

# New Lower Bounds for Van der Waerden Numbers

Alexey V. Komkov

January 10, 2017

## Abstract

This work contains certificates numbers Van der Waerden, was found using SAT Solver. These certificates establish the best currently known lower bounds of the numbers Van der Waerden  $W(7, 3)$ ,  $W(8, 3)$ ,  $W(9, 3)$ ,  $W(10, 3)$ ,  $W(11, 3)$ .

## 1 Introduction

Van der Waerden's theorem states that for any positive integers  $r$  and  $k$  there exists a positive integer  $N$  such that if the integers  $\{1, 2, \dots, N\}$  are colored, each with one of  $r$  different colors, then there are at least  $k$  integers in arithmetic progression all of the same color. The smallest such  $N$  is the van der Waerden number  $W(r, k)$ . Van der Waerden Numbers are quite difficult to calculate, at the time of writing the article, the exact values are known only for 7 van der Waerden numbers, and for the rest of the numbers only bounds are known. To prove the lower bound of a van der Waerden number it is sufficient to get a certificate of this number, i. e., a sequence of numbers  $\{1, 2, \dots, N\}$  of  $r$  colors, and not having same-coloured arithmetic progressions of length  $k$ . Then  $N+1$  will be the lower bound of  $W(r, k)$  number.

## 2 Certificates

Certificate  $W(7, 3) > 296$

535031154544040026246451356610100303342102016562263633535504634362164425255151130250524620041411  
616653413140243306066262215065603405532322424461321235031154544040026546451356610100303342102016  
512263633535504634362164425255151130250524620041411616653413140243306066262215065633405532324424  
46162125

Certificate  $W(8, 3) > 417$

755740655401747356577677643446375443706362454665665323352643326752513435545542122415322156414023  
244344310113042110453037121332332070027310073427260102212217677162077623161570711011065660517665  
120504676007007545574065540174735657767764344637544370636245466566532335264332675251343554554212  
241532215641402324434431011304211045303712133233207002731007342726010221221767716207762316157071  
101106566051766512050467600700757

Certificate  $W(9, 3) > 523$

```
766566553226204403081761850886554554421151833828706507487754434433100407227176854863766433233220
883861160657437526553221221187727500585463264154421101100766164884743521530433100800886550537736
324104283220887887754484266252130831721187767766433731551410287206100766566553226204403081761850
886554554421151833828706507487754434433100407227176854863766433233220883861160657437526553221221
187727500585463264154421101100766164884743521530433100800886550537736324104283220887887754484266
2521308317211877677664337315514102872061002
```

Certificate  $W(10, 3) > 641$

```
900100113778561831794056081847886112112248891729328051671929589972232233599028305388627820306900
833433446001894164027389314170119445445571129052751084904252312205565566822301638624950153639233
166766779334127497320612647403442778778804457385084317237585145538898899155634961954283486962566
499099002667450720653945970736775001001137785618317640560818478861121122488967294287516719295899
722322335990283053986278203069008334334460013941640973893141701194454455711240527510849042528122
055655668223016386219501536392331667667793341274973206126474034427787788044573850843172375851455
38898899155684961954283486962566499099002667950720683945970736775
```

Certificate  $W(11, 3) > 776$

```
4A094143721311443306A2A972042186A5239A890363122003323A5A19071A3907A931289772212211AA221290806
A60928A098208486791041009912018A7A794A8179A871A0675500A00AA8800A079696549706897609A5844AA9AA
9977AA9A685857386A57865A89153499899886699895747462759467549783423887887755887846363516483564386
72312776776644776735252405372453275612011656655336656241413A4261342164501A055455442255451303029
31502310534A09A443443311443402A2A18204A120A4239A893323322003323A1919071A3901A931289782212211A
A221290808A60928A098201786718011009911018A7A795A8179A871A0675600A00AA8800A079696849706897609A
5645AA9AA9977AA9A685857386A57865A8945349989988669989574743275946754978342288788775588784636151
648356438672312776776644776732252405372303275614016656455336656641413A42613A2164503AA5545544225
54413030293150231053A221A1
```

### 3 Results

| Number $W(r, k)$ | Old Lower Bounds | New Lower Bounds |
|------------------|------------------|------------------|
| $W(7, 3)$        | $> 273$ [1]      | $> 296$          |
| $W(8, 3)$        | $> 354$ [1]      | $> 417$          |
| $W(9, 3)$        | $> 454$ [1]      | $> 523$          |
| $W(10, 3)$       | $> 592$ [1]      | $> 641$          |
| $W(11, 3)$       | $> 731$ [1]      | $> 776$          |

### References

1. Alexey V. Komkov, "New Lower Bounds for Van Der Waerden Numbers Using Genetic Algorithm", 2016