Constructing a mathematical framework for the ensemble interpretation based on double-slit experiments

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The ensemble interpretation attributes the wave appearances of particles to their statistical characteristics. This has increasingly interested scientists. However, the ensemble interpretation is still not a scientific theory based on mathematics. Here, based on double-slit experiment, a mathematical framework for the ensemble interpretation is constructed. The Schrödinger equation and the de-Broglie equation are also deduced. Finally, a proof for the hypothesis of the Feynman path integral is provided. Analysis shows that the wave appearances of particles is caused by the statistical properties of these particles; the nature of the wave function is the average or the sum of the least action for the particles in a position.

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INTRODUCTION.

Double-slit experiments indicate that particles exhibit wave properties[1, 2, 3, 4]. Many interpretations have been proposed to explain the wave features appearing in double-slit experiments[5, 6]. However, scientists have not completely accepted any of these interpretations.

The Copenhagen Interpretation attributed the particle's wave appearances to particle's duality and regarded the wave as a probabilistic wave[7, 8, 9]. The remarkable feature of the Copenhagen Interpretation was that it denied a particle's classical trajectory. Most scientists have accepted the Copenhagen Interpretation's viewpoint of a "probabilistic wave", but its other viewpoints, a few scientists still doubt. American physicist Alfred Landé believed a successful interpretation should be classical[10]. Karl Popper believed there was no need to do away with the concept of a particle's classical trajectory[11]. Einstein was not satisfied with the Copenhagen Interpretation. He proposed the ensemble viewpoint[12] and believed the wave function described the properties of the ensemble. Leslie E. Ballentine improved and further developed the ensemble interpretation[13]. Max Jammer affirmed that, in practical work, physicists actually use the logic and terminology of the ensemble interpretation, whether or not they accepted it[14]. Today, more and more scientists have accepted the ensemble interpretation. However, the ensemble interpretation is a philosophical discussion not a theory based on principles of physics and mathematics.

In the micro world, many physical quantities are nonnegative, near-zero numbers. Particularly in an ensemble system formed by microscopic particles. Here, based on the features of the nonnegative, near-zero numbers, a mathematical description for particles' wave properties is provided. Also, Using the features of the non-negative near-zero numbers, other important conclusions conforming to quantum mechanics are obtained, such as the de-Broglie equation and the Schrödinger equation being deduced. Finally, a proof for the hypothesis of the Feynman path integral is provided.

DOUBLE-SLIT EXPERIMENT DESCRIPTION.

In FIG. 1(a), identical particles are sent out from source S. They pass through slit A or B arriving at screen x. O is the origin of screen x. S, O' and O are on one line with O' the mid point of slits A and B on board m. Board m is parallel to x with m and x perpendicular to SO. To simplify this problem, consider that the movement direction of particles, the double-slit, and screen x are on the same plane. Experiments have shown that when only one slit is opened, a diffraction phenomena exists on screen x as shown by either the solid curve or the dotted curve in (b) of FIG. 1. However, when the two slits are both opened simultaneously, an interference phenomenon occurs as in (c) of FIG. 1.

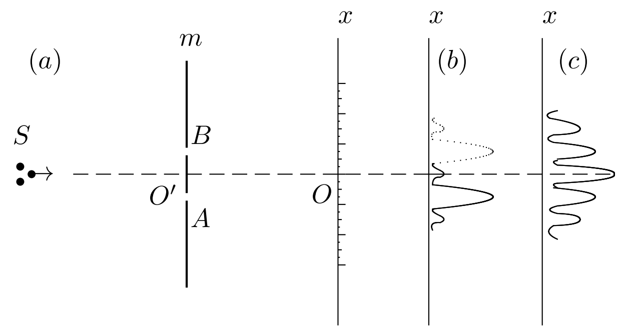

FIG. 1. Double-slit experiment with particles. *a*) Particles begin at S and pass through slit A or B arriving at screen x. *b*) When only slit A is open,

the particles' position (vertical orientation)-density (horizontal orientation) curve is described by the solid curve. When only slit B is open, the position-density curve is described by the dotted curve (the curve formed by slit A partially overlaps the dotted curve). c) When the two slits are opened simultaneously, the position-density curve appears as an interference pattern.

SOME LEMMAS.

Lemma 1. A nonnegative, near-zero number can be represented by one item of its complex Fourier series (symbols not included).

Proof. For a number $x > 0$, its Fourier series on the interval $[-a, a]$ (a is a constant and $a > 0$) is:

$$x = 2(\frac{\sin(\pi x/a)}{\pi/a} - \frac{\sin(2\pi x/a)}{2\pi/a} + \frac{\sin(3\pi x/a)}{3\pi/a} - \frac{\sin(4\pi x/a)}{4\pi/a} + ...) \quad (1)$$

Because $\lim_{x\to 0} \frac{\sin(n\pi x/a)}{n\pi x/a} = 1$, for $n \in Z^+$. So, when x is near-zero number, we have:

$$x = \frac{\sin(n\pi x/a)}{n\pi/a}$$

Due to $f(x) = x$ is an element of $L^2([-a, a])$, its Fourier series in complex form can be written as:

$$x = \sum_{n=-\infty}^{n=+\infty} \alpha_n e^{\frac{in\pi x}{a}} \quad (2)$$

Where $\alpha_n = \frac{1}{2a}\int_{-a}^{a} x e^{\frac{-in\pi x}{a}} dx$ for $n \in Z$. When $n = 0, \alpha_n = 0$; when $n = 1, 3, 5, 7, 9, ..., \alpha_n = -\frac{ia}{n\pi}$; and when $n = 2, 4, 6, 8, 10, ..., \alpha_n = \frac{ia}{n\pi}$.

Considering

$$Re(\alpha_n e^{\frac{in\pi x}{a}}) = \pm\frac{\sin(n\pi x/a)}{n\pi/a}$$

and

$$Re(\alpha_n e^{\frac{in\pi x}{a}}) = Re(\alpha_{-n} e^{\frac{-in\pi x}{a}}),$$

when $x \to 0$ and no considering signs, we can get:

$$x = Re(\alpha_n e^{\frac{in\pi x}{a}}) \quad (3)$$

The purpose of converting x into its Fourier series is to found out the changing law of x. When $x \to 0$, $\alpha_n e^{\frac{in\pi x}{a}}$ is the unique projection of x in orthogonal basis $\{\frac{1}{\sqrt{2a}}e^{ik\pi x/a}, k = ..., -1, 0, 1, ...\}$. So, in order to study the property of near-zero number x, it's enough to study the property of $\alpha_n e^{\frac{in\pi x}{a}}$.

On the other hand, when $x \to 0$, $\alpha_n \cos(\frac{n\pi x}{a})$ can be viewed as a constant. If we add a constant into x ($=\alpha_n \sin(\frac{n\pi x}{a})$), the changing law of x will keep no changing. So, when $x \to 0$, if just studying the changing law of x, we can use $\alpha_n e^{\frac{in\pi x}{a}}$ to replace x.

To sum up, when no account is taken of symbols, a nonnegative, near-zero number can be regarded as one item of its complex Fourier series.

Definition:

For a free particle, its action A is defined as:

$$A = \int_{t1}^{t2} (T - U)dt = \int_{t1}^{t2} \frac{1}{2}mV^2 dt \quad (4)$$

Where:

T and U are the kinetic energy and potential energy of the particle, respectively. In double slit experiment, supposing $U = 0$;

m and V are the mass and speed of the particle, respectively;

$t1$ and $t2$ are the start moment and end moment of the particle, respectively.

The principle of least action tells us that, a particle moves along its least action orbit.

In single slit diffraction experiment, by eq. (4), the average least action of particles in one diffraction fringe position is $L = \frac{1}{2}PR = \frac{1}{2}mVR$ or $L = Et$. Its component in screen direction is $\varepsilon = \frac{1}{2}pr = \frac{1}{2}mvr$ or $\ell = \varepsilon t$.

Where

P : average momentum of particles in one fringe position.

p : component of P in screen direction.

V : average speed of particles in one fringe position.

v : component of V in screen direction.

R : average displacement of particles in one fringe position.

r : component of R in screen direction.

E : average kinetic energy of particles in one fringe position.

ε : component of E in screen direction.

$t = t2 - t1$: average time particles taking from slit to one fringe position.

Discussion:

In Fig. 1, for single slit experiment, the least action of every particle at one diffraction fringe position is gotten by eq. (4). Usually, only their component in screen direction are near zero, nonnegative numbers and can be represented only one its Fourier item.

For example, for the electron single slit experiment, the component of the average least action of electrons in screen direction is $mvr/2$.
Where

$m = 9.1 \times 10^{-31} kg$;

v is the component of particle speed V in screen direction. If the angle between V and line $O'O$ is θ, then $v = V\sin(\theta)$. For the mid fringe position, usually θ has order of magnitude 10^{-3} radian and V has order of magnitude $10^3 m/s$. So, we can conclude that the order of magnitude for v is about 10^0.

r is the component of particle displacement in screen direction. Its value is equal to a quarter of the fringe width and has an order of magnitude $10^{-3}m$.

Thus, the estimated order of magnitude for $mvr/2$ is $10^{-34} J \cdot s$, it is a near-zero, nonnegative number and can be replaced by its one Fourier item.

Lemma 2. In Fig. 1, for particle single slit experiment, x^* is the average least action (or its component) for particles in one diffraction fringe position and x is the average least action (or its component) for particles in all fringe positions, if x^* can be replaced by one Fourier item of x , then the relative probability density of particles in the position can be described by $|x^*|^2$.

Proof [15]:

At one diffraction fringe position, supposing the least action of particle is a nonnegative, near-zero number group $X = [x_1, x_2, ..., x_N]$ with average value x^*. These numbers can be viewed as random variables with mathematical expectation value $E(X) = x^*$. By the definition of variance, the variance value $D(X)$ of these numbers is:

$$D(X) = \sum_{i=1}^{i=N} \frac{x_i^2}{N} - (x^*)^2$$

In above equation, if x^* increases Δx with $x_i (i = 1, 2, ..., N)$ increasing $\Delta x/N$, $(x^*)^2$ will become $(x^* + \Delta x)^2$ with increment $2x^*\Delta x + (\Delta x)^2$, and $1/N \sum_{i=1}^{i=N} x_i^2$ will become $1/N \sum_{i=1}^{i=N} (x_i + \Delta x/N)^2$ with increment $2x^*\Delta x/N + (\Delta x)^2/N^2$. When N is very big and x^* as well as Δx are very small, the increment of $(x^*)^2$ is far more than the increment of $1/N \sum_{i=1}^{i=N} x_i^2$.

So, when N is big enough and x^* is a near zero, nonnegative number, we can say, the greater $(x^*)^2$ is, the smaller $D(X)$ will be. The value of $D(X)$ reflects the degree of concentration for random variables : the less $D(X)$ is, the larger the degree of the number concentration will be. By the least action principle, the particle position is determined by the least action of the particle. Thus, we can say, in the diffraction fringe position, the larger $(x^*)^2$ is, the bigger the particle density will be. Meaning $(x^*)^2$ can describe the particle density in the fringe position. If x^* is represented as a complex number, we can say, $|x^*|^2$ describes the particle density in one fringe position.

Let's considering another side of the question. Let

$$f_N(x) = \sum_{k=-N}^{k=N} \alpha_k e^{\frac{ik\pi x}{a}}$$

as well as

$$g_N(x) = \sum_{k=-N}^{k=N} \beta_k e^{\frac{ik\pi x}{a}}$$

be the partial sum of the Fourier series of f and g, respectively. When $N \to \infty$, $f_N \to f$ and $g_N \to g$ in $L^2[-a, a]$, we have:

$$< f_N, g_N >= \sum_{k=-N}^{k=N} \sum_{n=-N}^{n=N} \alpha_k \overline{\beta_n} < e^{ik\pi x/a}, e^{in\pi x/a} >$$

Since $\{\frac{1}{\sqrt{2a}} e^{ik\pi x/a}, k = ..., -1, 0, 1, ...\}$ is orthogonal, therefore:

$$< f_N, g_N >= 2a \sum_{n=-N}^{n=N} \alpha_n \overline{\beta_n}$$

When $N \to \infty$, $< f_N, g_N > \to < f, g >$, so we have:

$$< f, g >= 2a \sum_{n=-\infty}^{n=\infty} \alpha_n \overline{\beta_n}$$

In the set of $L^2[-a,a]$, we can define the inner product as:

$$< f, g >= \int_{-a}^{a} f\overline{g}dx$$

Let $f(x) = g(x) = x$, then we get:

$$\int_{-a}^{a} x^2 dx = 2a \sum_{n=-\infty}^{n=\infty} |\alpha_n|^2$$

Thus,

$$\sum_{n=-\infty}^{n=\infty} |\frac{\sqrt{3}}{a}\alpha_n|^2 = 1$$

Let $\psi_n = \frac{\sqrt{3}}{a}\alpha_n e^{in\pi x/a}$, then:

$$\sum_{n=-\infty}^{n=\infty} |\psi_n|^2 = 1 \quad (5)$$

Due to $x_i^*(i \in Z)$ is one item of the Fourier series for x, we have:

$$\sum_{n=-\infty}^{n=\infty} |\frac{\sqrt{3}}{a}\alpha_n|^2 = \sum_{i=-\infty}^{i=\infty} |\frac{\sqrt{3}}{a}x_i^*|^2 = 1$$

Letting $\psi_i = \frac{\sqrt{3}}{a}x_i^*$, we get:

$$\sum_{i=-\infty}^{i=\infty} |\psi_i|^2 = 1$$

Above equation tells us, $|x_i^*|^2$ describes relative probability for particles falling in the fringe position. Therefor, when considering $|x_i^*|^2$ describes particle density, we can say, $|x_i^*|^2$ describes relative probability density for particles falling in the fringe position.

Discussion:

Due to $|\frac{\sqrt{3}}{a}x_i^*|^2$ presents particle probability density in one fringe position, we can use it to estimate the order of magnitude for a.

Taking the electron single-slit experiment as an example. When x_i^* represents the least action component in screen direction, x_i^* has the order of magnitude 10^{-34} and can be replaced by one Fourier item. In such situation, $|\frac{\sqrt{3}}{a}x_i^*|^2$ describes particle probability density.

In single slit experiment, 90 percent particles fall in the mid fringe position. Meaning, $|\frac{\sqrt{3}}{a}10^{-34}|^2 = 0.9$. Thus, the estimated order of magnitude for a should be 1.8×10^{-34}. It's dimension should be $kg \cdot m^2/s = J \cdot s$.

If x_i^* can not be replaced by one Fourier item, we can not say that $|\frac{\sqrt{3}}{a}x_i^*|^2$ describes particle probability density.

DOUBLE-SLIT EXPERIMENT ANALYSIS.

A. Mathematical description for particle probability density:

In single slit experiment, after passing slit, the average least action of particles from slit to one diffraction fringe position is $L = PR/2$. Usually, its component ℓ ($= 1/2pr$ or εt) in screen direction, is a near-zero, nonnegative number. By lemma 1, when $\ell \to 0$, ℓ can be replaced by its fourier item $\alpha_n e^{in\pi\ell/a}$. By lemma 2, the relative probability density of particles in the fringe position can be described by $|\alpha_n e^{in\pi\ell/a}|^2$.

Discussion:

(1)Generally, the average least action in the mid fringe position is larger than in the side fringe position. The reason is, when particles passing through slit, slit will decrease the kinetic energy of particles. The lose of kinetic energy for side particles is much larger than mid particles.

(2)In one fringe position, particle's least action is approximately equal.

B. Diffraction analysis: Our mathematical description should fit to the experiment. When $|\psi_n|^2 = |\alpha_n e^{in\pi\ell/a}|^2$ describe the relative probability density of particles in one diffraction fringe position, $\psi_n = \alpha_n e^{in\pi\ell/a}$ has the following properties:

(1) $|\psi_n|^2$ is the function of integers. This means the diffraction fringe positions of particles are discrete.

(2) $|\psi_n|^2 > |\psi_{n+1}|^2 (n = 0, 1, ...)$ corresponding to: particle probability density becoming smaller when leaving the mid fringe position.

(3) ψ_n is single valued.

(4) $\sum_{n=-\infty}^{n=\infty} |\psi_n|^2 = constant$, this property fits to: the sum of the particles' probability density on the screen is a constant.

It is worth noting that, in the diffraction experiment, the probability density distribution of particles is not continuous. So, $\psi_n = \alpha_n e^{in\pi\ell/a}$ makes sense only for integer n. In order to simplify analysis, we often view ψ_n as a continuous function by proper extension. But we should remember that, in the position between diffraction fringes, the probability density of particles is near zero.

C. Interference analysis: Particles which pass through slits can be classified as three types: Type (I), particles which only pass through slit A and arrive in the screen position x meaning the probability of passing through slit A is 1; Type (II), particles which only pass through slit B and arrive in the screen position x meaning the probability of passing through slit B is 1; Type (III), the probability of passing through slit A is $c_1 (0 < c_1 < 1)$, and the probability of passing through slit B is c_2 $(0 < c_2 < 1)$.

When only slit A is open, the particles in the neighborhood of x(x is interference position) consist of Type (I) particles. In this case, the particle's relative probability density in position x is $|\psi_A|^2$. When only slit B is open, the particles in the neighborhood of x consist of Type (II) particles. In this case, the particle's relative probability density in position x is $|\psi_B|^2$.

When A and B open simultaneously, Type (III) particles should be taken into account: if they pass through slit A, their particle relative probability density dedication to position x is $c_1^2|\psi_A|^2$; if they pass through slit B, their particle relative probability density dedication to position x is $c_2^2|\psi_B|^2$. The average dedication in the neighborhood of x should be the geometric average of $c_1^2|\psi_A|^2$ and $c_2^2|\psi_B|^2$, that is:

$$\sqrt{c_1^2|\psi_A|^2\, c_2^2|\psi_B|^2} = c_1 c_2 |\psi_A||\psi_B|.$$

So, when A and B open simultaneously, the relative probability density of particles in the neighborhood of x is:

$$|\psi_A|^2 + |\psi_B|^2 + c_1 c_2 |\psi_A||\psi_B|$$

The above analysis means that, when A and B open simultaneously, the particles density will increase, i.e., a interference phenomenon will appear.

If experimental conditions make $c_1 c_2 |\psi_A||\psi_B| = 2Re < \psi_A, \psi_B >$ hold, the following equation will hold:

$$|\psi_A|^2 + |\psi_B|^2 + c_1 c_2 |\psi_A||\psi_B| = |\psi_A + \psi_B|^2$$

This means that, under certain conditions, when slits A and B are opened simultaneously, $|\psi_1 + \psi_2|^2$ can be used to describe the particle relative probability density in the neighborhood of position x.

DERIVATION OF SCHRÖDINGER EQUATION

In single slit experiment, ℓ, the average particle least action component in the screen direction for one fringe position, is a nonnegative near-zero number and can be replaced by one Fourier item. By lemma 1, ℓ can be written as $\alpha_n e^{in\pi\ell/a}$.

For the average least action ℓ, its energy form is $(\alpha_n e^{i\pi n\varepsilon t/a})$ and its momentum form is $(\alpha_n e^{i\pi\frac{1}{2}npr/a})$.

To simplify equation form, $n\varepsilon$ can be merged into ε, then ε should correspond to $1\varepsilon, 2\varepsilon, 3\varepsilon, ...$; and $\frac{1}{2}nr$ can be merged into r, this means that, when calculating, r should be discrete and should be replaced by $\frac{n}{2}r$. Obviously, when we say the least action of the particles in one diffraction fringe position, the average value of the energy form and momentum form should be taken into account simultaneously. Here, we use the geometric average value of the two forms of least action:

$$\sqrt{\alpha_n e^{i\pi pr/a}\overline{\alpha_n e^{i\pi\varepsilon t/a}}} = \alpha_n e^{i\pi(pr-\varepsilon t)/(2a)}$$

or

$$\sqrt{\alpha_n e^{i\pi\varepsilon t/a}\overline{\alpha_n e^{i\pi pr/a}}} = \alpha_n e^{i\pi(\varepsilon t-pr)/(2a)}$$

Supposing $\psi(r,t)$ is the average least action of particles in fringe position r, then, we have:

$$\psi(r,t) = Ce^{i\pi(pr-\varepsilon t)/(2a)} \tag{6}$$

or

$$\psi(r,t) = Ce^{i\pi(\varepsilon t-pr)/(2a)} \tag{6$'$}$$

Where $C = \alpha_n$ $(n \in Z)$.

In (a) of FIG. 1, for particle single slit experiment, when the screen moves forward or backward a small distance along line $O'O$, the time particles take to arrive in the neighborhood of a diffraction fringe position x will increase or decrease. After particles passing the slit, for one fringe position, using eq. (6), the average least action $\psi(r,t)$ is

$$\psi(r,t) = Ce^{i(pr-\varepsilon t)\pi/(2a)}.$$

When the screen moves forward or backward a small distance, the change of the least action is:

$$\delta\psi = \frac{\partial\psi}{\partial r}dr + \frac{\partial\psi}{\partial t}dt$$

By the principle of least action, we get:

$$\delta\psi = 0.$$

Thus,

$$\frac{\partial\psi}{\partial t} = -\frac{\partial\psi}{\partial r}\frac{dr}{dt}.$$

Using equation (6), we get $\frac{\partial\psi}{\partial t} = \frac{\varepsilon\pi}{2ai}\psi$ and $\frac{\partial}{\partial r}(\frac{\partial\psi}{\partial r}) = -(\frac{p\pi}{2a})^2\psi$. So, the relationship between $\frac{\partial\psi}{\partial t}$ and $\frac{\partial\psi}{\partial r}$ is:

$$\frac{\partial\psi}{\partial t} = \frac{ia}{\pi m}\frac{\partial}{\partial r}(\frac{\partial\psi}{\partial r}).$$

Letting $\hbar = \frac{2a}{\pi}$, the above equation can be written as:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial r^2}. \tag{7}$$

Comparing equation (7) with the Schrödinger equation for free particles, the same form and properties are found. Both of them describe the relationship between the particle position, momentum and kinetic energy. So, equation (7) corresponds to the Schrödinger equation for free particles.

Discussion:

(1) In single slit experiment, taking the mid fringe position as an example, if the angle between particle average momentum P and $O'O$ is θ, then, $P = p/\sin(\theta)$ and $R = r/\sin(\theta)$ hold. Particle average kinetic energy $E = \varepsilon/\sin^2(\theta)$ holds. So, particle average least action $\Psi(R,t) = \psi(r,t)/\sin^2(\theta)$ also holds. Thus, we can get the same equation as eq. (7):

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2}. \tag{7'}$$

Above equation means that, when particles exhibit wave feature, both average least action and its average component satisfy eq. (7). Obviously, $|\Psi|^2$ also describes particle relative probability density.

(2) When we use particle average least action $\Psi(R,t) = C'e^{i(PR-Et)\pi/(2a)}$ to derive eq. (7'), we just need $\hbar = \frac{2a}{\pi}$ holds.

(3) If particles in potential field $U(r)$ appear diffraction fringe, particles in the same fringe position will get an equal additional displacement. In this case, if we view $\Psi(R,t) = C'e^{i(PR-(E-U)t)\pi/(2a)}$ as particle average least action and letting $\hbar = \frac{2a}{\pi}$, we can get the following equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2} - U\Psi$$

If we select a proper zero potential energy point for $U(r)$, above equation can be written as:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2} + U\Psi$$

NEW UNDERSTANDING ON DE-BROGLIE EQUATION

In eq. (6), $|\psi(r,t)|^2$ describes the relative probability density of particles in one diffraction fringe position. That is, the greater $|\psi|^2$ is, the bigger the particle relative probability density will be. Also, we can say: the greater $|\psi|^2$ is, the smaller the relative displacement between a particle's position and the particles's mean position will be. Noticing that, when $|\psi|^2$ becomes bigger, $-|\psi|$ will be smaller, the relative probability density of particles will become larger. Thus, the smaller $-|\psi|$ is, the lesser the relative displacement of particles will be. So, $-|\psi|$ can describe relative displacement of particles. Usually, we think ψ can reflect the relative displacement between a particle's actual screen position and the particles's mean position.

Let's compare ψ with the classical harmonic wave function.

Supposing a classical simple harmonic wave with wavelength λ and frequency ν is propagating in the positive direction of coordinate x, then after time t, the relative displacement of an infinitesimal quantity of the medium to the balance position x is:

$$Y(x,t) = Ae^{i\pi(\frac{2}{\lambda}x-2\nu t)} \tag{8}$$

When comparing equation (6) with the classical simple harmonic wave function above, many similar characteristics are found. These include: 1) reflecting the relative displacement between particles screen position and the mean position versus describing the relative displacement between an infinitesimal quantity of the medium and the balance position, 2) having the same equation form, and 3) causing the same physical phenomena (diffraction and interference). Therefore, equation (6) has the properties of equation (8), and can be considered a classical simple harmonic wave function.

Comparing eq. (6) and eq. (8), by dimension corresponding principle, $\frac{p\pi}{2a}$ corresponds to $\frac{2\pi}{\lambda}$ and $\frac{\varepsilon\pi}{2a}$ corresponds to $2\pi\nu$. If the two corresponding relationships are regarded as equal. We have:

$$\frac{p\pi}{2a} = \frac{2\pi}{\lambda}, \quad \frac{\varepsilon\pi}{2a} = 2\pi\nu,$$

or

$$\lambda = \frac{4a}{p}, \quad \nu = \frac{\varepsilon}{4a}. \tag{9}$$

In equation (6), the dimension for the constant a is $J{\cdot}s$ which has the same dimension as Planck's constant $h(J{\cdot}s)$. When comparing the order of magnitude for $4a$ and Planck constant h, we can found that they have same value. Also, comparing equations (9) and (6), the value of ε in equation (9) should be: 0, $4a\nu(1\varepsilon)$, $8a\nu(2\varepsilon)$,... which agrees with Planck's hypothesis of "quanta". Therefor, we can view eq. (9) as de-Broglie equation.

Discussion:

(1) When particle appears wave appearance, the momentum of a particle can be viewed as $p = \frac{4a}{\lambda}$. In this situation, p refers to the average momentum of particles who was involved in wave show. For particles which was not involved in wave exhibation, their momentum is mv.

(2) Different from classical wave, when colliding with materials, the frequency of probability wave formed by particles will change.

(3)If eq. (6) holds, then, not changing the relative particle probability density, we can use

$$\frac{\psi}{sin^2(\theta)} = \frac{c}{sin^2(\theta)} e^{i(PR-Et)\pi/(2a)}$$

to replace eq. (6) and get

$$\Psi(R,t) = Ce^{i(PR-Et)\pi/(2a)}$$

Where θ is the angle between the average velocity of particles and line $O'O$. Above equation can be viewed as a classical wave with wave lenth $\lambda = \frac{4a}{P}$ and wave frequency $\nu = \frac{E}{4a}$.

(4) Eq. (7) and eq. (9) are derived independently. In eq. (7), if letting $h = 4a$, we can get the Schrödinger equation. In eq. (9), if letting $h = 4a$, we can get the de-Broglie equation. Thus, eq. (7) can be associated with eq. (9) by h.

DERIVATION OF HYPOTHESIS OF PATH INTEGRALS

By the hypothesis of the Feynman path integral [16], if free particles emit from particle source A to arrive at a screen, then the probability of finding particles at screen position x should be:

$$|C|^2 \,|\sum_{all\,paths} e^{is(r)/\hbar}|^2.$$

where C is a constant. By the hypothesis, there are many possible trajectories for a particle to arrive at x. If the particle passes along a possible trajectory r arriving at x, then $s(r)$ in the above formula represents the particle's action and "*all paths*" means the particle's all possible trajectories should be taken into account when calculating $s(r)$. The following proof is based on classical mechanics.

Proof:

In single slit experiment, if free particles start from particle source A, passing through the slit to arrive at a diffraction fringe position x on the screen, then, by equation $(6')$, in the neighborhood of x, the average least action of particles can be written as

$$\psi(r,t) = Ce^{-ipr\pi/(2a)} e^{i\varepsilon t\pi/(2a)}$$

Supposing the number of particles in the neighborhood of x is N. Particle $j (j = 1, 2, ..., N)$ with kinetic energy E_j takes time t_j to move along its least action trajectory r_j arriving in the neighborhood of x, then, by eq. (4), its least action will be $\psi_j = E_j t_j$. In the neighborhood of x, all particles have approximate equivalent least action and close to the average value $\psi(r,t)$. Therefor, in the neighbourhood of x, the sum of the least action for all particles is:

$$\psi_s = \psi_1 + \psi_2 + ... + \psi_N = \sum_{j=1}^{N} Ce^{-ipr\pi/(2a)} e^{iEt\pi/(2a)}$$

For all particles in the neighbourhood of x, $e^{-ipr\pi/(2a)}$ can be looked as a constant. For particle j, $E_j t_j = Et$, so above equation can be written as:

$$\psi_s = Ce^{-ipr\pi/(2a)} \sum_{j=1}^{N} e^{iE_j t_j \pi/(2a)}$$

In (7) and (9), we have knew that $\hbar = \frac{2a}{\pi}$ holds, letting $s_j(r_j) = E_j t_j$, then, we get:

$$\psi_s = Ce^{-ipr/\hbar} \sum_{j=1}^{N} e^{is_j(r_j)/\hbar} \tag{10}$$

In equation (7), ψ describes the average least action of particles with $|\psi|^2$ presenting the relative probability density of particles. In equation (10), ψ_s is the sum of every particle's least action in the neighbourhood of x. Does $|\psi_s|^2$ also describe the relative probability density of particles in the neighbourhood of x?

In fact, in the neighbourhood of x, every particle's least action is approximately equivalent to ψ. So, $\psi_s = N\psi$. Due to $|\psi|^2$ describes the relative probability density of particles in the neighbourhood of x, so, $|\psi_s|^2$ also describes particle's relative probability density and can be written as:

$$|\psi_s|^2 = |C|^2 |\sum_{j=1}^{N} e^{is_j(r_j)/\hbar}|^2 \tag{11}$$

When just calculating the relative probability density of particles in the neighbourhood of x, the following two statements are equivalent: (1) for particle 1, 2, ..., N, every particle falls in the neighbourhood of x travelling along its least action orbit, the probability of finding particles in the neighbourhood of x is determined by eq. (11); (2) one particle has N possible orbits falling in the neighbourhood of x, the probability of finding a particle in position x is determined by the following equation:

$$|\psi_s|^2 = |C|^2 |\sum_{all\,paths} e^{is(r)/\hbar}|^2 \tag{12}$$

Where "all paths" means a particle's possible trajectory is N. At a diffraction fringe position, the number of particles is finite, i. e., N is finite. So the right side of eq. (12) is convergent.

This proof is slightly different from Feynman's hypothesis, as 1)the screen position x should be a diffraction fringe position, 2) this proof holds only for statistical occasion, 3) $s(r)$ is a particle's least action, not its action, and 4) every particle has only one least action trajectory.

Discussion:

In single-slit experiment, eq. (12) is applicable for the component of particle average least action. If we look $\Psi(R,t) = C'e^{-iPR\pi/(2a)}e^{iEt\pi/(2a)}$ as the average least action for particles in one fringe position, then, Ψ_s,the sum of the least action for every particle is:

$$\Psi_s = C'e^{-iPR/\hbar}\sum_{j=1}^{N} e^{iS_j(R_j)/\hbar}$$

Thus, we get

$$|\Psi_s|^2 = |C'|^2 |\sum_{j=1}^{N} e^{iS_j(R_j)/\hbar}|^2 \tag{13}$$

Comparing eq. (13) with (11), we can get:

$$|\frac{\psi_s}{\Psi_s}|^2 = |\frac{C}{C'}|^2 |\frac{\sum_{j=1}^{N} e^{is_j(r_j)/\hbar}}{\sum_{j=1}^{N} e^{iS_j(R_j)/\hbar}}|^2$$

In the neighbourhood of x, the least action of every particle is very close, so we can believe that $\sum_{j=1}^{N} e^{is_j(r_j)/\hbar} = Ne^{is_j(r_j)/\hbar}$ and $\sum_{j=1}^{N} e^{iS_j(R_j)/\hbar} = Ne^{iS_j(R_j)/\hbar}$ hold. Therefor, we have:

$$|\frac{\psi_s}{\Psi_s}|^2 = |\frac{C}{C'}|^2 \frac{|Ne^{is_j(r_j)/\hbar}|^2}{|Ne^{iS_j(R_j)/\hbar}|^2} = |\frac{C}{C'}|^2$$

Obviously, $|\Psi_s|^2$ also portray particle relative probability density. Above equation means that, in single slit experiment, when wave feature appears, particle relative probability density can be described by $|\psi_s|^2$ or $|\Psi_s|^2$.

CONCLUSION

Based on the ensemble interpretation viewpoints (particles move in classical trajectory and the wave phenomena are rooted in the statistical behavior of particles), we have inferred eq. (7), (9) and (12) uncovering the relationships among particle position, momentum, kinetic energy and least action. These equations have the same form and physical meaning as the Schr$\ddot{o}$dinger equation, the de-Brogile equation, and the hypothesis of the Feynman path integral. In the real world, it is impossible for particles obeying two kinds of law of motion simultaneously. So, eq. (7), (9) and (12) should be equivalent to the Schr$\ddot{o}$dinger equation, the de-Brogile equation and the hypothesis of the Feynman path integral, respectively.

Analysis revealed that, the nature of wave function is the average or the sum of least action for particles in one position, and quantization is the feature of an ensemble system formed by micro-particles.

When particles appear as wave properties, both the least action of particles and their component in screen direction satisfy equation (7) and (12). In this case, particles in one fringe position can be viewed as a classical wave: wave length and wave frequency can be described by eq. (9), wave function can be described by eq. (6); the square modulus of the wave function can represent the relative probability density of particles.

For the hypothesis of the Feynman path integral, analysis exposed the another side of the wave function: it describes the sum of the least action for particles in one diffraction position with the square of its module also portraying the relative probability density of particles in that position.

Since "action" is considered a physical attribute of every kinetic particle, this conclusion is applicable to all statistical ensemble systems formed by micro-particles.

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