Annals of Pure and Applied Mathematics
Vol. 11, No. 1, 2016, 99-104
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 February 2016
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Characterization of Smarandache-Soft Neutrosophic Near-Ring by Soft Neutrosophic Quasi-Ideals

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Received 10 January 2016; accepted 28 January 2016

Abstract. In this paper, we introduce the Smarandache-2-algebraic structure of soft neutrosophic near-ring, namely Smarandache-soft neutrosophic near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N, such that there exists a proper subset M of N which is embedded with a stronger algebraic structure S_2. A stronger algebraic structure means satisfying more axioms, that is $S_1 << S_2$, and by proper subset one can understand a subset different from the empty set. We also define Smarandache-soft neutrosophic near-ring and obtain its characterization through soft neutrosophic quasi-ideals.

Keywords: Soft Neutrosophic Near-Ring, Soft Neutrosophic Near-Field, Smarandache-Soft Neutrosophic Near-Ring, Soft Neutrosophic Quasi-Ideals.

AMS Mathematics Subject Classification (2010): 46C20, 15A09

1.Introduction

New notions were introduced in algebra by Florentin Smarandache [2], to better study the congruence in number theory. By <proper subset> of a set A, it is considered a set P included in A, but different from A, different from the empty set, and from the unit element in A, if any of them rank the algebraic structures using an order relationship. We have the algebraic structures $S_1 << S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms than S_1 laws, or S_2 has more laws than S_1 . For example: Semi group<< Monoid <<group<< ring<<field, or Semi group<< to commutative semi group, ring<< unitary ring etc. A general special structure was defined to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where $SM << SN$. In addition, we have published [9,10,11,12]. For basic concepts of near-ring, we refer to Pilz, for quasi-ideals, we refer Lwao Yakabe, and for soft neutrosophic algebraic structure, we refer to Muhammed Shabir, Mumtaz Ali, Munazza Naz, and Florentin Smarandache.

2. Preliminaries

Definition 2.1. Let $\langle N\, UI \rangle$ be a neutrosophic near-ring and (F, A) be a soft set over $\langle N\, UI \rangle$.Then (F, A) is called soft neutrosophic near-ring if and only if F(a) is a neutrosophic sub near-ring of $\langle N\, UI \rangle$ for all $a \in A$.

Definition 2.2. By a soft neutrosophic near-ring, we mean a non-empty set *(F,A)* in which an addition + and multiplication $*$ are defined such that:

(a) ((F,A),+) is a soft neutrosophic group
(b) ((F,A),$*$) is a soft neutrosophic semigroup
(c) $(F(n_1) + F(n_2))F(n) = F(n_1)F(n) + F(n_2)F(n)$ where $F(n), F(n_1), F(n_2)$ in (F,A).

In dealing with general soft neutrosophic near-rings, the neutral element of ((F,A),+) will be denoted by F(0).

Definition 2.3. Let $K(I) = \langle KUI \rangle$ be a neutrosophic near-field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic near-field if and only if $F(a)$ is a neutrosophic sub near-field of $K(I)$ for all $a \in A$.

Definition 2.4. Let (F,A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$. We say that (F,A) is soft neutrosophic zero-symmetric if F(n)F(0) = F(0) for every element F(n) of (F,A).

Definition 2.5. An element F(d) of soft neutrosophic near-ring (F,A) over $\langle N \cup I \rangle$ is called soft neutrosophic distributive if $F(d)(F(n_1) + F(n_2)) = F(d)F(n_1) + F(d)F(n_2)$ for all elements $F(n_1), F(n_2)$ of (F,A).

Definition 2.6. Let (H,A) and (G,B) be two non-empty soft neutrosophic subsets of (F,A). We shall define two types of products:

$$(H,A)(G,B) = \{ \sum H(a_i)G(b_i) \;/\; H(a_i) \text{ in } (H,A),\ G(b_i) \text{ in } (G,B) \} \text{ and}$$

$(H,A)*(G,B) = \{ \sum H(a_i)\,(H(a_i') + G(b_i)) - H(a_i)H(a_i')) \;/\; H(a_i), H(a_i') \text{ in } (H,A),\ G(b_i) \text{ in } (G,B) \}$ where $\sum$ denotes all possible additions of finite terms. In the case when (G,B) consists of a single element G(b), we denote (H,A)(G,B) by (H,A)G(b), and so on.

Definition 2.7. A soft neutrosophic subgroup (H,A) of ((F,A),+) is called a (F,A)-subgroup of (F,A) if $(F,A)(H,A) \subset (H,A)$. For instance, (F,A)F(a) is a (F,A)-subgroup of (F,A) for every element F(a) in (F,A).

Definition 2.8. Let (F,A) be the soft neutrosophic near-ring over $\langle N \cup I \rangle$. The set $(F,A)_0$ = { F(n) in (F,A) / F(n)F(0) = F(0)} is called a soft neutrosophic zero symmetric part of (F,A); $(F,A)_c$ = {F(n) in (F,A) / F(n)F(0) = F(n)} is called a soft neutrosophic constant part of (F,A).

Now we introduce the basic concept, the **Smarandache-soft neutrosophic-near ring.**

Definition 2.9. A soft neutrosophic-near ring is said to be a Smarandache-soft neutrosophic–near ring, if a proper subset of it is a soft neutrosophic–near field with respect to the same induced operations.

Definition 2.10. A soft neutrosophic subgroup (L_Q,A) of ((F,A),+) is called a soft neutrosophic quasi-ideal of (F,A), if (L_Q,A) (F,A) ∩ (F,A) (L_Q,A) ∩ (F,A) ∗ (L_Q,A) ⊂ (L_Q,A). For instance, every (F,A)-sugroup of (F,A) and F(d)(F,A) with a distributive element F(d) of (F,A) are soft neutrophic quasi-ideals of (F,A).

Clearly, {F(0)} and (F,A) are soft neutrosophic quasi-ideals of (F,A). If (F,A) has no soft neutrosophic quasi-ideals except {F(0)} and (F,A), we say that (F,A) is L_Q- simple.

We recall the following properties of soft neutrosophic quasi-ideals:

(a) The intersection of any set of soft neutrosophic quasi-ideals of (F,A) is a soft neutrosophic quasi-ideal of (F,A).

(b) Suppose that (F,A) is soft neutrosophic zero-symmetric.Then a soft neutrosophic subgroup (L_Q,A) of ((F,A),+) is a soft neutrosophic quasi-ideal of (F,A) if and only if (L_Q,A)(F,A) ∩ (F,A) (L_Q,A) ⊂ (L_Q,A).

3. Characterization of Smarandache-soft neutrosophic near-ring

A soft neutrosophic near ring (F,A) over $\langle N \cup I \rangle$ is called a soft neutrosophic near-field, if its non-zero elements form a group with respect to the multiplication defined in (F,A). In this section, we exclude those soft neutrosophic near-fields which are isomorphic to the soft neutrosophic near-field.

So, every soft neutrosophic near-field is zero-symmetric and L_Q-simple.

We characterize now the zero-symmetric soft neutrosophic near-rings, which are soft neutrosophic near-fields. We start with the following lemma:

Lemma 3.1. Let F(n) be a right cancellable element of a Smarandache-soft neutrosophic near-ring (F,A) over $\langle N \cup I \rangle$ contained in the (F,A) – subgroup (F,A)F(n), then (F,A) has a right identity element F(e) such that F(n) = F(e)F(n) = F(n)F(e). In particular, if F(n) is a cancellable element of (F,A) contained in (F,A)F(n), then (F,A) has a two-sided identity element.

Proof: Since F(n) is contained in (F,A)F(n), there exists an element F(e) in (F,A) such that F(e)F(n) = F(n). Then F(x)F(e)F(n) = F(x)F(n) for every element F(x) of (F,A), whence F(x)F(e) = F(x), that is F(e) is right identity element of (F,A) such that F(n) = F(e)F(n) = F(n)F(e).

If F(n) is a cancellable element contained in (F,A)F(n), then the equations F(n) = F(e)F(n) = F(n)F(e) imply that F(x)F(e) = F(x) and F(e)F(x) = F(x) for every element F(x) in (F,A).

Now we characterize the soft neutrosophic zero-symmetric near-rings which are soft neutrosophic near-fields.

Theorem 3.1. Let (F,A) be a Smarandache-soft neutrosophic near-ring over $\langle N \cup I \rangle$ which is soft neutrosophic zero-symmetric with more than one element. Then (H,A) is a soft neutrosophic near-field if and only if (H,A) has a cancellable and distributive element contained in a minimal soft neutrosophic quasi-ideal of (F,A), where (H,A) is a proper subset of (F,A), which is soft neutrosophic near-field.

Proof: Assume that (H,A) is a soft neutrosophic near-field. Then (H,A) is a minimal soft neutrosophic quasi-ideal of (F,A) and (H,A) has a two-sided identity element which is cancellable and distributive.

Conversely, assume that the soft neutrosophic zero-symmetric near-ring (H,A) has a cancellable and distributive element H(n) contained in a minimal soft neutrosophic quasi-ideal (L_Q,A) of (F,A). Then, H(n) (H,A) ∩ (H,A) H(n) is a soft neutrosophic quasi-ideal of (H,A) and it contains the non-zero element $H(n)^2$.

Moreover, H(n) (H,A) ∩ (H,A) H(n) ⊂ (L_Q,A)(H,A) ∩ (H,A) (L_Q,A) ⊂ (L_Q,A). Hence, we have (L_Q,A) = H(n)(H,A) ∩ (H,A)H(n). Therefore, (L_Q,A) ⊂ (H,A)H(n). So, by Lemma, (H,A) has a two-sided identity element H(e).

On the other hand, $H(n)^2$(H,A) ∩ (H,A)$H(n)^2$ is also soft neutrosophic quasi-ideal of (H,A), since $H(n)^2$ is distributive. Moreover, it contains the non-zero element $H(n)^3$ and is contained in the minimal soft neutrosophic quasi-ideal (L_Q,A). Hence, we have (L_Q,A) = $H(n)^2$(H,A) ∩ (H,A)$H(n)^2$. Thus, H(n) in (L_Q,A) ⊂ (H,A)$H(n)^2$ and H(n) = H(e)H(n) = H(x)$H(n)^2$ for some H(x) of (H,A). Therefore, H(e) = H(x)H(n) in (H,A)H(n). Dually, we obtain that H(e) in H(n)(H,A). So, H(e) in H(n)(H,A) ∩ (H,A)H(n) = (L_Q,A), whence (H,A) = H(e)(H,A) ∩ (H,A)H(e) ⊂ (L_Q,A) ,that is (H,A) = (L_Q,A). This relation and the minimality of (L_Q,A) imply that (H,A) is L_Q – simple. So, (H,A) is a soft neutrosophic near-field.

Theorem 3.2. Let (F,A) be a Smarandache-soft neutrosophic near-ring over $\langle N \cup I \rangle$ which is soft neutrosophic zero-symmetric with more than one element. Then, the followings are equivalent:

(i) (H,A) is a soft neutrosophic near-field;

(ii) (H,A) has a cancellable element contained in a minimal soft neutrosophic (H,A) – subgroup of $_{(H,A)}$(H,A);

(iii) (H,A) has a cacellable element contained in a minimal soft neutrosophic quasi-ideal of (H,A), where (H,A) is a soft neutrosophic near-field.

Proof: The implications (i) ⇒ (ii) and (i) ⇒ (iii) are equivalent.

(ii) ⇒ (i)

Assume H(n) to be a cancellable element contained in a minimal soft neutrosophic (H,A) – subgroup (H_1,A) of $_{(H,A)}$(H,A). Then, (H,A)H(n) is a (H,A) – subgroup of $_{(H,A)}$(H,A) containing the non-zero element $H(n)^2$ and (H,A)H(n) ⊂ (H,A)(H_1,A) ⊂ (H_1,A). So, (H,A)H(n) = (H_1,A), by the minimality of (H_1,A) and (H_1,A), has a two-sided identity element H(e) by the lemma.

On the other hand, (H,A)$H(n)^2$ is an soft neutrosophic (H,A) – subgroup of $_{(H,A)}$(H,A) containing the non-zero element $H(n)^2$ and (H,A)$H(n)^2$ ⊂ (H,A)H(n) = (H_1,A). So (H,A)$H(n)^2$ = (H_1,A) by the minimality of (H_1,A). This implies H(n) in (H_1,A) = (H,A)$H(n)^2$. Thus, H(e)H(n) = H(n) = H(x)$H(n)^2$ for some H(x) of (H,A).

Therefore, H(e) = H(x)H(n) in (H,A)H(n) = (H_1,A), that is (H_1,A) = (H,A). This relation and the minimality of (H_1,A) imply that $_{(H,A)}$(H,A) is (H,A) – simple. So (H,A) is a soft neutrosophic near-field.

(iii)⇒ (i)

Assume H(n) to be a cancellable element contained in a minimal soft neutrosophic quasi-ideal (L_Q,A) of (H,A). Then, (H,A)H(n) is a soft neutrophic quasi-ideal of (H,A) containing a non-zero element $H(n)^2$, and $H(n)^2$ in (L_Q,A).

So, $(L_Q,A) \cap$ (H,A)H(n) is a non-zero soft neutrosophic quasi-ideal of (H,A) contained in the minimal soft neutrosophic quasi-ideal (L_Q,A), whence $(L_Q,A) = (L_Q,A) \cap$ (H,A)H(n). Thus H(n) in $(L_Q,A) \subset$ (H,A)H(n) and (H,A) has a two-sided identity element H(e) by lemma.

On the other hand, $(H,A)H(n)^2$ is also a soft neutrosophic quasi-ideal of (H,A) containing a non-zero element $H(n)^2$. Similarly to the above consideration we obtain H(n) in $(L_Q,A) \subset (H,A)H(n)^2$ and H(e)H(n) = H(n) = $H(x)H(n)^2$ for some H(x) of (H,A).

Therefore H(e) = H(x)H(n) and H(e) = H(n)H(x) because H(e) is a two-sided identity element and H(n) is cancellable. Thus H(e) = H(n)H(x) = H(x)H(n) in $(L_Q,A)(H,A) \cap (H,A)(L_Q,A) \subset (L_Q,A)$, that is (L_Q,A) = (H,A). This relation and the minimality of (L_Q,A) imply that (H,A) is L_Q – simple. So (H,A) is a soft neutrosophic near-field.

Proposition 3.1. Let (F,A) be a Smarandache-soft neutrosophic near-ring $\langle N \cup I \rangle$ is L_Q - simple, then either (F,A) is soft neutrosophic zero-symmetric or (F,A) is constant.

Proof: Since the soft neutrosophic zero-symmetric part $(F,A)_0$ of (F,A) is a soft neutrosophic quasi-ideal of (F,A), either $(F,A)_0$ = (F,A) or $(F,A)_0$ = {F(0)}, that is, either (F,A) is soft neutrosophic zero-symmetric or (F,A) is constant.

Theorem 3.3. Let (F,A) be a Smarandache-soft neutrosophic near-ring over $\langle N \cup I \rangle$ with more than one element. Then, the following conditions are equivalent:

(i) (H,A) is a soft neutrosophic near-field;

(ii) (H,A) is L_Q – simple and (H,A) has a left identity;

(iii) (H,A) is L_Q – simple, H(d) ≠ {H(0)} and for each non-zero element H(n) of (H,A) there exists an element $H(n_1)$ of (H,A) such that $H(n_1)H(n) \neq H(0)$, where (H,A) is a proper subset of (H,A).

Proof:

(i)⇒(ii)

Clearly (H,A) has a left identity and (H,A) is soft neutrosophic zero-symmetric. Let (L_Q,A) be a soft neutrosophic quasi-ideal of (H,A) and $L_Q(a)$ a non-zero element of (L_Q,A), then (H,A) = $L_Q(a)$(H,A) = (H,A) $L_Q(a)$. Hence (H,A) = $L_Q(a)(H,A) \cap (H,A) L_Q(a) \subseteq (L_Q,A)(H,A) \cap (H,A)(L_Q,A) \subseteq (L_Q,A)$, whence (L_Q,A) = (H,A).

(ii) ⇒ (iii)

If (H,A) has a left identity H(e), then H(e) is non-zero and distributive. Hence H(d) ≠ {H(0)} and H(e)H(n) = H(n) ≠ H(0) for every non-zero element H(n) of (H,A).

(iii)⇒(i)

H(d) ≠ {H(0)} implies that (H,A) is not constant. Hence (H,A) is soft neutrosophic zero-symmetric by Proposition 3.1. Moreover, let H(n) be a non-zero element of (H,A), then (H,A)H(n) is a soft neutrosophic quasi-ideal of (H,A) and $H(n_1)H(n)$ in (H,A)H(n), where $H(n_1)$ is an element of (H,A) such that $H(n_1)H(n) \neq H(0)$. Hence, (H,A)H(n) = (H,A). Therefore, (H,A) is a soft neutrosophic near-field.

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