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On a Q-Smarandache Fuzzy Commutative Ideal of a Q-Smarandache BH-algebra

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Abstract

In this paper, the notions of Q-Smarandache fuzzy commutative ideal and Q-Smarandache fuzzy sub-commutative ideal of a Q-Smarandache BH-Algebra are introduced, examples and related properties are investigated. Also, the relationships among these notions and other types of Q-Smarandache fuzzy ideal of a Q-Smarandache BH-Algebra are studied.

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1 Introduction

The concept of BCK-algebra was introduced by Y. Imai and K. Iseki [18]. In 1995 the concept of n-fold commutative BCK-algebras has been introduced [7]. In 1998, Y.B. Jun, E.H. Roh and H.S. Kim introduced the notion of BH-algebra, which is a generalization of BCH/BCI/BCK-algebra [15]. In 2005, Y.B. Jun introduced the notion of a Smarandache BCI-algebra, Smarandache ideal of a Smarandache BCI-algebra [13]. In 2009, A.B. Saeid and A. Namdar, introduced the notion of a Q-Smarandache BCH-algebra and Q-Smarandache ideal of Q-Smarandache BCH-algebra [1]. In 2015, H.H. Abbass and H.K. Gatea introduced the notion Q-Smarandache Sub-Commutative ideal of a Q-Smarandache BH-Algebra [4]. In this paper we introduce the notion of Q-Smarandache fuzzy Commutative ideal and Q-Smarandache fuzzy Sub-Commutative ideal of a Q-Smarandache BH-Algebra. In this paper X denotes Q-Smarandache BH-Algebra.

2 Preliminary Notes

In this section, some basic concepts about a BH-algebra, a Q-Smarandache BH-algebra, a Q-Smarandach ideal in ordinary and fuzzy sences, Q-Smarandache sub-commutative ideal and Q-Smarandache commutative ideal of a Q-Smarandache BH-algebra are given.

Definition 2.1. *[14]. A BCI-algebra is an algebra* $(X, *, 0)$ *of type* $(2, 0)$*, where* X *is a nonempty set,* $*$ *is a binary operation and* 0 *is a constant, satisfying the following axioms: for all* $x, y, z \in X$*:*

i. $((x * y) * (x * z)) * (z * y) = 0$,

ii. $(x * (x * y)) * y = 0$,

iii. $x * x = 0$,

iv. $x * y = 0$ *and* $y * x = 0$ *imply* $x = y$.

Definition 2.2. *[11]. BCK-algebra is a BCI-algebra satisfying the axiom:* $0 * x = 0$ *for all* $x \in X$.

Definition 2.3. *[15]. A BH-algebra is a nonempty set* X *with a constant* 0 *and a binary operation * satisfying the following conditions:*

i. $x * x = 0, \forall x \in X$.

ii. $x * y = 0$ *and* $y * x = 0$ *imply* $x = y, \forall x, y \in X$.

iii. $x * 0 = x, \forall x \in X$.

Definition 2.4. *[2].*
A BCK-algebra X is called commutative if $x * (x * y) = y * (y * x), \forall x, y \in X$.

Lemma 2.5. *[2]*
In a BCI-algebra X the following conditions are equivalent:

i. $x * y = x * (y * (y * x)), \qquad \forall x, y \in X.$

ii. *X is a commutative BCK-algebra*

Definition 2.6. *[6]. A Q-Smarandache BH-algebra is defined to be a BH-algebra X in which there exists a proper subset Q of X such that*

i. $0 \in Q$ *and* $|Q| \geq 2$.

ii. *Q is a BCK-algebra under the operation of X.*

Definition 2.7. *[4] A Q-Smaradache BH-algebra is said to be a Q-Smaradache implicative BH- algebra if it satisfies the condition,* $(x*(x*y))*(y*x)=y*(y*x)$. $\forall\, x,\, y \in Q$

Definition 2.8. *[4] A Q-Smarandache BH-algebra X is called a Q-Smarandache medial BH-algebra if* $x * (x * y) = y$, $\forall\, x, y \in Q$

Definition 2.9. *[6]. A nonempty subset I of X is called a Q-Smarandache ideal of X, denoted by a Q-S.I of X if it satisfies:*

(J_1) $0 \in I$.

(J_2) $\forall\, y \in I$ *and* $x * y \in I \Longrightarrow x \in I, \forall\, x \in Q$.

Definition 2.10. *[4].A subset I of a BH-algebra X is called commutative ideal of X if it satisfies (J_1) and :*

(J_3) $(x * y) * z \in I$ *and* $z \in I \Rightarrow x * (y * (y * x)) \in I, \forall x\,, y\,, z \in X$.

Definition 2.11. *[4]. A subset I of a Q-Smarandache BH-algebra X is called a Q- Smarandache commutative ideal of X if it satisfies (J_1) and :*

(J_4) $(x * y) * z \in I$ *and* $z \in I \Rightarrow x * (y * (y * x)) \in I, \forall\, x\,, y \in Q$ *and* $z \in X$.

Definition 2.12. *[4].*
A nonempt subset I of a Q-Smarandache BH -algebra X is called a Q-Smarandache sub-commutative ideal of X if it satisfies (J_1) and :

(J_6) $(y*(y*(x*(x*y))))*z \in I$ *and* $z \in I$ *imply* $x*(x*y) \in I, \forall\, x, y \in Q, z \in X$

Definition 2.13. *[12] A fuzzy subset A of a BH-algebra X is said to be a fuzzy ideal if and only if:*

(I_1) $A(0) \geq A(x)$, $\forall\, x \in X$.

(I_2) $A(x) \geq \min\{A(x * y), A(y)\}$, $\forall\, x, y \in X$.

Definition 2.14. *[16] Let X be a BCK-algebra. A fuzzy set A in X is called a fuzzy commutative ideal of X if it satisfies*(I_1)*and*

(I_3) $A((x * (y * (y * x))) \geq \min\{((x * y) * z), (z)\} \quad \forall\; x,\; y,\; z \in X$.

We generalize the concept of a Q-Smarandache fuzzy commutative ideal to the Q-Smarandache BH-algebra.

Definition 2.15. *A fuzzy subset A of a BH-algebra X is called a fuzzy commutative ideal of X, denoted by a F.C.I if it satisfies*(I_1)*and*

(I_4) $A((x * (y * (y * x))) \geq \min\{((x * y) * z), (z)\} \quad \forall\; x,\; y,\; z \in X$.

Definition 2.16. *[10]. Let A be a fuzzy set in X,* $\forall\, \alpha \in [0, 1]$, *the set.*
A_α=$\{x \in X,\; A(x) \geq \alpha\}$ *is called a level subset of A.*
Note that, A_α *is a subset of X in the ordinary sense.*

Definition 2.17. *[6]. A fuzzy subset A of X is said to be a Q-Smarandache fuzzy ideal of X, denoted by a Q-S.F.I of X:*

(F_1) $A(0) \geq A(x)$, $\forall\, x \in X$.

(F_2) $A(x) \geq \min\{A(x * y), A(y)\}$, $\forall\, x, \in Q,\; y \in X$.

3 Main results

In this section, we introduce the concepts of a Q-Smarandache fuzzy commutative ideal and Q-Smarandache fuzzy sub-commutative ideal of a Q-Smarandache BH-algebra, and also we study some properties of them.

Definition 3.1. *A fuzzy subset A of a X is called a Q-Smarandache fuzzy commutative ideal of X, denoted by a Q-S.F.C.I if it satisfies* (F_1) *and,*

(F_3) $A(x * (y * (y * x))) \geq \min\{A((x * y) * z), A(z)\}$, *for all* $x, y \in Q,\; z \in X$.

Example 3.2. *Consider* $X = \{0, 1, 2, 3\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3
0	0	0	0	3
1	1	0	2	3
2	2	2	0	3
3	3	3	3	0

where $Q = \{0, 1\}$ *is a BCK-algebra. The fuzzy subset A defined by*
$A(0) = A(1) = A(2) = 0.6$ *and* $A(3) = 0.3$.

Proposition 3.3. *Every Q-S.F.C.I of X is Q-S.F.I of X*

Proof. Let A be Q-S.F.C.I of X ,to prove that A is a Q-S.F.I. by Definition (3.1) the condition (F_1) is satisfied .Now, let $x \in Q$ and $y \in X$. we have $x = x * (0 * (0 * x))$it follows that $A(x) = A(x * (0 * (0 * x))) \geq \min\{A(x * 0) * y), A(y)\}$[*by* 0*x=0 *and* $x * 0 = x$] implies that $A(x) \geq \min\{A(x * y), A(y)\}$. Hence A is Q-S.F.I of X. ∎

Remark 3.4. *In the following example, we see that the converse of theorem 3.3 may not be true in general.*

Example 3.5. *Consider* $X = \{0, 1, 2, 3, 4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	1	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	4	0

Where Q={0,2,3} is a BCK-algebra. The fuzzy subset A defined by $A(0) = 0.7, A(1) = 0.5$ *and* $A(2) = A(3) = A(4) = 0.3$ *A is a Q-S.F.I of X, but A is not a Q-S.F.C.I since if if x=2, y=3, z=0, then*

$$A(2 * (3 * (3 * 2))) = 0.3 \ngeq \min\{A((2 * 3) * 0), A(0)\} = 0.7$$

Theorem 3.6. *Let* A *be a Q-S.F.I of* X*. Then* A *is a Q-S.F.C.I of* X *if and only if the level subset* A_α*is a Q-S.C.I of* X, $\forall\ \alpha \in [0, A(0)]$, *such that* $A(0) = \sup_{x \in X} A(x)$

Proof. Let A be a Q-S.F.C.I of X and $\alpha \in [0, A(0)]$. To prove A_α is a Q-S.C.I of X.It is clear that $A(0) \geq \alpha$. So $0 \in A_\alpha$. Hence A_α satisfies I_1 .Now, let $x, y \in Q$, $z \in X$ such that $(x * y) * z \in A_\alpha$ and $z \in A_\alpha$, it follows that $A((x * y) * z) \geq \alpha$ *and* $A(z) \geq \alpha$ thus $\min\{A((x * y) * z), A(z)\} \geq \alpha$. But $A(x * (y * (y * x))) \geq \min\{A((x * y) * z), A(z)\}$ [*Since A is a Q-S.F.C.I of X. By definition 3.1*(F_3)] so $A(x * (y * (y * x))) \geq \alpha \Rightarrow (x * (y * (y * x))) \in A_\alpha$ Therefore, A_α is a Q-S.C.I of X.
Conversely,
Let A_α be a Q-S.C.I. of X,and $\forall\ \alpha \in [0, A(0)]$. *It is clear that* $A(0) \geq A(x)\ \ \forall x \in X$. Now, Let $x, y \in Q, z \in X$ $\alpha = \min\{A((x * y) * z), A(z)\}$ *.Then* $A((x*y)*z) \geq \alpha$ *and* $A(z) \geq \alpha$, *it follows that* $((x*y)*z) \in A_\alpha$ *and* $z \in A_\alpha$, thus $(x*(y*(y*x))) \in A_\alpha$[*Since* A_α *is a Q-S.C.I of X*]$\Rightarrow A(x*(y*(y*x))) \geq \alpha$, we get $A(x * (y * (y * x))) \geq \min\{A((x * y) * z), A(z)\}$. Therefore, A is a Q-S.F.C.I of X. ∎

Proposition 3.7. *Let A be a Q-S.F.I of X .Then A is a Q-S.F.C.I if and only if* $\forall\ x,\ y \in Q$; $\quad A(x*(y*(y*x))\ \geq A(x*y) \qquad (b_1)$

Proof. Let A be a Q-S.F.C.I.Then $A(x*(y*(y*x) \geq \min\{A((x*y)*0), A(0)\}$. [*By definition*3.1($F_3$)]. We obtain $A(x*(y*(y*x) \geq A(x*y)$[*Since* $x*0 = x$ *and* $A(0) \geq A(x) \quad \forall x \in X$]. *Hence the condition* (b_1) *is satisfied*
Conversely,
Let A be a Q-S.F.I and $x, y \in Q, z \in X$.Then $A(x*y) \geq \min\{A(x*y)*z), A(z)\}$ [*A is a Q-S.F.I*] $\Rightarrow A(x*(y*(y*x)) \geq \min\{A(x*y)*z), A(z)\}$ [*By condition* (b_1)] . Therefore, A is a Q-S.F.C.I of X . ∎

Theorem 3.8. *Let A be a Q-S.F.I of a commutative Q- Smarandache BH-algebra X such that Q is a commutative BCK-algebra . Then A is a Q-S.F.C.I of X.*

Proof. Let A be a Q-S.F.I of X.To prove that A is Q-S.F.C.I. By Definition (2.17) the condition (F_1) is satisfied. Now, let $x, y \in Q$ and $z \in X$. Then $A(x*y) \geq \min\{A((x*y)*z), A(z)\}$[*From Definition* 2.17($F_2$)] implies that $A(x*(y*(y*x))) \geq \min\{A((x*y)*z), A(z)\}$ [*Since Q is commutative BCK-algebra*,by Lemma 2.5(i)].Hence*A is a Q-S.F.C.I of X.* ∎

Definition 3.9. *Let n be a positive integer. A nonempty subset I of X is called a Q-Smarandache n-fold commutative ideal of X. denoted by a Q-S .n-fold C.I of X if it satisfies* (J_1) *and :*

(J_5) $(x*y^n)*z \in I$ *and* $z \in I \Rightarrow x*(y^n*(y^n*x)) \in I, \forall\ x\ , y\ \in Q$ *and* $z \in X$.

Example 3.10. *Consider* $X = \{0, 1, 2, 3, 4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

where Q={0,1} is a BCK-algebra . Then $I = \{0, 1, 2\}$ *is A is a Q-S.2-fold.C.I*

Definition 3.11. *Let n be a positive integer. A fuzzy subset A of a X is called a Q-Smarandache fuzzy n-fold commutative ideal of X, denoted by a Q-S.F .n-fold.C.I of X if it satisfies* (F_1) *and,*

(F_4). $A(x*(y^n*(y^n*x)) \geq \min\{A(x*y^n)*z), A(z)\}$, *for all* $x, y \in Q$, $z \in X$

Example 3.12. *Consider* $X = \{0,1,2,3,4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

where Q={0,1} is a BCK-algebra. The fuzzy subset A defined by .
$A(0) = A(1) = A(2) = 0.8$ *and* $A(3) = A(4) = 0.5$ *A is a Q-S.F.2-fold.C.I.*

Proposition 3.13. *Every Q-S.n-fold.F.C.I of X is Q-S.F.I of X*

Proof. let A Q-S.F.C.I of X To prove that A is Q-S.F.I. by Defintion (3.11) the condition (F_1) is satisfied .Now, let $x \in Q$ and $y \in X$. we have $x = (x*(0^n*(0^n*x))$ it follows that $A(x) = A(x*(0^n*(0^n*x)) \geq \min\{A(x*0^n)*y), A(y)\}[by\ \ 0*x = 0\, and\ \ x*0 = x]$ implies that $A(x) \geq \min\{A(x*y), A(y)\}$. Hence A is Q-S.F.I of X. ∎

Remark 3.14. *In the following example, we see that the converse of Proposition 3.13 may not be true in general.*

Example 3.15. *Consider* $X = \{0,1,2,3,4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	0	4
2	2	2	0	1	4
3	3	3	3	0	4
4	4	4	4	4	0

where Q={0,1,2} is a BCK- algebra. The fuzzy subset A defined by .
$A(0) = 0.8\,\text{and}\, A(1) = A(2) = A(3) = A(4) = 0.5$ *Is Q-S.F.I of X, but it is not 1-fold Q-S.F.C.I of X. Since x=1, y=2, z=0*

$$A(1*(2*(2*1) = 0.5 \ngeq \min\{A((1*2)*0), A(0)\} = 0.8$$

Theorem 3.16. *Let A be a Q-S.F.I of X .Then A is a Q-S.F.n-fold C.I if and only if*
$\forall\, x,\ y \in Q, \quad A(x*(y^n*(y^n*x))) \geq A(x*y^n) \qquad (b_2)$

Proof. Let A be a Q-S.F.n-fold C.I of X and $x, y \in Q$

$$A(x*(y^n*(y^n*x) \geq \min\{A((x*y^n)*0), A(0)\}.[\text{By definition } 3.11\ (F_4)]$$
$$\Longrightarrow A(x*(y^n*(y^n*x) \geq A(x*y^n)[\text{Since } x*0=x, A(0) \geq A(x).\forall x \in X)]$$
$$\Longrightarrow \text{The condition } (b_2) \text{ is satisfied.}$$

Conversely,
let A be a Q-S.F.I of X, $x, y \in Q$ and $x \in X$. Then

$$A(x*y^n) \geq \min\{A((x*y^n)*z), A(z)\}[\text{Since } A \text{ is a Q-S.F.I of } X\ ]$$
$$\Longrightarrow A(x*(y^n*(y^n*x)) \geq \min A\{((x*y^n)*z), A(z)\}\ [\text{By condition}(b_2)]$$

Therefore, A is a Q-S.F.n-fold .C.I of X ∎

Definition 3.17. *A fuzzy subset A of X is called a Q-Smarandache fuzzy sub-commutative ideal of X, denoted by a Q-S.F.S.C.I of X if it satisfies* (F_1) *and,*

(F_5) $A(x*(x*y)) \geq \min\{A(y*(y*(x*(x*y)))*z), A(z)\}\ \forall\ x,\ y \in Q,\ z \in X.$

Example 3.18. *Consider* $X = \{0, 1, 2, 3\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

where Q={0,1} is a BCK-algebra. The fuzzy subset A defined by $A(0) = A(1) = A(2) = 0.6$ *and* $A(3) = 0.3$ *A is Q-S.F.C.I of X .*

Theorem 3.19. *Let A be a Q-S.F.S.C.I of X. Then A is a Q-S.F.I of X.*

Proof. Let A be a Q-S.F.S.C.I of X.It is clear that the condition (F_1) is satisfied .Now, let $x \in Q$ and $y \in X$, we have $A(x*(x*x)) \geq \min\{A(x*(x*(x*(x*x)))*y), A(y)\}$,[*By Definition* 3.17$(F_5)$] it follows that $A(x*0) \geq \min\{A(x*(x*(x*0)*y), A(y)\}$[*Since Q is a BCK-algebra* $x*x=0$] implies that $A(x) \geq \min\{A(x*y), A(y)\}$[*Since Q is a BCK-algebra* $x*0=x$] . Hence*A is a Q-S.F.I of X.* ∎

Remark 3.20. *In the following example shows that the converse of theorem 3.19 may not be true in general.*

Example 3.21. *Consider* $X = \{0, 1, 2, 3\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	1
3	3	3	3	0

Where Q={0,1,2} is a BCK-algebra. The fuzzy subset A defined by $A(0) = A(3) = 0.9$, *and* $A(1) = A(2) = 0.5$ *A is a Q-S.F.I of X, but it is not a Q-S.F.S.C.I. Since, x=1, y=2, z=0*

$$A(1 * (1 * 2)) \ngeq \min\{A(2 * (2 * (1 * (1 * 2))) * 0), A(0)\}$$

Theorem 3.22. *Let A be a Q-S.F.I of X. Then A is a Q-S.F.S.C.I of X if and only if it is* $\forall\ x, y \in Q, \quad A(x * (x * y)) \geq A(y * (y * (x * (x * y)))$ (b_3)

Proof. *Suppose A is a Q-S.F.S.C.I of X. Let* $x, y \in Q$.Then $A(x * (x * y)) \geq \min\{A(y * (y * (x * (x * y)) * 0)), A(0)\}$ [*By definition* 3.17(F_5)]it follows that $A(x * (x * y)) = \min\{A(y * (y * (x * (x * y)))), A(0)\}$ [Since X ;$x * 0 = x$]implies that $A(x * (x * y)) \geq A(y * (y * (x * (x * y)))$ [$A(0) \geq A(x)\ \forall x \in X$]. *By definition* 3.17($F_1$)]. Hence *The condition* (b_3) *is satisfied.*
Conversely,
Let A be a Q-S.F.I of X and the condition (b_2) satisfied.To prove that A is Q-S.F.S.C.I. By Definition (2.17) the condition (F_2) is satisfied. Now, let $x, y \in Q$ *and* $z \in X$ we have $A(y * (y * (x * (x * y)))) \geq \min\{A(y * (y * (x * (x * y))) * z), A(z)\}$[*Since A is a Q-S.F.I of X, by Definition* 2.17 (F_2)] implies that $A(x * (x * y)) \geq \min\{A(y * (y * (x * (x * y))) * z), A(z)\}$[$By(b_3)$].Hence *A is a Q-S.F. S.C.I of X.* ∎

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