Constructing a mathematical framework for the ensemble interpretation based on double-slit experiments

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The ensemble interpretation attributes the wave appearances of particles to their statistical characteristics. This has increasingly interested scientists. However, the ensemble interpretation is still not a scientific theory based on mathematics. Here, based on the double-slit experiment, a mathematical framework for the ensemble interpretation is constructed. The Schrödinger equation and the de-Broglie equation are also deduced. Analysis shows that the wave appearance of particles is caused by the statistical properties of these particles; the nature of the wave function is the average least action for the particles in a position.

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INTRODUCTION.

Double-slit experiments indicate that particles exhibit wave properties[1, 2, 3, 4]. Many interpretations have been proposed to explain the wave features appearing in double-slit experiments[5, 6]. However, scientists have not completely accepted any of these interpretations.

The Copenhagen Interpretation attributed the particle's wave appearances to a particle's duality and regarded the wave as a probabilistic wave[7, 8, 9]. The remarkable feature of the Copenhagen Interpretation was that it denied the particle's classical trajectory. Most scientists have accepted the Copenhagen Interpretation's viewpoint of a "probabilistic wave", but its other viewpoints, a few scientists still doubt. American physicist Alfred Landé believed a successful interpretation should be classical[10]. Karl Popper believed there was no need to do away with the concept of a particle's classical trajectory[11]. Einstein was not satisfied with the Copenhagen Interpretation. He proposed the ensemble viewpoint[12] and believed the wave function described the properties of the ensemble. Leslie E. Ballentine improved and further developed the ensemble interpretation[13]. Max Jammer affirmed that, in practical work, physicists actually use the logic and terminology of the ensemble interpretation, whether or not they accepted it[14]. Today, more and more scientists have accepted the ensemble interpretation. However, the ensemble interpretation is a philosophical discussion not a theory based on principles of physics and mathematics.

In the micro world, many physical quantities are nonnegative, near-zero numbers, particularly in an ensemble system formed by microscopic particles. Since the physical behavior of particles depends on their basic physical quantities such as momentum, displacement, energy, and least action, the wave feature of these particles should be closely linked to the near zero, nonnegative physical quantities of these particles. Based on this thought, the universal least action principle, and the ensemble interpretation, this work provided a mathematical analysis for the wave feature of particles: explained the single-slit and double-slit experiments, revealed the reason of why energy can be quantized, analyzed why the modular square of the wave function can represent the relative particle probability density, and deduced the Schrödinger equation, etc..

DOUBLE-SLIT EXPERIMENT DESCRIPTION.

In FIG. 1(a), identical particles are sent out from source S. They pass through slit A or B arriving at screen x. O is the origin of screen x. S, O' and O are on one line with O' the midpoint of slits A and B on board m. Board m is parallel to x with m and x perpendicular to SO. To simplify this problem, consider that the movement direction of particles, the double-slit, and screen x are on the same plane. Experiments have shown that when only one slit is opened, a diffraction phenomenon exists on screen x as shown by either the solid curve or the dotted curve in (b) of FIG. 1. However, when the two slits are both opened simultaneously, an interference phenomenon occurs as in (c) of FIG. 1.

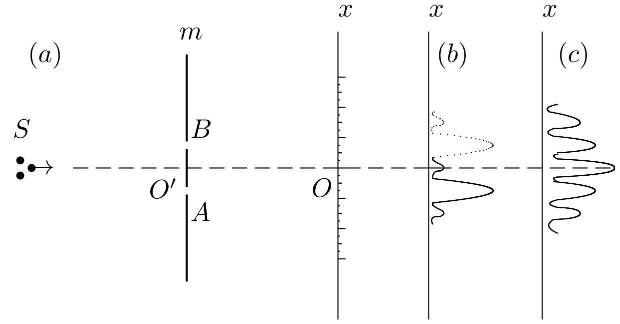

FIG. 1. Double-slit experiment with particles. *a*) Particles begin at S and pass through slit A or B arriving at screen x. *b*) When only slit A is open, the particles' position (vertical orientation)-density (horizontal orientation) curve is described by the solid curve. When only slit B is open, the position-density curve is described by the dotted curve (the curve formed by slit A partially overlaps the dotted curve). *c*) When the two slits are opened simultaneously, the position-density curve appears as an interference pattern.

SOME LEMMAS.

Lemma 1. A nonnegative, near-zero number can be represented by one term of its complex Fourier series (symbols not included).

Proof:

For a number $x > 0$, its Fourier series on the interval $[-a, a]$ (a is a constant and $a > 0$) is:

$$x = 2\left(\frac{\sin(\pi x/a)}{\pi/a} - \frac{\sin(2\pi x/a)}{2\pi/a} + \frac{\sin(3\pi x/a)}{3\pi/a} - \frac{\sin(4\pi x/a)}{4\pi/a} + ...\right) \tag{1}$$

Because $\lim_{x\to 0} \frac{\sin(n\pi x/a)}{n\pi x/a} = 1$, for $n \in Z^+$. Thus, when x is a near-zero number, we have:

$$x \approx \frac{\sin(n\pi x/a)}{n\pi/a}$$

Since $f(x) = x$ is an element of $L^2([-a, a])$, its Fourier series in complex form can be written as:

$$x = \sum_{n=-\infty}^{n=+\infty} \alpha_n e^{\frac{in\pi x}{a}} \tag{2}$$

where $\alpha_n = \frac{1}{2a}\int_{-a}^{a} x e^{\frac{-in\pi x}{a}} dx$ for $n \in Z$. When $n = 0, \alpha_n = 0$; when $n = 1, 3, 5, 7, 9, ..., \alpha_n = -\frac{ia}{n\pi}$; and when $n = 2, 4, 6, 8, 10, ..., \alpha_n = \frac{ia}{n\pi}$.

Considering

$$Re(\alpha_n e^{\frac{in\pi x}{a}}) = \pm\frac{\sin(n\pi x/a)}{n\pi/a}$$

and

$$Re(\alpha_n e^{\frac{in\pi x}{a}}) = Re(\alpha_{-n} e^{\frac{-in\pi x}{a}}),$$

when $x \to 0$ and when not considering signs, we can get:

$$x \approx Re(\alpha_n e^{\frac{in\pi x}{a}}) \tag{3}$$

The purpose of converting x into its Fourier series is to find out the changing law of x. When $x \to 0$, $\alpha_n \cos(\frac{n\pi x}{a})$ can be viewed as a constant. If we add this constant to $i\alpha_n \sin(\frac{n\pi x}{a})$, the varying pattern of $i\alpha_n \sin(\frac{n\pi x}{a})$ will be the same as $i\alpha_n \sin(\frac{n\pi x}{a}) + \alpha_n \cos(\frac{n\pi x}{a})$. So, when $x \to 0$, if studying the changing law of x, we can use $\alpha_n e^{\frac{in\pi x}{a}}$ to replace x.

To sum up, when no account is taken of symbols, a nonnegative, near-zero number can be regarded as one term of its complex Fourier series.

In a single slit experiment, the least action of a free particle is often defined as:

$$L = \int_{t1}^{t2} \frac{1}{2} mV^2 dt = \frac{1}{2} PR = Et \tag{4}$$

The component of L in the screen direction is $\ell = \frac{1}{2}pr$ or $\ell = \varepsilon t$,

where

P : represents the momentum of the particle in the screen position,
p : represents the component of P in the screen direction,
V : represents the speed of the particle in the screen position,
v : represents the component of V in the screen direction,
R : represents displacement of the particle from one slit (after passing through) to the screen position,
r : represents the component of R in the screen direction,
E : represents the kinetic energy of the particle in the screen position,
ε : represents the component of E in the screen direction, and
$t = t2 - t1$: represents the time the particle takes to travel from one slit (after passing through) to the screen position.

Lemma 2. In a single slit experiment, if the least action component(in the screen direction) of every particle in the screen can be replaced by one term of its Fourier series, then the position of the particles is discrete.

Proof:

For particle j, if its least action ℓ_j can be replaced by one term of ℓ_j's Fourier series, i.e., by $\alpha_{n_j} e^{in_j\pi\ell_j/a}$, then, the average least action of all particles (the number of particles is N), x, is:

$$x = \sum_{j=1}^{j=N} \frac{\alpha_{n_j} e^{in_j\pi\ell_j/a}}{N} = \sum_{j=1}^{j=N} C_j e^{in_j\pi\ell_j/a}$$

By Eq. (2), x can also be written as:

$$x = \sum_{n=-\infty}^{n=\infty} \alpha_n e^{\frac{in\pi x}{a}}$$

At the same screen position, the least action of particles is equal. In the screen position of particle j, if the number of particles is M, then, the least action proportion over x in this position can be viewed as $MC_j e^{in_j \pi \ell_j /a}$. By the uniqueness of the Fourier series, $MC_j e^{in_j \pi \ell_j /a}$ must be one term of the Fourier series of x, meaning the least action of every particle in the screen can be replaced by one Fourier term of x. In this case, for the same single slit experiment, the least action of particles will depend only on integer n. Since n is discrete, the least action of particles in the screen is discrete. By the principle of least action, the position of a particle in the screen is determined by its least action. Thus, the position of the particles in the screen is discrete.

Lemma 3. In a single slit experiment, if the average least action (in the screen direction) for particles in one diffraction fringe position is x^*, then, the relative probability density of particles in the position can be described by $|x^*|^2$.

Proof [15]:

By lemma 2, when particles show a diffraction appearance, the least action of every particle in the screen can be replaced by one Fourier term of x, where x is the average least action of all particles in the screen. So, at one diffraction fringe position, the least action of particles can form a nonnegative, near-zero number group $X = [x_1, x_2, ..., x_N]$ with an average value x^*. These numbers can be viewed as random variables with a mathematical expectation value $E(X) = x^*$. By the definition of variance, the variance value $D(X)$ of these numbers is:

$$D(X) = \sum_{i=1}^{i=N} \frac{x_i^2}{N} - (x^*)^2$$

In the above equation, if x^* increases Δx with $x_i (i = 1, 2, ..., N)$ increasing $\Delta x/N$, $(x^*)^2$ will become $(x^* + \Delta x)^2$ with an increment of $2x^*\Delta x + (\Delta x)^2$, and $1/N \sum_{i=1}^{i=N} x_i^2$ will become $1/N \sum_{i=1}^{i=N} (x_i + \Delta x/N)^2$ with an increment of $2x^*\Delta x/N + (\Delta x)^2/N^2$. When N is very big and x^* as well as Δx are very small, the increment of $(x^*)^2$ is far more than the increment of $1/N \sum_{i=1}^{i=N} x_i^2$.

In a single slit experiment, usually N is very big and x^* is a near zero, nonnegative number. So, we can conclude that, the greater $(x^*)^2$ is, the smaller $D(X)$ will be. The value of $D(X)$ reflects the degree of concentration for random variables : the less $D(X)$ is, the larger the degree of the number concentration will be. By the least action principle, the position of a particle is determined by the least action of the particle. Thus, we can say, in the diffraction fringe position, the larger $(x^*)^2$ is, the greater the particle density will be, meaning $(x^*)^2$ can describe the particle density in the fringe position. If x^* is represented as a complex number, we should say, $|x^*|^2$ describes the particle density in one fringe position.

Let us considering another side of the question. Let

$$f_N(x) = \sum_{k=-N}^{k=N} \alpha_k e^{\frac{ik\pi x}{a}}$$

as well as

$$g_N(x) = \sum_{k=-N}^{k=N} \beta_k e^{\frac{ik\pi x}{a}}$$

be the partial sum of the Fourier series of f and g, respectively. When $N \to \infty$, $f_N \to f$ and $g_N \to g$ in $L^2[-a, a]$, we have:

$$< f_N, g_N >= \sum_{k=-N}^{k=N} \sum_{n=-N}^{n=N} \alpha_k \overline{\beta_n} < e^{ik\pi x/a}, e^{in\pi x/a} >$$

Since $\{\frac{1}{\sqrt{2a}} e^{ik\pi x/a}, k = ..., -1, 0, 1, ...\}$ is orthogonal, therefore:

$$< f_N, g_N >= 2a \sum_{n=-N}^{n=N} \alpha_n \overline{\beta_n}$$

When $N \to \infty$, $< f_N, g_N > \to < f, g >$, so we have:

$$< f, g >= 2a \sum_{n=-\infty}^{n=\infty} \alpha_n \overline{\beta_n}$$

In the set of $L^2[-a, a]$, we can define the inner product as:

$$< f, g >= \int_{-a}^{a} f\overline{g} dx$$

Letting $f(x) = g(x) = x$, we get:

$$\int_{-a}^{a} x^2 dx = 2a \sum_{n=-\infty}^{n=\infty} |\alpha_n|^2$$

Thus,

$$\sum_{n=-\infty}^{n=\infty} |\frac{\sqrt{3}}{a} \alpha_n|^2 = 1$$

If we Let $\psi_n = \frac{\sqrt{3}}{a}\alpha_n e^{in\pi x/a}$, then:

$$\sum_{n=-\infty}^{n=\infty} |\psi_n|^2 = 1 \tag{5}$$

If x represents the average least action of all particles in the screen, then, by lemma 2, in one diffraction position, $x_i^*(i \in Z)$ can be viewed as one term of the Fourier series of x. Therefore, we have:

$$\sum_{n=-\infty}^{n=\infty} |\alpha_n|^2 = \sum_{i=-\infty}^{i=\infty} |x_i^*|^2$$

and

$$\sum_{n=-\infty}^{n=\infty} |\frac{\sqrt{3}}{a}\alpha_n|^2 = \sum_{i=-\infty}^{i=\infty} |\frac{\sqrt{3}}{a}x_i^*|^2 = 1$$

Letting $\psi_i = \frac{\sqrt{3}}{a}x_i^*$, we get:

$$\sum_{i=-\infty}^{i=\infty} |\psi_i|^2 = 1$$

The above equation tells us, $|\frac{\sqrt{3}}{a}x_i^*|^2$ describes the probability of particles falling in the fringe position, and $|x_i^*|^2$ describes the relative probability of particles falling in the fringe position. Therefore, when considering that $|x_i^*|^2$ describes particle density, we can say, $|x_i^*|^2$ describes the relative probability density of particles falling in the fringe position.

DOUBLE-SLIT EXPERIMENT ANALYSIS.

A. Basic analysis for particles in a single-slit experiment:

In a single slit experiment, after passing the slit, the least action of a particle from one slit to the screen position is $L = PR/2$. Usually, its component ℓ ($= 1/2pr$ or εt) in the screen direction is a near-zero, nonnegative number. By lemma 1, when $\ell \to 0$, ℓ can be replaced by its fourier term $\alpha_n e^{in\pi\ell/a}$. In this situation, if x represents the average least action of all particles in the screen, then, by lemma 2, ℓ can be replaced by one Fourier term of x and the position of particles in the screen are discrete forming diffraction fringes. Accordingly, in one diffraction fringe position, if the average least action of particles in this position is x^*, then, by lemma 3, the relative probability density of particles in the position can be described by $|\alpha_n e^{in\pi x^*/a}|^2$. Generally, in the same diffraction position, x^* is close to ℓ. So, $|\alpha_n e^{in\pi\ell/a}|^2$ can be used to describe the relative probability density of particles in the diffraction fringe position.

Discussion:

Generally, the average least action in the mid fringe position is larger than in the side fringe position. The reason is, when particles pass through a slit, the slit will decrease the momentum of the particles. The loss of the momentum for the side particles is much greater than the mid particles.

B. Diffraction analysis:

In a single slit experiment, usually the mass of particles is very small. So the least action component in the screen direction is often nonnegative, near-zero numbers. If the average least action component in the screen is x, then, by lemma 2, the least action component in the screen of every particle can be replaced by one Fourier term of x and its position in the screen is discrete. This means that, in a single slit experiment, particles tend to arrive at discrete positions in the screen.

For one diffraction fringe position, $|\psi_n|^2 = |\alpha_n e^{in\pi\ell/a}|^2$ can be used to describe the relative probability density of particles in the position. In this case, we can say $\psi_n = \alpha_n e^{in\pi\ell/a}$ describes the state of particles in the position. Thus, ψ_n provides two messages: the position of particles and the particle relative probability density in that position. In this sense, ψ_n is equivalent to the wave function of particles.

Therefore, ψ_n has the following properties:

(1) ψ_n is discrete,

(2) ψ_n is single valued, and

(3) $\sum_{n=-\infty}^{n=\infty} |\frac{\sqrt{3}}{a}\psi_n|^2 = 1$. This means $|\psi_n|^2$ only describes the relative probability density of particles.

C. Interference analysis: see [16]

DERIVATION OF THE SCHRÖDINGER EQUATION

In a single slit experiment, if the average particle least action component in the screen direction for one fringe position, ℓ, is a nonnegative near-zero number and can be replaced by one Fourier term, then by lemma 1, ℓ can be written as $\alpha_n e^{in\pi\ell/a}$.

For the average least action ℓ in one diffraction position, its energy form is $(\alpha_n e^{i\pi n\varepsilon t/a})$ and its momentum form is $(\alpha_n e^{i\pi \frac{1}{2} npr/a})$.

To simplify equation forms, $n\varepsilon$ can be merged into ε, with ε corresponding to $1\varepsilon, 2\varepsilon, 3\varepsilon, ...$; and $\frac{1}{2}nr$ can be merged into r, meaning that, when calculating, r should be discrete and should be replaced by $\frac{n}{2}r$. Obviously, when we say the least action of the particles in one diffraction fringe position, the average value of the energy form and momentum form should be taken into account simultaneously. Here, we use the geometric average value of the two forms of least action:

$$\sqrt{\alpha_n e^{i\pi pr/a} \overline{\alpha_n e^{i\pi\varepsilon t/a}}} = \alpha_n e^{i\pi(pr-\varepsilon t)/(2a)}$$

or

$$\sqrt{\alpha_n e^{i\pi\varepsilon t/a} \overline{\alpha_n e^{i\pi pr/a}}} = \alpha_n e^{i\pi(\varepsilon t-pr)/(2a)}$$

Supposing $\psi(r,t)$ represents the average least action of particles in the screen direction for diffraction fringe position r, then, we have:

$$\psi(r,t) = Ce^{i\pi(pr-\varepsilon t)/(2a)} \tag{6}$$

or

$$\psi(r,t) = Ce^{i\pi(\varepsilon t-pr)/(2a)} \tag{6'}$$

where $C = \alpha_n$ $(n \in Z)$.

In (a) of FIG. 1, for a particle single slit experiment, the particle's displacement R, its projection in the screen direction r, and $O'O$ satisfy equation $\overrightarrow{R} + \overrightarrow{r} = \overrightarrow{O'O}$. Thus, when the screen moves forward or backward a small distance Δ along line $O'O$, R and r will change, and the time particles take to arrive in the neighborhood of a diffraction fringe position will increase or decrease.

After particles pass through the slit, for one fringe position, using Eq. (6), the average least action component in the screen direction $\psi(r,t)$ is

$$\psi(r,t) = Ce^{i(pr-\varepsilon t)\pi/(2a)}.$$

In order to make the above equation continuous, we can redefine $\psi(r,t)$ as:

$$\psi(r,t) = \begin{cases} \psi(r,t) & : \text{ r is in a diffraction position,} \\ K & : \text{ r is not in a diffraction position} \end{cases}$$

where K is a constant. After redefining, $\psi(r,t)$ becomes a continuous function.

When the screen moves forward or backward a small distance, the change of the least action in the screen direction will be:

$$\delta\psi = \frac{\partial\psi}{\partial r}dr + \frac{\partial\psi}{\partial t}dt$$

By the principle of least action, we have:

$$\delta\psi = 0.$$

Thus,

$$\frac{\partial\psi}{\partial t} = -\frac{\partial\psi}{\partial r}\frac{dr}{dt}.$$

On the other hand, $\psi(r,t)$ satisfies $\frac{\partial\psi}{\partial t} = \frac{\varepsilon\pi}{2ai}\psi$ and $\frac{\partial}{\partial r}(\frac{\partial\psi}{\partial r}) = -(\frac{p\pi}{2a})^2\psi$. So, the relationship between $\frac{\partial\psi}{\partial t}$ and $\frac{\partial\psi}{\partial r}$ is:

$$\frac{\partial\psi}{\partial t} = \frac{ia}{\pi m}\frac{\partial}{\partial r}(\frac{\partial\psi}{\partial r}).$$

Letting $\hbar = \frac{2a}{\pi}$, the above equation can be written as:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial r^2}. \tag{7}$$

Comparing Eq. (7) with the Schrödinger equation for free particles, the same form and properties are found. Both of them describe the relationship among the particle position, momentum, and kinetic energy. So, Eq. (7) corresponds to the Schrödinger equation for free particles.

Discussion:

(1) In a single slit experiment, taking the mid fringe position as an example, if the angle between particle average momentum P and $O'O$ is θ, then, $P = p/\sin(\theta)$ and $R = r/\sin(\theta)$ hold. In addition, particle average kinetic energy $E = \varepsilon/\sin^2(\theta)$ holds. So, particle average least action $\Psi(R,t) = \psi(r,t)/\sin^2(\theta)$ also holds. Thus, we can get the same equation as Eq. (7):

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2}. \tag{7'}$$

The above equation means that, when particles exhibit a wave feature, both average least action and its average component satisfy Eq. (7). Obviously, $|\Psi|^2$ also describes particle relative probability density.

(2) Thus, in our daily work, we can use

$$\Psi(R,t) = C'e^{i(PR-Et)\pi/(2a)}$$

and Eq. (7') to replace Eq. (6) and Eq. (7).

(3) In a single slit experiment showing diffraction fringes, if particles are added to a potential field $U(r)$, the particles in the same fringe position will get an equal additional displacement. In this case, if we view $\Psi(R,t) = C'e^{i(PR-(E-U)t)\pi/(2a)}$ as the particle average least action in the diffraction fringe position and let $\hbar = \frac{2a}{\pi}$, we can get the following equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2} - U\Psi$$

If we select a proper zero potential energy point for $U(r)$, the above equation can be written as:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial R^2} + U\Psi$$

NEW UNDERSTANDING ON THE DE-BROGLIE EQUATION

In Eq. (6), $|\psi(r,t)|^2$ describes the relative probability density of particles in one diffraction fringe position. That is, the greater $|\psi|^2$ is, the greater the particle relative probability density will be. Also, we can say: the greater $|\psi|^2$ is, the smaller the relative displacement between a particle's position and the particle's mean position will be. Noticing that, when $|\psi|^2$ becomes larger, $-|\psi|$ will be smaller, and the relative probability density of particles will become larger. Thus, the smaller $-|\psi|$ is, the less the relative displacement of particles will be. So, $-|\psi|$ can describe relative displacement of particles. Therefore, we can view ψ as reflecting the relative displacement between a particle's actual screen position and the mean position of particles.

Let's compare ψ with the classical harmonic wave function.

Suppose a classical simple harmonic wave with wavelength λ and frequency ν is propagating in the positive direction of coordinate x. Then after time t, the relative displacement of an infinitesimal quantity of the medium to the balance position x is:

$$Y(x,t) = Ae^{i\pi(\frac{2}{\lambda}x - 2\nu t)} \tag{8}$$

When comparing Eq. (6) with the classical simple harmonic wave function above, many similar characteristics are found. These include: 1) reflecting the relative displacement between a particle's screen position and the mean position versus describing the relative displacement between an infinitesimal quantity of the medium and the balance position, 2) having the same equation form, and 3) causing the same physical phenomena (diffraction and interference). Therefore, Eq. (6) has the properties of Eq. (8), and can be considered a classical simple harmonic wave function.

Comparing Eq. (6) and Eq. (8), by the dimension corresponding principle, $\frac{p\pi}{2a}$ corresponds to $\frac{2\pi}{\lambda}$ and $\frac{\varepsilon\pi}{2a}$ corresponds to $2\pi\nu$. If the corresponding relationship is regarded as an equal relation, we have:

$$\frac{p\pi}{2a} = \frac{2\pi}{\lambda}, \quad \frac{\varepsilon\pi}{2a} = 2\pi\nu,$$

or

$$\lambda = \frac{4a}{p}, \quad \nu = \frac{\varepsilon}{4a}. \tag{9}$$

In Eq. (6), the dimension for the constant a is $J\cdot s$ which has the same dimension as Planck's constant $h(J\cdot s)$. When comparing Eq. (9) and Eq. (6), the value of ε in Eq. (9) should be: 0, $4a\nu$, $8a\nu$,... which agrees with Planck's hypothesis of "quanta". Therefore, we can view Eq. (9) as the de-Broglie equation.

Discussion:

(1) When particles have a wave appearance, the average momentum of the particles can be viewed as $p = \frac{4a}{\lambda}$. For particles not involved in the wave exhibition, their momentum is mv.

(2) Differing from a classical wave, when colliding with materials, the frequency of the "probability wave" formed by particles will change.

(3) If Eq. (6) holds, then, without changing the relative particle probability density, we can use

$$\frac{\psi}{sin^2(\theta)} = \frac{c}{sin^2(\theta)}e^{i(PR-Et)\pi/(2a)}$$

to replace Eq. (6) and get

$$\Psi(R,t) = Ce^{i(PR-Et)\pi/(2a)}$$

where θ is the angle between the average velocity of particles and line $O'O$. The above equation can be viewed as a new classical wave with wave length $\lambda = \frac{4a}{P}$ and wave frequency $\nu = \frac{E}{4a}$. Such treatment for Eq. (6) does not affect the particle relative probability density in the diffraction fringe position.

(4) Eq. (7) and Eq. (9) are derived independently. In Eq. (7), letting $h = 4a$ and $\hbar = \frac{h}{2\pi}$, we can get the Schrödinger equation. In Eq. (9), letting $h = 4a$, we can get the de-Broglie equation. Thus, we can conclude that Plank's constant $h = 4a$ holds.

CONCLUSION

Based on the ensemble interpretation viewpoints (a particle moving in a classical trajectory and the wave phenomena are rooted in the statistical behavior of particles), we have inferred Eqs. (7) and (9) uncovering the relationships among particle position, momentum, kinetic energy and least action. These equations have the same form and physical meaning as the Schrödinger equation and the de-Brogile equation. In the real world, it is impossible for particles to obey two laws of motion simultaneously. So, Eqs. (7) and (9) should be equivalent to the Schrödinger equation and the de-Brogile equation, respectively.

Analysis revealed that, the nature of a wave function is the average least action of particles in one position, and quantization is the feature of an ensemble system formed by micro-particles.

Since "action" is considered a physical attribute of every kinetic particle, the analysis in this paper is applicable to all statistical ensemble systems formed by micro-particles.

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