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Dual Curves of Constant Breadth According to Bishop Frame in Dual Euclidean Space

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Abstract: In this work, curves of constant breadth are defined and some characterizations of closed dual curves of constant breadth according to Bishop frame are presented in dual Euclidean space. Also, it has been obtained that a third order vectorial differential equation in dual Euclidean 3-space.

Keywords: Dual Euclidean space, dual curves, curves of constant breadth, Bishop frame

1 Introduction

The curves of constant breadth were introduced by :[1]. [2] had obtained a problem to determine whether there exist space curve of constant breadth or not, and he defined breadth for space curves on a surface of constant breadth. Furthermore, [3] defined the curve of constant breadth on the sphere. In [5], some geometric properties of plane curves of constant breadth are given. In recently work [4], these properties are studied in the Euclidean 3-space E^3. Moreover, In [18], this kind curves are studied in four dimensional Euclidean space E^4. In [6] expressed some characterizations of timelike curves of constant breadth in Minkowski 3-space and partially null curves of constant breadth in semi-Riemannian space.. In recently works this topic were studied and further characterizations related to different geometries were obtained, see [7], [8], [18], characterizations of Curves of Constant Breadth in Galilean 3-Space G^3[10].

2 Dual Curves of Constant Breadth According to Dual Bishop Frame in D^3

[11] introduced dual numbers with the set

$$D=\left\{\hat{x}=x+\varepsilon x^{*};x,x^{*}\in\mathbb{R}\right\}$$

The symbol ε designated the dual unit with the property $\varepsilon^2=0$ for $\varepsilon\neq 0$. Thereafter, a good amount of research work has been done on dual numbers, dual functions and as well as dual curves [12].

Then dual angle is introduced, which is defined as $\overset{\wedge}{\theta}=\theta+\varepsilon\theta^{*}$, where θ is the projected angle between two spears and θ^{*} is the shortest distance between them. In recent years, dual numbers have been applied to study the motion of a line in space; they seem even to be most appropriate way for this end and they have triggered use of dual numbers in kinematical problems.

The theory of relativity opened a door for using of degenerate submanifolds, and the researchers treated some of classical differential geometry topics, extended to Lorentzian manifolds. In light of the existing literature, Yılmaz deal with the timelike dual curves of constant breadth in dual Lorentzian space, see [16].

The set D of dual numbers is commutative ring with the operations $+$ and $\cdot$ The set

$$D^3=D\times D\times D=\left\{\overset{\wedge}{\varphi}=\varphi+\varepsilon\varphi^{*};\varphi,\varphi^{*}\in E^3\right\}$$

is a module over the ring D, Let us denote $\hat{a}=a+\xi a^{*}$ and $\hat{b}=b+\xi b^{*}$. The Euclidean inner product of $\hat{a}$ and $\hat{b}$ is defined by

$$<\hat{a},\hat{b}>=<a,b>+\varepsilon(<a^{*},b>+<a,b^{*}>).$$

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For $\hat{\varphi} \neq 0$, the norm $\left\|\hat{\varphi}\right\|$ of $\hat{\varphi}$ is defined by

$$\left\|\hat{\varphi}\right\| = \sqrt{<\hat{\varphi},\hat{\varphi}>}.$$

Let $\left\{\hat{T},\hat{N}_1,\hat{N}_2\right\}$ be dual Bishop frame of the differential dual space curve in the dual space D^3. Then the dual Bishop frame equations are

$$\begin{aligned} \hat{T}' &= \hat{k}_1.\hat{N}_1 + \hat{k}_2.\hat{N}_2 \\ \hat{N}_1' &= -\hat{k}_1.\hat{T} \\ \hat{N}_2' &= -\hat{k}_2.\hat{T} \end{aligned} \tag{1}$$

where $\hat{k}_1 = k_1 + \varepsilon k_1^*$ and $\hat{k}_2 = k_2 + \varepsilon k_2^*$ are nowhere pure dual natural curvatures and

$$\hat{k} = k + \varepsilon k^* = \sqrt{k_1^2 + k_2^2 + 2\varepsilon(k_1 k_1^* + k_2 k_2^*)},$$

$$\hat{\theta}(s) = \theta + \varepsilon\theta^* = Arc\tan(\frac{\hat{k}_2}{\hat{k}_1}),\ \hat{\tau}(s) = \frac{d\hat{\theta}}{ds}.$$

Let $\varphi = \varphi(s)$ be a simple closed dual curve in D^3. These curves will be denoted by C. The normal plane at every point P on the curve meets the curve at a single point Q which is different from P. We call the point Q the opposite point of P. We consider dual curve in the class Γ as in [2] having parallel tangents $\hat{T}$ and $\hat{T}_\xi$ in opposite directions at the opposite points $\hat{\varphi}$ and $\hat{\xi}$ of the curve. A simple closed dual curve of constant breadth having parallel tangents in opposite directions at opposite points can be represented with respect to dual Frenet frame by the equation

$$\hat{\xi} = \hat{\varphi} + \hat{\gamma}.\hat{T} + \hat{\delta}.\hat{N}_1 + \hat{\lambda}.\hat{N}_2 \tag{2}$$

where $\hat{\gamma},\hat{\delta}$ and $\hat{\lambda}$ are arbitrary functions of s. Differentiating both sides of equation (2), we get

$$\frac{d\hat{\xi}}{ds_\xi}\cdot\frac{ds_\xi}{ds} = (\frac{d\hat{\gamma}}{ds} - \hat{\delta}.\hat{k}_1 - \hat{\lambda}.\hat{k}_2 + 1)\hat{T} + (\hat{\gamma}.\hat{k}_1 + \frac{d\hat{\delta}}{ds})\hat{N}_1 + (\hat{\gamma}.\hat{k}_2 + \frac{d\hat{\lambda}}{ds})\hat{N}_2$$

consider $\hat{T} = -\hat{T}_\xi$ we have the following system of equations

$$\begin{aligned} \frac{d\hat{\gamma}}{ds} &= \hat{\delta}.\hat{k}_1 + \hat{\lambda}.\hat{k}_2 - 1 - \frac{ds_\xi}{ds} \\ \frac{d\hat{\delta}}{ds} &= -\hat{\gamma}.\hat{k}_1 \\ \frac{d\hat{\lambda}}{ds} &= -\hat{\gamma}.\hat{k}_2. \end{aligned} \tag{3}$$

If we call $\hat{\theta}$ as the angle between the tangent of the curve C at point $\hat{\varphi}$ with a given direction and consider $\frac{d\hat{\theta}}{ds} = \hat{\tau}$ and $\frac{d\hat{\theta}}{ds_\xi} = \hat{\tau}^*$, then we have (3) as follow;

$$\begin{aligned} \frac{d\hat{\gamma}}{d\hat{\theta}} &= \hat{\delta}.\frac{\hat{k}_1}{\hat{\tau}} + \hat{\lambda}.\frac{\hat{k}_2}{\hat{\tau}} - f(\hat{\theta}) \\ \frac{d\hat{\delta}}{d\hat{\theta}} &= -\hat{\gamma}.\frac{\hat{k}_1}{\hat{\tau}} \\ \frac{d\hat{\lambda}}{d\hat{\theta}} &= -\hat{\gamma}.\frac{\hat{k}_2}{\hat{\tau}} \end{aligned} \tag{4}$$

where $f(\hat{\theta}) = \frac{1}{\hat{\tau}} + \frac{1}{\hat{\tau}^*}$

Let $\hat{K}_1 = \frac{\hat{k}_1}{\hat{\tau}}$ and $\hat{K}_2 = \frac{\hat{k}_2}{\hat{\tau}}$, and this case using system of ordinary differential equations (4), we have the following dual third order differential equation with respect to $\hat{\gamma}$ as;

$$\begin{aligned} &\frac{d^3\hat{\gamma}}{d\hat{\theta}^3} + (\hat{K}_1 + \hat{K}_2)\frac{d\hat{\gamma}}{d\hat{\theta}} \\ &+ [\frac{d}{d\hat{\theta}}(\hat{K}_1)^2 + \hat{K}_1\frac{d}{d\hat{\theta}}(\hat{K}_1)\frac{d}{d\hat{\theta}}(\hat{K}_2)]\hat{\gamma} \\ &+ (\int_0^{\hat{\theta}}\frac{\hat{K}_1}{\hat{\tau}}d\hat{\theta}).\frac{d^2}{d\hat{\theta}^2}(\hat{K}_1) + \frac{d^2 f}{d\hat{\theta}^2} = 0 \end{aligned} \tag{5}$$

Corollary 2.1 : The obtained dual differential equation of third order (5) is a characterizations for the simple closed dual curve ξ. By means of solution of it, position vector of a simple closed dual curve can be determined.

Let us investigate the solution of the equation (5) in a special case let $\hat{K}_1,\hat{K}_2$ and $f(\hat{\theta})$ be constant. Then equation (5) has the form

$$\frac{d^3\hat{\gamma}}{d\hat{\theta}^3} + (\hat{K}_1 + \hat{K}_2)\frac{d\hat{\gamma}}{d\hat{\theta}} = 0. \tag{6}$$

The solution of the equation (6) yields the components

$$\hat{\gamma} = A + B\cos[(\hat{K}_1^2 + \hat{K}_2)\hat{\theta}] + C\sin[(\hat{K}_1^2 + \hat{K}_2)\hat{\theta}]$$

$$\hat{\delta} = -\int_0^{\hat{\theta}} \{(A+B\cos[(\hat{K}_1^2+\hat{K}_2)\hat{\theta}]+C\sin[(\hat{K}_1^2+\hat{K}_2)\hat{\theta}])\frac{\hat{k}_1}{\hat{\tau}}\}d\hat{\theta}$$

$$\hat{\lambda} = -\int_0^{\hat{\theta}} \{(A+B\cos[(\hat{K}_1^2+\hat{K}_2)\hat{\theta}]+C\sin[(\hat{K}_1^2+\hat{K}_2)\hat{\theta}])\frac{\hat{k}_2}{\hat{\tau}}\}d\hat{\theta} \tag{7}$$

Corollary 2.2: Position vector a simple closed curve with constant dual curvature and constant dual torsion can be obtained in terms of $(7)_1$, $(7)_2$ and $(7)_3$.

If the distance between opposite points of $\hat{\varphi}$ and $\hat{\xi}$ is constant, then we can write that

$$\left\| \hat{\xi} - \hat{\varphi} \right\| = \hat{\gamma}^2 + \hat{\delta}^2 + \hat{\lambda}^2 = \text{constant} \tag{8}$$

Differentiating with respect to $\hat{\theta}$

$$\hat{\gamma}\frac{d\hat{\gamma}}{d\hat{\theta}} + \hat{\delta}\frac{d\hat{\delta}}{d\hat{\theta}} + \hat{\lambda}\frac{d\hat{\lambda}}{d\hat{\theta}} = 0 \tag{9}$$

By virtue of (4), the differential equation (9) yields

$$\hat{\gamma}.f(\hat{\theta}) = 0 \tag{10}$$

Case 1: $\hat{\gamma} = 0$. Then we have other components $\hat{\delta}$ and $\hat{\lambda}$ which are constant.

Since the following invariant of dual curves of constant breadth can be written as

$$\hat{\xi} = \hat{\varphi} + \hat{M}_1.\hat{N}_1 + \hat{M}_2.\hat{N}_2 \tag{11}$$

where $\hat{\delta} = \hat{M}_1$, $\hat{\lambda} = \hat{M}_2$, $\hat{M}_1$ and $\hat{M}_2$ are constant.

Case 2: $f(\hat{\theta}) = 0$. We have a relation among radii of curvatures as

$$\frac{1}{\hat{\tau}} + \frac{1}{\hat{\tau}^*} = 0 \tag{12}$$

Using other components, we easily have a third order variable coefficient differential equation with respect to $\hat{\gamma}$

$$\frac{d^3\hat{\gamma}}{d\hat{\theta}^3} + (\hat{K}_1 + \hat{K}_2)\frac{d\hat{\gamma}}{d\hat{\theta}}$$

$$+[\frac{d}{d\hat{\theta}}(\hat{K}_1^2) + \hat{K}_1\frac{d}{d\hat{\theta}}(\hat{K}_1)\frac{d}{d\hat{\theta}}(\hat{K}_2)]\hat{\gamma} \tag{13}$$

$$+(\int_0^{\hat{\theta}} \frac{\hat{K}_1}{\hat{\tau}} d\hat{\theta}).\frac{d^2}{d\hat{\theta}^2}(\hat{K}_1) = 0$$

This equation is characterizations for the components. However, the general solution of it has not been found. Due to this, we investigate in a special case, let us suppose $\hat{K}_1 = \hat{K}_2 = 0$ and $\hat{\tau} \neq 0$.

In this case, we rewrite (13)

$$\frac{d^3\hat{\gamma}}{d\hat{\theta}^3} = 0.$$

By this way, we have such that the components

$$\hat{\gamma} = \frac{s^2}{2} + cs$$

$$\hat{\delta} = \text{constant}$$

$$\hat{\lambda} = \text{constant}.$$

3 Conclusion

In this work we extened the curves of constant breadth concept to dual curves of dual Euclidean space according to type-2 Bishop frame. Thereafter, we determined relations using dual Bishop derivate formulau. In the light of the obtained result, we characterized dual constant breadth of the curves according to some special cases. We also express some open problems and theorems for further studies.
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References

[1] Euler L., De Curvis Trangularibus, Acta Acad Petropol, 3-30 (1870).
[2] Fujivara M., On Space Curves of Constant Breadth, Tohoku Math. J.**5** 179-184 (1914).
[3] Solow R.M., Quarterly Journal of Economics **70**, 65-94 (1956).
[4] Köse Ö., Some Properties of Ovals and Curves of Constant Width in a Plane, Doğa Turk Math. J. **8** 119-126 (1984).
[5] Mağden A. and Köse Ö., On The Curves of Constant Breadth, Tr. J. of Mathematics 227-284 (1997).
[6] Yılmaz S. and Turgut M., Partially null curves of constant breadth in semi-Riemannian space, Modern Applied Science **3** 60-63 (2009).
[7] Kocayiğit H. and Önder M., Space curve of Constant Breadth in Minkowski 3-Space, Annali di Matematica **192** 805-814 (2013).
[8] Yılmaz S., Özyılmaz E. and Savcı Ü. Z., Contributions to Differential Geometry of Partially Null Curves in Semi Euclidean Space, International J. Math. Combin.**1** 6-12 (2014).

[9] Yılmaz S. and Turgut M., On the time-like curves of constant breadth in Minkowski 3-Space, International J. Math. Combin.**3** 34-39 (2008).
[10] Yılmaz S., Savcı Ü. Z., and Turgut M., Characterizations of Curves of Constant Breadth in Galilean 3-Space G^3, Journal of Advanced Resarch in Pure Mathematics **6** 1-6 (2014).
[11] Clifford W.K., Preliminary Sketch of B. Quaternions, Proceedings of London Math. Soc. **4** 361-395 (1873).
[12] Baky R. A. A., An Explicit Characterization of Dual Spherical Curve, Commun. Fac Sci, Univ. Ank. Series A **51** 1-9 (2002).
[13] Ali A.T., Special Smarandache Curves in The Euclidean Space, Math. Cobin. Bookser **2** 30-36 (2010)..
[14] Veldkamp G.R., On The Use of Dual Numbers, Vectors and Matrices in Instantaneous Spatial Kinematics, Mech. Math. Theory **11** 141-156 (1976).
[15] Yang A.T., Application of Quaternion Algebra and Dual Number to the Analysis of Spatial Mechanical, Doctoral Dissertation, Columbia Universty, 1963.
[16] Yılmaz S., Timelike Dual Curves of Constant Breadth in Dual Lorentzian Space, Sci. J. of International Black Sea University **2** 129-136 (2008).
[17] Turgut M., Smarandache breadth pseudo null curves in Minkowski space-time. International J. Math. Combin. **1** 46-49 (2009).
[18] Mağden A. and Yılmaz S., On the Curves of Constant Breadth in Four Dimensional Galilean Space, Int. Math. For. **9**(25) 1229-1236 (2014).

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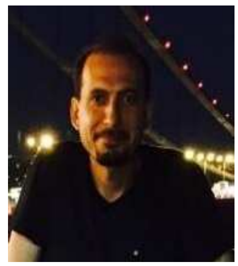

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