommended value that has an unusually high reported uncertainty, resulting in a lower limit that is in near perfect agreement with result (25).

State of the art techniques and instrumentation for the laboratory determination of  $G$  are capable of reasonable internal precision. Not known, however, is the absolute accuracy of these results, the reported uncertainties actually being a statement of precision in the determination and not deviations from the absolute value of  $G$ . It is not improbable that the most precise determinations contain positive errors due to the operation of extraneous influences. Therefore, one might advance a plausible argument for a lesser value of  $G$ . Supporting this are three relative recent experimental determinations designed to minimize extraneous influences, as reported by very reputable experimenters. A Russian group [5] reports obtaining a mantissa of 6.6650, while a New Zealand group [6] reports a result of 6.6659, and the German laboratory of Bugh Wuppertal [7] a value of 6.6685. While still greater than theory, these determinations

nevertheless approach result (25) quite closely within the probable margins of error. Upon inspection of Eq. (12), we see that  $G$  is directly proportional to the square of  $\lambda_\phi$ . Thus, the resonance wavelength needs be only 0.00088468 angstroms longer to bring about absolute agreement between the 2014 CODATA recommended value of  $G$  and that as calculated from Eq. (12).

### III. THE RESONANCE WAVELENGTH AND QUANTUM ENERGY STATES OF THE HYDROGEN ATOM

In deference to Eq. (17), the ground state Bohr orbital radius must be equal to one-half that of the resonance wavelength. What could be the significance, if any, of this apparent relationship? Obviously, gravity, per se, can have no direct effect upon the orbital mechanics of the electron. Nevertheless, I propose that resonance waves might interact with electrons in such fashion that they are “shepherd”, so to speak, into specific regions wherein interference between wave fronts emanating from the nucleus and an orbiting electron is occurring, and only in these regions can the electron physically exist and are electron orbits possible. More specifically, I propose that the only allowable orbital regions are those that meet the interference condition that the distance between consecutive radii must be a whole number multiple of the resonance wavelength plus one-half of a wavelength, as follows:

$$[r_{(n+1)} - r_n] = n\lambda_\phi + \lambda_\phi/2. \quad (26)$$

The only function in terms of  $\lambda_\phi$  and  $n$  expressing the distance between consecutive radii that will result in said condition is:

$$(n + 1)^2 \frac{\lambda_\phi}{2} - n^2 \frac{\lambda_\phi}{2}$$

this, upon expansion and collection of terms yields the right hand member of Eq. (26).

Therefore:

$$[r_{(n+1)} - r_n] = (n + 1)^2 \frac{\lambda_\phi}{2} - n^2 \frac{\lambda_\phi}{2}$$

which fixes the orbital radii as proportional to the square of  $n$ , just as in the Bohr derivation, such that:

$$r = n^2 a_o = n^2 \frac{\lambda_\phi}{2} \quad (27)$$

the equivalent of Eq. (17).

Throughout the following, one needs to be mindful of the fact that the resonance wavelength is the independent operator and that in relationships involving permissible radii, it is the former that determines the latter.

First, the simplest case of the Bohr atom will be considered, wherein the central mass is assumed the common center of mass of the atom and  $r_n$  and  $a_n$  are identical.

Given that the total energy of an orbiting electron is:

$$E = -e^2/2r$$

the electron orbital radius should continually decrease as the atom radiates energy thereby, in accordance with classical theory, producing a continuous spectrum. To account for the observed fact that it does not, as well as to account for the integers that appear in the Rydberg empirical formula, Bohr introduced his first and second postulates into his derivations, leading to his arriving at a total energy of the orbiting electron of:

$$E = -e^2/2r = -\frac{2\pi^2 m_e e^4}{n^2 \hbar^2}.$$

Applying Eqs. (18) and (27), along with  $\alpha$  in terms of  $e^2/\hbar c$ , results in:

$$E = -\frac{e^2}{2r} = -\frac{e^2}{2a_n} = -\frac{e^2}{n^2 \lambda_\phi} = -\frac{m_e e^4}{2n^2 \hbar^2} = -\frac{2\pi^2 m_e e^4}{n^2 \hbar^2} \quad (28)$$

Thus derived independently of his first postulate is Bohr's equation for the allowable energy states of the hydrogen atom. If an orbiting electron exists physically only in regions meeting the interference requirement of Eq. (26), then the electron is trapped and cannot continuously absorb energy and expand its radius, nor can its orbital radius continuously decay resulting in a continuous spectrum. When sufficient energy  $h\nu$  has been absorbed, the electron effectively ceases to exist for a near infinitesimal increment of time, possible the Planck time, tunneling through the prohibited regions until it re-emerges in an allowed region of higher energy, the difference in energy states corresponding to the energy absorbed. When the electron radiates energy, the exact reverse scenario must occur. The electron being in a thermodynamically less stable state of higher energy spontaneously radiates whole quanta of energy and tunnels back through the prohibited regions until it re-emerges in an allowed region of lower energy, the difference in energy states being equal to the quanta of energy radiated in accordance with Bohr's second postulate. Thus, from Eq. (28) results:

$$E_i - E_f = \frac{e^2}{\lambda_\phi} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = h\nu$$

leading to:

$$\frac{1}{\lambda} = \frac{e^2}{hc\lambda_\phi} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \frac{2\pi^2 m_e e^4}{ch^3} = R_\infty \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

Bohr's wave number equation, derived independently, wherein the Rydberg constant  $R_\infty$  for infinite mass is  $e^2/hc\lambda_\phi$ . From the above, a simple relationship of the Rydberg constant and the resonance wavelength follows:

$$R_\infty = \alpha/2\pi\lambda_\phi.$$

Accepting that the only permissible orbital radii are as given by Eq. (27) then Bohr's first postulate follows as an inescapable consequence as follows, in sequence, without comment, except to define  $\rho$  as the angular momentum of the electron in its orbit.

$$m_e v_n^2 / r_n = e^2 / r_n^2$$

$$\begin{aligned}
m_e^2 v_n^2 r_n^2 &= (m_e e^2)(n^2 \lambda_\phi / 2) = \rho^2 \\
\rho^2 &= (n^2 m_e e^2)(h / 2\pi \alpha m_e c) = \\
n^2 m_e (\alpha h c / 2\pi)(h / 2\pi \alpha m_e c) &= \\
n^2 h^2 / 4\pi^2.
\end{aligned}$$

Thus, Bohr's first postulate:

$$\rho = nh/2\pi.$$

Now the real world scenario, wherein the nucleus is not fixed, and  $r_n$  and  $a_n$  are not identical, will be considered. Removing the restriction of a fixed nucleus introduces a serious dilemma. Namely, the inescapable conclusion that the resonance wavelength must of necessity decrease with the isotopic mass of the nucleus. This because both the nucleus and electron rotate about their common center of mass with common angular momentum  $\omega$ , the new axis of rotation being on the line joining the nucleus and the electron, and dividing this line in the inverse ratio of their masses. If  $a$  is the distance of the electron from the axis of rotation and  $A$  the distance of the nucleus from the same axis, then:

$$\frac{a}{A} = \frac{M}{m_e} \quad (29)$$

and  $r$ , the distance of the electron from the nucleus is  $a + A$ , from which in conjunction with Eq. (29), follows:

$$r = \frac{a(M + m_e)}{M}.$$

Beginning with  $^1H_1$ , it is apparent that  $A$  will be at its maximum, as will  $r$ . As the isotopic mass of the nucleus increases, the common center of mass, along with  $A$  and  $a$  will all move closer to the nucleus and  $r$  will continuously decrease, until at infinite mass,  $A$  is zero, and the common center of mass and the nucleus become identical, as do  $r$  and  $a$ . Therefore, it should be obvious that if one assumes central mass emanation of the resonance wave, its length will be equal to  $2r_o$  for  $^1H_1$ , but will decrease approaching  $2a_o$  as the isotopic mass of the nucleus increases, an untenable scenario. Upon the other hand, however, the length  $a$  is always equal  $a_o$ , regardless of the isotopic mass. Thus, assuming common center of mass emanation of the resonance wave resolves the problem, as  $a_o$ , and therefore,  $\lambda_\phi$ , remain independent of the isotopic mass.

#### IV. Conclusions

Demonstrated is the achievement of the major objectives as set forth in the introduction. In addition, as set forth in the text, other interesting relationships have resulted, such as symbolic and numerical results for the Planck mass, length, and time, all independent of  $G$ . Additionally found is that, in theory, a large pure number is associated with  $G$ , expressed in terms of  $\alpha$  and  $\pi$ , that essentially is numerically identical with Avogadro's number.

The calculated  $G$ -value that results from the theory presented herein does not agree well with any but one of the CODATA recommended values from 1998 to 2014. These experimental values, for the most part, are reasonably precise; however their possible absolute error values are unknown. Therefore, the worth of an absolute comparison is debatable.

Because of the seeming disparity in experimental determinations of  $G$ , one might conclude that a highly accurate theoretical value, such as provided herein, is desirable. To measure  $G$  as accurately as other physical constants it may be necessary to design and carry out an earth-orbit determination to eliminate the gravitational gradient and other problems associated with earthbound measurements. Most likely NASA would be reluctant to fund such an enterprise, and some argue there is no practical or scientific reasons why anybody needs to know  $G$  any better than the current CODATA accepted value; even so, Nobili [8] presents the reasons and proposed methodology for doing so onboard the International Space Station.

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