

CONJECTURE ON KISSING NUMBERS AND UNIFORM n -POLYTOPES

PABLO ÁLVAREZ DOMÍNGUEZ

CANGAS DO MORRAZO, 25TH OF DECEMBER, 2016

ABSTRACT

The main objectives of this little work is to propose a conjecture about a condition that every Kissing Number must satisfy and to study a little bit its most basic direct consequences if it were proven true. It can seem that nowadays there is not enough knowledge to conjecture it (mainly because the little information we have about Kissing Numbers). However, the few known examples we have about this type of numbers satisfy it.

It is a trivial deduction that any counterexample would nullify the statement.

THE CONJECTURE

Conjecture 1:

The n -th Kissing Number correspond to the number of vertices of at least one uniform (or regular, in some cases for $n \leq 4$) n -polytope.

Conjecture 2:

Let an n -sphere be *covered* by the maximum possible amount of kissing n -spheres. Then, those kissing n -spheres can be arranged in a way that their centers are the vertices of an uniform (or a regular, in some cases for $n \leq 4$) n -polytope.

Corollary:

Conjecture 1 and Conjecture 2 are equivalent.

DIRECT IMPLICATIONS

Theorem 1:

Let $k(n)$ be the n -th kissing number. Then, if Conjecture 1 holds,

n	$k(n)$
1	2
2	6
3	12
4	24
5	40
6	72
7	126 or 128
8	240

Proof:

Results obtained just by applying the bounds on [1], [2], [3], [4], [5] and [6] and looking for a possible number of vertices of the corresponding n -polypode that fits them

REFERENCES

- [1] O. R. Musin (2003). "*The problem of the twenty-five spheres*". Russ. Math. Surv. 58 (4): 794–795.
- [2] Pfender, Florian; Ziegler, Günter M. (September 2004). "*Kissing numbers, sphere packings, and some unexpected proofs*". Notices of the American Mathematical Society: 873–883.
- [3] Levenshtein, Vladimir I. (1979). "*О границах для упаковок в n -мерном евклидовом пространстве*". Doklady Akademii Nauk SSSR. **245** (6): 1299–1303.

- [4] Odlyzko, A. M., Sloane, N. J. A. '*New bounds on the number of unit spheres that can touch a unit sphere in n dimensions*'. J. Combin. Theory Ser. A 26 (1979), no. 2, 210—214.
- [5] В. А. Зиновьев, Т. Эрикссон (1999). '*Новые нижние оценки на контактное число для небольших размерностей*'. Пробл. Передачи Информ. 35 (4): 3–11.