La Grangian Multiplier equation in Pre Planckian space-time early universe and the Cosmological constant.

Andrew Walcott Beckwith

Physics Department, Chongqing University, College of Physics, Chongqing University Huxi Campus, No. 44 Daxuechen Nanlu, Shapinba District, Chongqing 401331, People's Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract

We look at two action integrals for the early universe. One action integral as specified by Ambjorn, et.al, 2010 is part of a quantum gravity as a sum over space-time results and another is by Padmanabhan, in 2005 gives an action integral in terms of the Cosmological Constant prominently in, as a way to obtain the Einstein Equations in general relativity. The procedure in our derivation is to say that both first integrals in the Pre-Planckian space-time are giving the same 'information' and from there to utilize an equivalence between these two first integrals as to interpret what the Lagrangian multiplier in the Abjorn et.al. first integral is saying. In addition we interpret the Ricci scalar, in the Padmabhan first integral in terms of a treatment given by Majumbdar, 2015, which has the value of being rendered in terms of scalar factors a(t). In doing so, we utilize the physical interpretation of a Lagrangian multiplier, as given by Karrabulet which is in terms of minimum conditions needed for affecting the physics (of cosmological expansion of the universe) from first principles

Key words, Ricci scalar, inflaton physics.

1. Laying out the foundations. The Two First Integrals diagrammed out.

In order to most efficiently do this problem, we will be making the assertion that in the Pre Planckian space-time that we will be examining a 1 -1 and onto relationship between the two first Integrals of references [1, 2], as given and that we will initially describe the inputs into these two First integrals. This sort of analysis will draw upon material given in [3,4,5,6, 7]

We will first discuss the first Integral of [1] with the reader told to examine the [6,7] ideas of what a Lagrangian Multiplier does, in Mechanics. Roughly put, according to [6,7] a Lagrangian multiplier invokes a constraint of how a "minimal surface" is obtained by constraining a physical process so as to use the idea of [6,7,8] which invokes the idea of minimization of a physical processes. In the case of [1], the minimization process is implicitly that, if $a(t)$ were a scale factor as defined by Roos, [9] and if g_{tt} were a time component of a metric tensor, which we will later define via [4, 10]

$$\begin{aligned}
&\int dt \sqrt{g_{tt}} V_3(t) = V_4(t) \sim 8\pi^2 r^4 / 3 \\
&\& V_3(t) = 2\pi^2 a(t)^3 / 3 \\
&\& k_2 = 9\left(2\pi^2\right)^{2/3}
\end{aligned} \tag{1}$$

Here, the subscripts 3 and 4 in the volume refer to 3 and 4 dimensional spatial dimensions, and this will lead to us writing, via [1] a 1st integral as defined by [8], in the form , if G is the gravitational constant, that

$$S_1 = \frac{1}{24\pi G} \cdot \left(\int dt \sqrt{g_{tt}} \left(\frac{g_{tt} \dot{V}_3^2}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right) \right) \tag{2}$$

This should be compared against the Padmabhan 1st integral [2] of the form , with the third entry of Eq. (3) having a Ricci scalar defined via [3] and usually the curvature $\aleph$ set as extremely small,

$$\begin{aligned}
&S_2 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4 x \cdot \left(\Re - 2\Lambda \right) \\
&\& -g = -\det g_{uv} \\
&\& \Re = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right)
\end{aligned} \tag{3}$$

Also, the variation of $\delta g_{tt} \approx a_{\min}^2 \phi$ as given by [4,10] will have an inflaton, ϕ given by [2]

$$
\begin{aligned}
& a \approx a_{\min} t^{\gamma} \\
& \Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \\
& \Leftrightarrow V \approx V_0 \cdot \exp\left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}
\end{aligned}
\tag{4}
$$

Leading to [2]

$$
\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \tag{5}
$$

The innovation we will be looking at will be in comparing a 1-1 and onto equivalence, i.e. an information based isomorphism between 1st integrals [11, 12, 13]

$$
S_1 \cong S_2 \tag{6}
$$

We will be making a simple equivalence between the two first integrals via Eq. (6) assuming that even in the Pre Planck-Planck regime that curvature $\aleph$ will be a very small part of Ricci scalar $\Re$ and that to first approximation even in the Plank time regime, that to first order

$$
\Re = 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{\aleph}{a^2} \right) \sim 6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right) \tag{7}
$$

This last approximation will make a statement as to applying Eq. (6) far easier may not be defensible, but we will use it for the time being.

2. Comparison between Eq. (2) and Eq. (3) with Eq.(5), Eq. (6) and Eq. (7)

In order to obtain maximum results, we will be stating that the following will be assumed to be equivalent.

$$
\sqrt{g_{tt}} \left(\frac{g_{tt} \dot{V}_3^2}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right) \sim \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^3x \cdot \left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right) - 2\Lambda \right) \tag{8}
$$

i.e.

$$
\sqrt{g_{tt}} \left(\frac{g_{tt} \dot{V}_3^2}{V_3(t)} + k_2 V_3^{1/3}(t) \right) \sim \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^3x \cdot \left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right) \right) \tag{9}
$$

And

$$
\sqrt{g_{tt}} \left(\lambda V_3(t) \right) \sim \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^3x \cdot (2\Lambda) \tag{10}
$$

If the term Λ is indeed a constant (.i.e. we avoid Quinessence, and the vacuum energy is invariant), then Eq. (10) puts a profound restriction upon g_{tt} which will be elaborated upon in the next section. I.e. for the sake of Argument we will make the following assumptions which may be debatable, i.e.

$$\sqrt{-g} \text{ is approximately a constant} \tag{11}$$

For extremely small time intervals (in the boundary between Pre Planckian to Planckian physics boundary regime).

$$g_{tt} \sim \delta g_{tt} \approx a_{\min}^2 \phi \tag{12}$$

The next section will be investigating the physical implications of such assumptions.

3. What we can extract in physics, if Eq. (9), Eq. (10), Eq. (11) and Eq.(12) hold ?

Simply put a relationship of the Lagrangian multiplier giving us the following:

$$\lambda \sim \frac{1}{\kappa}\sqrt{\frac{-g}{\left(\delta g_{tt} \approx a_{\min}^2 \phi\right)}} \cdot \Lambda \tag{13}$$

If the following is true, i.e. in a Pre Plankian to Planckian regime of space-time

$$\sqrt{\frac{-g}{\left(\delta g_{tt} \approx a_{\min}^2 \phi\right)}} \approx \text{ constant} \tag{14}$$

Then what has been done is to conflate the Lagrangian as equivalent to Λ which if Λ is also a constant is implying that the cosmological constant is obtaining for us the consomological constant value chosen as a precursor for (DE ?) expansion of the universe, as given in the scale factors as of Eq. (9) and Eq. (8). i.e. what we are inferring then is similar to a result assumed by Padmanabhan, in [14]

.

3 . Conclusions

But what is noticeable, is that the inflaton equation as given by Padmanabhan [2] hopefully will not be incommensurate with the physics of the Corda Criteria given in the Gravity's breath document [15]. Keep in mind the importance of the result from reference [16] below which forms the core of Eq. (15) below

$$N_{e-foldings} = -\frac{8\pi}{m_{Planck}^2} \cdot \int_{\phi 1}^{\phi 2} \frac{V(\phi)}{\left(\frac{\partial V(\phi)}{\partial \phi}\right)} d\phi \geq 65 \quad (15)$$

Furthermore, we should keep in mind the physics incorporated in [16,17], i.e. as to the work of LIGO. i.e. it is important to keep in mind that in addition, that [18] has confirmed that a subsequent analysis of the event GW150914 by the LSC constrained the graviton Compton wavelength of those alternative theories of gravity in which the graviton is massive and placed a level of 90\% confidence on the lower bound of 10^{13} km for a Compton wavelength of the graviton. Doing these sort of vetting protocols in in line with being consistent with investigation as to a real investigation as to the fundamental nature of gravity. This is a way of also look I.e. is this a way to show if general relativity is the final theory of gravitation. I.e., if massive gravity is confirmed, as given in [20], then GR is perhaps to be replaced by a scalar-tensor theory, as has been shown by Corda.

We can say though that if we do confirm Eq. (13) and Eq. (14) that such observations may enable a more precise rendering of settling the issues brought up by references [15], and [20].

6.. **Acknowledgements**

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

References

[1] Ambjorn, J. Jurkiewicz, J. and Loll, R., "Quantum Gravity as Sum over Space-times", pp, 59-124, of New Paths Towards Quantum Gravity, ed. Boob-Bavnbek, B., Esposito, G. Lesch, M. Lecture Notes in Physics 807, Springer-Verlag, Berlin-Heidelberg, Federal Republic of Germany, 2010

[2] Padmanabhan, T., "Understanding Our Universe; Current Status, and Open Issues", pp 175-204, of , " 100 Years of Relativity , Space-Time, Structure: Einstein and Beyond", World Scientific, P.T.E. LTD, Singapore, Republic of Singapore, 2005. http://arxiv.org/abs/gr-qc/0503107

[3] Majumdar, D. "Dark Matter, An Introduction", CRC Press, Boca Racon, LA., USA 2016

[4] Beckwith, A. (2016) Gedanken Experiment for Refining the Unruh Metric Tensor Uncertainty Principle via Schwarzschild Geometry and Planckian Space-Time with Initial Nonzero Entropy and Applying the Riemannian-Penrose Inequality and Initial Kinetic Energy for a Lower Bound to Graviton Mass (Massive Gravity). *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 106-124. doi: 10.4236/jhepgc.2016.21012

[5] Camara, C.S., de Garcia Maia, M.R., Carvalho, J.C. and Lima, J.A.S. (2004) Nonsingular FRW Cosmology and Non Linear Dynamics. Arxiv astro-ph/0402311 Version 1, Feb 12.

[6] Karabulut, H., "The physical meaning of Lagrange multipliers" , **European Journal of Physics, 27, 709-718 (2006),** https://arxiv.org/abs/0705.0609

[7] Spiegel, M. "Theory and Problem of Theoretical Mechanics:, Schaum's Outline Series, McGraw Hill, San Francisco, CA., USA, 1980

[8] L D Landau , E.M. Lifshitz ; "Mechanics, 3rd edition", in Course in Theoretical Physics, Volume 1, Elsevier Books, Boston, Massachusetts, 2005 (printing dates)

[9] Roos, M., "Introduction to Cosmology, 3rd edition, Wiley Scientific, Hoboken, NJ, USA, 2003

[10] Giovannini, M. (2008) A Primer on the Physics of the Cosmic Microwave Background. World Press Scientific, Hackensack.New Jersey, USA http://dx.doi.org/10.1142/6730

[11] *Awodey, Steve (2006). "Isomorphisms". Category theory. Oxford University Press. p. 11. ISBN 9780198568612.*

[12] *Vinberg, Ėrnest Borisovich (2003). A Course in Algebra. American Mathematical Society. p. 3. ISBN 9780821834138.*

[13] Judson, T. "Abstract Algebra, Theory and Applications", P:WS P:Ublishing Company 20 Park Plaza, Boston, MA, USA, 1994

[14] Padmanabhan, T., https://www.physics.rutgers.edu/grad/690/Padmanabhan-2003.pdf

[15] . Corda, C. " Gravity's Primordial Breath, " Electronic Journal of Theoretical physics, EJTP 9, No. 26 (2012) 1–10; http://www.ejtp.com/articles/ejtpv9i26p1.pdf

[16] Freese, K. "Natural Inflation", pp 408- 428, of "Particles, Strings, and Cosmology, Northeastern University , March 25-30, 1991, edited by Nath, P., and Recucroft, S. World Scientific Publishing company, Pte. Ltd, Singapore, Republic of Singapore, 1992

[17] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) ***; "***Observation of Gravitational Waves from a Binary Black Hole Merger"**;** Phys. Rev. Lett. **116**, 061102 – Published 11 February 2016

[18] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)***; "***GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence", Phys. Rev. Lett. **116**, 241103 – Published 15 June 2016

[19] B. P. Abbott et al. "Tests of general relativity with GW150914", https://arxiv.org/pdf/1602.03841.pdf

[20] Corda, C. "Interferometric detection of gravitational waves: the definitive test for General Relativity", Int. J. Mod. Phys. D18:2275-2282,2009; https://arxiv.org/abs/0905.2502