Numbers in base b that generate primes with help the Luhn function of order ω

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Abstract

The number $12_{(8)} = 10_{(10)}$ generated the prim number $389_{(10)}$ with help the Luhn function of order 2. Indeed, we have $(12_{(8)})^2 + (21_{(8)})^2 = 605_{(8)} = 389_{(10)}$ and $389_{(10)}$ is a prime number.

We put the problem to determine the sets of integers in base $b \geq 2$ that generate primes with using a function.

1 Introduction

Let us denote the set of prime numbers by $\mathbb{P}$. We put the problem to determine a function to generate as many primes. As is well known, the Euler polynomial

$$P_E(n) = n^2 + n + 41 \ ,$$

generates 40 primes for $n = 0, 1, \ldots, 39$. We can say that $P_E(n) \in \mathbb{P}$ for any $n = 0, 1, \ldots, 39$.

There are many polynomials generating prime numbers, for example polynomial's Dress, Laudreau and Gupta

$$P_{DLG}(n) = \left| \frac{n^5 - 133n^4 + 6729n^3 - 158379n^2 + 1720294n - 6823316}{4} \right| \ ,$$

which for $n \in \{0, 1, \ldots, 56\}$ generates 57 primes, [2]. Then we can say that $P_{DLG}(n) \in \mathbb{P}$ for any $n \in \{0, 1, \ldots, 56\}$.

Let be the function, $R : \mathbb{N} \to \mathbb{N}$, reverse the number $n_{(b)} = \overline{d_1 d_2 \ldots d_m}$, where $d_k \in \{0, 1, \ldots, b-1\}$, $R(n_{(b)}) = \overline{d_m d_{m-1} \ldots, d_1}$, A056964.

We define Luhn function of the order ω with reference to base of numeration b,

$$L_{b,\omega} : \mathbb{N} \to \mathbb{N}, \qquad L_{b,\omega}(n) = (n_{(b)})^{\omega} + (R(n_b))^{\omega} \ . \tag{1}$$

Definition 1. The numbers $n \in \mathbb{N}$, that satisfy the condition $L_{b,\omega}(n) \in \mathbb{P}$, are called *integers that generates primes* with help the function $L_{b,\omega}$.

To easier track the results, we convert the numbers in base $b \neq 10$ in numbers in base $b = 10$.

We agree that the numbers in base 10, do not indicate numeration basis. For example, we say that the number $12_{(8)} = 10$ generates on 389, prime number, using the Luhn function $L_{8,2}$ because $(12_{(8)})^2 + (21_{(8)})^2 = 605_{(8)} = 389 \in \mathbb{P}$. We prefer to work with function Luhn, $L_{b,\omega}$, given following form:

$$L(n, b, \omega) = n^{\omega} + \Big(\big(R(n_{(b)})\big)_{(10)}\Big)^{\omega} \ . \tag{2}$$

The function defined by (1) is equivalent to the function given (2).

2 Programs for the Luhn function

Program 2. *The program counting digits of n in base b.*

$$nrd(n, b) := 1 + floor\big(log(n, b)\big) \ .$$

Program 3. *The program determine digits the number n in base b.*

$$dn(n,b) := \left| \begin{array}{l} return\ (0)\ if\ n{=}0 \\ for\ k \in 1..nrd(n,b) \\ \quad \left| \begin{array}{l} t \leftarrow Trunc(n,b) \\ cb_k \leftarrow n - t \\ n \leftarrow \dfrac{t}{b} \end{array} \right. \\ return\ cb \end{array} \right.$$

Program 4. *The program determine reverse of the number n, given in base b, and returns the reverse in base 10.*

$$R(n,b) := \left| \begin{array}{l} cb \leftarrow dn(n,b) \\ m \leftarrow length(cb) \\ for\ k \in 1..m \\ \quad Rn \leftarrow Rn + cb_k \cdot b^{m-k} \\ return\ Rn \end{array} \right.$$

Program 5. *Program for Luhn function, given by (2).*

$$L(n, b, \omega) := n^{\omega} + R(n, b)^{\omega} \ .$$

3 Theorems

Theorem 6. *The sum of the number, in base b with an even number of digits, with their reverse is not prime*

Proof.

1. Case $b = 2\beta$

   (a) n with 2 digits

   $$n_{(2\beta)} + R(n_{(2\beta)}) = (d_1 \cdot 2\beta + d_2) + (d_2 \cdot 2\beta + d_1) = (2\beta+1)(d_1+d_2) \notin \mathbb{P} \ ,$$

   if $d_1 + d_2 \neq 1$.

   (b) n with 4 digits

   $$\begin{aligned} & n_{(2\beta)} + R(n_{(2\beta)}) \\ & \quad = [d_1(2\beta)^3 + d_2(2\beta)^2 + d_3(2\beta) + d_4] + [d_4(2\beta)^3 + d_3(2\beta)^2 + d_2(2\beta) + d_1] \\ & \qquad = (2\beta+1)[(2\beta^2+1)(d_1+d_4) - 2\beta(d_1 - d_2 - d_3 + d_4) + d_1 + d_4] \notin \mathbb{P} \ , \end{aligned}$$

   because

   $$(2\beta^2+1)(d_1+d_4) - 2\beta(d_1 - d_2 - d_3 + d_4) + d_1 + d_4 \neq 1 \ .$$

   For any $d_1, d_2, d_3, d_4 \in \{0, 1, \ldots, 9\}$ and $d_1 \neq 0$ does not exist β, $\beta \in \mathbb{N}$, solution of the equation

   $$(2\beta^2+1)(d_1+d_4) - 2\beta(d_1 - d_2 - d_3 + d_4) + d_1 + d_4 = 1 \ .$$

   (c) n with 6 digits $\ldots$.

2. Case $b = 2\beta + 1$

   (a) n with 2 digits

   $$\begin{aligned} & n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\ & \qquad = [d_1(2\beta+1) + d_2] + [d_2(2\beta+1) + d_1] = 2(\beta+1)(d_1+d_2) \notin \mathbb{P} \ . \end{aligned}$$

   (b) n with 4 digits

   $$\begin{aligned} & n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\ & \qquad = [d_1(2\beta+1)^3 + d_2(2\beta+1)^2 + d_3(2\beta+1) + d_4] \\ & \qquad + [d_4(2\beta+1)^3 + d_3(2\beta+1)^2 + d_2(2\beta+1) + d_1] \\ & \qquad\quad = 2(\beta+1)[4\beta^2(d_1+d_4) + (2\beta+1)(d_1+d_2+d_3+d_4)] \notin \mathbb{P} \ . \end{aligned}$$

(c) n with 6 digits … .

$$\begin{aligned}
&n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\
&= [d_1(2\beta+1)^5 + d_2(2\beta+1)^4 + d_3(2\beta+1)^3 + d_4(2\beta+1)^2 + d_5(2\beta+1) + d_6] \\
&+ [d_6(2\beta+1)^5 + d_5(2\beta+1)^4 + d_4(2\beta+1)^3 + d_3(2\beta+1)^2 + d_2(2\beta+1) + d_1] \\
&= 2(\beta+1)[16\beta^4(d_1+d_6) + 8\beta^3(3d_1+d_2+d_5+3d_6) \\
&+ 4\beta^2(4d_1+2d_2+d_3+d_4+2d_5+4d_6) \\
&+ (4\beta+1)(d_1+d_2+d_3+d_4+d_5+d_6)] \notin \mathbb{P}
\end{aligned}$$

The proof is analogous for numbers with 8, 10, …, $2m$ digits . □

Remark 7. The theorem 6 has an exception case when $d_1 = 1$ and $d_2 = 0$, then if $b = 2\beta$ we have

$$n_{(2\beta)} + R(n_{(2\beta)}) = (d_1 \cdot 2\beta + d_2) + (d_2 \cdot 2\beta + d_1) = (2\beta+1)(d_1+d_2) = 2\beta+1 \ ,$$

where $2\beta+1$ may be a prime number. See case $10 + 01 = 11 \in \mathbb{P}$.

Theorem 8. *The sum of numbers, in odd base, with their reverse are not primes.*

Proof.

1. n with 2 digits

$$\begin{aligned}
&n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\
&= [d_1(2\beta+1) + d_2] + [d_2(2\beta+1) + d_1] = 2(\beta+1)(d_1+d_2) \notin \mathbb{P} \ .
\end{aligned}$$

2. n with 3 digits

$$\begin{aligned}
&n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\
&= [d_1(2\beta+1)^2 + d_2(2\beta+1) + d_3] + [d_3(2\beta+1)^2 + d_2(2\beta+1) + d_1] \\
&= 2[2\beta^2(d_1+d_2) + (2\beta+1)(d_1+d_2+d_3)] \notin \mathbb{P} \ .
\end{aligned}$$

3. n with 4 digits

$$\begin{aligned}
&n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\
&= [d_1(2\beta+1)^3 + d_2(2\beta+1)^2 + d_3(2\beta+1) + d_4] \\
&+ [d_4(2\beta+1)^3 + d_3(2\beta+1)^2 + d_2(2\beta+1) + d_1] \\
&= 2(\beta+1)[4\beta^2(d_1+d_4) + (2\beta+1)(d_1+d_2+d_3+d_4)] \notin \mathbb{P}
\end{aligned}$$

4. n with 5 digits

$$
\begin{aligned}
&n_{(2\beta+1)} + R(n_{(2\beta+1)}) \\
&\qquad = [d_1(2\beta+1)^4 + d_2(2\beta+1)^3 + d_3(2\beta+1)^2 + d_4(2\beta+1) + d_5] \\
&\qquad + [d_5(2\beta+1)^4 + d_4(2\beta+1)^3 + d_3(2\beta+1)^2 + d_2(2\beta+1) + d_1] \\
&= 2\big[8\beta^4(d_1+d_5) + 4\beta^3(4d_1+d_2+d_4+4d_5) + 2\beta^2(6d_1+3d_2+2d_3+3d_4+6d_5) \\
&\qquad + (4\beta+1)(d_1+d_2+d_3+d_4+d_5)]\big] \notin \mathbb{P} \ ,
\end{aligned}
$$

5. $\dots$.

The proof is analogous for numbers with 6, 7, ..., m digits . □

Theorem 9. *The function $L_{b,\omega}$, given by (1), does not generate primes for any $n \in \mathbb{N}$, $n > 1$, if $\omega \in \mathbb{N}$ and $\omega \neq 2^\eta$, where $\eta \in \mathbb{N}$.*

Proof. We have one identity

$$\alpha^{2m+1} + \beta^{2m+1} = (\alpha+\beta) \cdot \sum_{j=0}^{2m} (-1)^j \alpha^{2m-j} \beta^j \ .$$

1. If $\omega = 2m+1$, then for $\forall n \in \mathbb{N}$ and $\forall m \in \mathbb{N}$ then

$$L_{b,\omega}(n) = n^\omega + R(n)^\omega = n^{2m+1} + R(n)^{2m+1} \notin \mathbb{P} \ ,$$

   then it follows that $L_{b,\omega}(n) \notin \mathbb{P}$.

2. If ω is an even number, then:

   (a) $\omega = 2^{\eta-k}(2m+1)$, cu $1 \leq k \leq \eta - 1$, then

$$
\begin{aligned}
\alpha^\omega + \beta^\omega &= (\alpha^{2^{\eta-k}})^{2m+1} + (\beta^{2^{\eta-k}})^{2m+1} \\
&= (\alpha^{2^{\eta-k}} + \beta^{2^{\eta-k}}) \cdot \sum_{j=0}^{2m} (-1)^j (\alpha^{2^{\eta-k}})^{2m-j} (\beta^{2^{m-k}})^j \ ,
\end{aligned}
$$

   therefore for $\forall n \in \mathbb{N}$ and $\forall m \in \mathbb{N}$ result that $L_{b,\omega}(n) = n^\omega + R(n)^\omega \notin \mathbb{P}$.

   (b) $\omega = 2^\eta$, then we can not say anything about $\mathbb{L}_{b,\omega}(n)$.

□

Theorem 10. *For $\omega \geq 4$ primes generated by the Luhn functions $L_{b,\omega}$ have the last digit 1 or 7.*

Proof. Let $n = \overline{d_1 \ldots d_m}$ be a natural number with the first digit d_1 and the last digit d_m. The last digit of the number $L_{b,\omega}(n) = n^\omega + R(n)^\omega$ is the sum $d_1^\omega + d_m^\omega$, where d_1 and $d_m \in \{0, 1, 2, \ldots, 9\}$. The functions that give us the first and last digit of the number n are:

$$f(n) = trunc\big(n \cdot 10^{-floor(\lg(n))}\big)$$

and

$$\ell(n) = mod(n, 10) \ .$$

The last digit of the number $L_{b,\omega}(n)$ is given by

$$u(L_{b,\omega}(n)) = \ell\big(f(L_{b,\omega}(n))^\omega + \ell(L_{b,\omega}(n))^\omega\big) \ ,$$

where $\text{mod}(a, b)$ is the function "*rest of the division*" a to b. Analyzing all possible cases result as

$f(n)$	1	1	1	1	1	2	2	2	2	2	3	3	3
$\ell(n)$	0	2	4	6	8	1	3	5	7	9	0	2	4
$u(L_{b,\omega}(n))$	1	7	7	7	7	7	7	1	7	7	1	7	7

3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	6
6	8	1	3	5	7	9	2	4	6	8	1	3	5	7	9
7	7	7	7	1	7	7	1	1	1	1	7	7	1	7	7

7	7	7	7	7	8	8	8	8	8	9	9	9	9	9
0	2	4	6	8	1	3	5	7	9	0	2	4	6	8
1	7	7	7	7	7	7	1	7	7	1	7	7	7	7

.

□

4 The sets $\mathbb{G}$

We denote $\mathbb{G}_{b,\omega} = \{n \mid n \in \mathbb{N},\ L(n, b, \omega) \in \mathbb{P}\}$. We determine the $\mathbb{G}_{b,\omega}$ with $1 \le n \le 10^3$ set of numbers that have the property that $L_{b,\omega}(\mathbb{G}_{b,\omega}) \subset \mathbb{P}$.

The number $1_{(b)} = 1 \in \mathbb{G}_{b,\omega}$ for any $b$and ω because $L_{b,\omega}(1) = 2 \in \mathbb{P}$ for any b base of numeration and $\omega \in \mathbb{N}^*$. According the theorem 8, the sets $\mathbb{G}_{b,\omega}$ with base b odd number are empty sets. According the theorem 9 the Luhn functions $L_{b,\omega}$ with $\omega \ne 2^m$ not generate primes. Therefore we have the following sets of integers which generates primes with help the Luhn function (1):

1. Sets $\mathbb{G}$ of order 1: $\mathbb{G}_{2,1}$, $\mathbb{G}_{4,1}$, $\mathbb{G}_{6,1}$, $\mathbb{G}_{8,1}$, $\mathbb{G}_{10,1}$, $\mathbb{G}_{12,1}$, $\mathbb{G}_{14,1}$, $\mathbb{G}_{16,1}$, $\ldots$,

2. Sets $\mathbb{G}$ of order 2: $\mathbb{G}_{2,2}$, $\mathbb{G}_{4,2}$, $\mathbb{G}_{6,2}$, $\mathbb{G}_{8,2}$, $\mathbb{G}_{10,2}$, $\mathbb{G}_{12,2}$, $\mathbb{G}_{14,2}$, $\mathbb{G}_{16,2}$, $\ldots$,

3. Sets $\mathbb{G}$ of order 4: $\mathbb{G}_{2,4}$, $\mathbb{G}_{4,4}$, $\mathbb{G}_{6,4}$, $\mathbb{G}_{8,4}$, $\mathbb{G}_{10,4}$, $\mathbb{G}_{12,4}$, $\mathbb{G}_{14,4}$, $\mathbb{G}_{16,4}$, $\ldots$,

4. Sets $\mathbb{G}$ of order 8: $\mathbb{G}_{2,8}$, $\mathbb{G}_{4,8}$, $\mathbb{G}_{6,8}$, $\mathbb{G}_{8,8}$, $\mathbb{G}_{10,8}$, $\mathbb{G}_{12,8}$, $\mathbb{G}_{14,8}$, $\mathbb{G}_{16,8}$, $\ldots$,

5. Sets $\mathbb{G}$ of order 16: $\mathbb{G}_{2,16}$, $\mathbb{G}_{4,16}$, $\mathbb{G}_{6,16}$, $\mathbb{G}_{8,16}$, $\mathbb{G}_{10,16}$, $\mathbb{G}_{12,16}$, $\mathbb{G}_{14,16}$, $\mathbb{G}_{16,16}$, $\ldots$,

6. Sets $\mathbb{G}$ of order 32: $\mathbb{G}_{2,32}$, $\mathbb{G}_{4,32}$, $\mathbb{G}_{6,32}$, $\mathbb{G}_{8,32}$, $\mathbb{G}_{10,32}$, $\mathbb{G}_{12,32}$, $\mathbb{G}_{14,32}$, $\mathbb{G}_{16,32}$, $\ldots$,

7. Sets $\mathbb{G}$ of order 64: $\mathbb{G}_{2,64}$, $\mathbb{G}_{4,64}$, $\mathbb{G}_{6,64}$, $\mathbb{G}_{8,64}$, $\mathbb{G}_{10,64}$, $\mathbb{G}_{12,64}$, $\mathbb{G}_{14,64}$, $\mathbb{G}_{16,64}$, $\ldots$,

8. Sets $\mathbb{G}$ of order 128: $\mathbb{G}_{2,128}$, $\mathbb{G}_{4,128}$, $\mathbb{G}_{6,128}$, $\mathbb{G}_{8,128}$, $\mathbb{G}_{10,128}$, $\mathbb{G}_{12,128}$, $\mathbb{G}_{14,128}$, $\mathbb{G}_{16,128}$, $\ldots$,

9. $\ldots$.

Ignoring case number 1 which generates the prime number 2 for any b and ω we can to say that just sets $\mathbb{G}_{b,\omega}$ for b even number and $\omega = 2^m$, $m \in \mathbb{N}^*$ are possible nonempty.

5 The sets $\mathbb{G}$ of order 1

5.1 The set $\mathbb{G}_{2,1}$

Sample numbers from the set $\mathbb{G}_{2,1}$:

- $n = 1 = 1_{(2)}$, $L_{2,1}(n) = 1_{(2)} + R(1_{(2)}) = 10_{(2)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{2,1}$,
- $n = 2 = 10_{(2)}$, $L_{2,1}(n) = 10_{(2)} + R(10_{(2)}) = 11_{(2)} = 3 \in \mathbb{P} \Rightarrow 2 \in \mathbb{G}_{2,1}$,
- $n = 4 = 100_{(2)}$, $L_{2,1}(n) = 100_{(2)} + R(100_{(2)}) = 101_{(2)} = 5 \in \mathbb{P} \Rightarrow 4 \in \mathbb{G}_{2,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,1}$ is: 1, 2, 4, 16, 26, 76, 86, 88, 106, 116, 118, 256, 274, 278, 284, 296, 298, 308, 314, 326, 332, 334, 338, 344, 356, 358, 368, 394, 400, 404, 406, 418, 424, 440, 452, 454, 460, 464, 466, 484, 494 .

Table 1: The set $\mathbb{G}_{2,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{2,1}$	$L_{2,1}(n) \in \mathbb{P}$
1	2
2	3
4	5
16	17
26	37
76, 88	101
86, 116	139
106	149
118	173
256	257
296	337

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$n \in \mathbb{G}_{2,1}$	$L_{2,1}(n) \in \mathbb{P}$
284, 308, 344, 368	397
274, 400	419
332, 356	433
298, 424	467
278, 338, 404, 464	487
314, 440	499
326, 452	523
394, 418	557
334, 358, 460, 484	563
406, 466	617
454	653
494	733

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,1}$ contains 41 integers with generates 23 very probable primes using the Luhn function $L_{2,1}$.

5.2 The set $\mathbb{G}_{4,1}$

Sample numbers from the set $\mathbb{G}_{4,1}$:

- $n = 1 = 1_{(4)}$, $L_{4,1}(n) = 1_{(4)} + R(1_{(4)}) = 2_{(4)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{4,1}$,
- $n = 4 = 10_{(4)}$, $L_{4,1}(n) = 10_{(4)} + R(10_{(4)}) = 11_{(4)} = 5 \in \mathbb{P} \Rightarrow 4 \in \mathbb{G}_{4,1}$,
- $n = 16 = 100_{(4)}$, $L_{4,1}(n) = 100_{(4)} + R(100_{(4)}) = 101_{(4)} = 17 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{4,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,1}$ is: 1, 4, 16, 22, 26, 28, 37, 41, 43, 47, 52, 56, 58, 62, 256, 262, 266, 268, 292, 296, 298, 302, 304, 308, 316, 322, 326, 328, 352, 356, 358, 362, 364, 368, 376, 386, 388, 416, 418, 422, 424, 428, 436, 448, 482, 484, 488, 496, 508, 517, 521, 527, 539, 553, 557, 559, 563, 577, 581, 587, 599, 613, 617, 619, 623, 641, 647, 659, 671, 673, 677, 679, 683, 703, 707, 719, 731, 737, 739, 743, 763, 767, 772, 776, 782, 794, 808, 812, 814, 818, 832, 836, 842, 854, 868, 872, 874, 878, 896, 902, 914, 926, 928, 932, 934, 938, 958, 962, 974, 986, 992, 994, 998 .

Table 2: The set $\mathbb{G}_{4,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{4,1}$	$L_{4,1}(n) \in \mathbb{P}$
1	2

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$n \in \mathbb{G}_{4,1}$	$L_{4,1}(n) \in \mathbb{P}$
4	5
16	17
28	41
22, 37, 52	59
26, 41, 56	67
43, 58	101
47, 62	109
256	257
304	353
292, 352	389
308, 368	421
296, 356, 416	457
268, 328, 388, 448	461
316, 376, 436, 496	557
364, 424, 484	593
428, 488	661
508	761
262, 322, 517, 577, 772, 832	839
266, 326, 386, 521, 581, 641, 776, 836, 896	907
298, 358, 418, 553, 613, 673, 808, 868, 928	971
302, 362, 422, 482, 557, 617, 677, 737, 812, 872, 932, 992	1039
563, 818	1381
539, 599, 659, 794, 854, 914	1453
527, 587, 647, 707, 782, 842, 902, 962	1489
559, 619, 679, 739, 814, 874, 934, 994	1553
623, 683, 743, 878, 938, 998	1621
671, 731, 926, 986	1657
719, 974	1693
703, 763, 958	1721
767	1789

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,1}$ contains 113 integers with generates 31 primes using the Luhn function $L_{4,1}$.

5.3 The set $\mathbb{G}_{6,1}$

Sample numbers from the set $\mathbb{G}_{6,1}$:

- $n = 1 = 1_{(6)}$, $L_{6,1}(n) = 1_{(6)} + R(1_{(6)}) = 2_{(6)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{6,1}$,

- $n = 6 = 10_{(6)}$, $L_{6,1}(n) = 10_{(6)} + R(10_{(6)}) = 11_{(6)} = 7 \in \mathbb{P} \Rightarrow 6 \in \mathbb{G}_{6,1}$,
- $n = 36 = 100_{(6)}$, $L_{6,1}(n) = 100_{(6)} + R(100_{(6)}) = 101_{(6)} = 37 \in \mathbb{P} \Rightarrow 36 \in \mathbb{G}_{6,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,1}$ is: 1, 6, 36, 46, 48, 54, 64, 66, 81, 83, 89, 99, 101, 116, 118, 124, 134, 136, 151, 153, 159, 169, 171, 186, 188, 194, 204, 206 .

Table 3: The set $\mathbb{G}_{6,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{6,1}$	$L_{6,1}(n) \in \mathbb{P}$
1	2
6	7
36	37
48	61
54	73
66	97
46, 81, 116, 151, 186	197
64, 99, 134, 169, 204	233
83, 118, 153, 188	271
89, 124, 159, 194	283
101, 136, 171, 206	307

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,1}$ contains 28 integers with generates 11 primes using the Luhn function $L_{6,1}$.

5.4 The set $\mathbb{G}_{8,1}$

Sample numbers from the set $\mathbb{G}_{8,1}$:

- $n = 1 = 1_{(8)}$, $L_{8,1}(n) = 1_{(8)} + R(1_{(8)}) = 2_{(8)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{8,1}$,
- $n = 80 = 120_{(8)}$, $L_{8,1}(n) = 120_{(8)} + R(120_{(8)}) = 141_{(8)} = 97 \in \mathbb{P} \Rightarrow 80 \in \mathbb{G}_{8,1}$,
- $n = 88 = 130_{(8)}$, $L_{8,1}(n) = 130_{(8)} + R(130_{(8)}) = 161_{(8)} = 113 \in \mathbb{P} \Rightarrow 88 \in \mathbb{G}_{8,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,1}$ is: 1, 74, 80, 82, 86, 88, 92, 94, 100, 116, 122, 137, 143, 145, 149, 151, 155, 157, 163, 179, 185, 200, 206, 208, 212, 214, 218, 220, 226, 242, 248, 269, 275, 277, 281, 283, 289, 305, 311, 319, 332, 338, 340, 344, 346, 352, 368, 374, 382, 395, 401, 403, 407, 409, 437, 439, 445, 458, 464, 466, 470, 472, 500, 502, 508 .

Table 4: The set $\mathbb{G}_{8,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{8,1}$	$L_{8,1}(n) \in \mathbb{P}$
1	2
80	97
88	113
74, 137, 200	211
82, 145, 208	227
122, 185, 248	307
92, 155, 218, 281, 344	373
100, 163, 226, 289, 352	389
116, 179, 242, 305, 368	421
86, 149, 212, 275, 338, 401, 464	487
94, 157, 220, 283, 346, 409, 472	503
143, 206, 269, 332, 395, 458	601
151, 214, 277, 340, 403, 466	617
311, 374, 437, 500	811
319, 382, 445, 508	827
407, 470	877
439, 502	941

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,1}$ contains 65 integers with generates 17 primes using the Luhn function $L_{8,1}$.

5.5 The set $\mathbb{G}_{10,1}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,1}$ is: 1, 10, 100, 116, 118, 140, 142, 146, 158, 166, 170, 172, 178, 182, 188, 190, 196, 215, 217, 229, 239, 241, 245, 257, 265, 269, 271, 277, 281, 287, 295, 299, 314, 316, 328, 338, 340, 344, 356, 364, 368, 370, 376, 380, 386, 394, 398, 413, 415, 427, 437, 439, 443, 455, 463, 467, 469, 475, 479, 485, 493, 497, 499, 512, 514, 526, 536, 538, 542, 554, 562, 566, 568, 574, 578, 584, 592, 596, 598, 611, 613, 625, 635, 637, 641, 653, 661, 665, 667, 673, 677, 683, 691, 695, 697, 710, 712, 724, 734, 736, 740, 752, 760, 764, 766, 772, 776, 782, 790, 794, 796, 811, 823, 833, 835, 839, 851, 863, 865, 871, 875, 881, 889, 893, 895, 910, 922, 932, 934, 938, 950, 962, 964, 970, 974, 980, 988, 992, 994 . See figure 1.

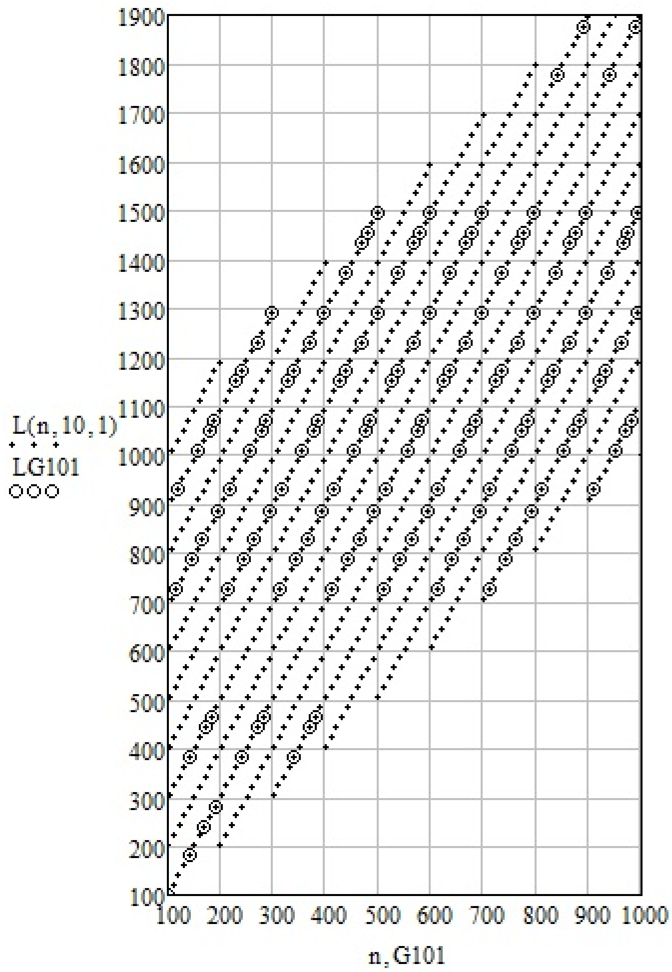

Figure 1: The function $L_{10,1}$ and the set $LG101 = L_{10,1}(\mathbb{G}_{10,1})$

Table 5: The set $\mathbb{G}_{10,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{10,1}$	$L_{10,1}(n) \in \mathbb{P}$
1	2
10	11
100	101
140	181
170	241
190	281
142, 241, 340	383
172, 271, 370	443

Continued on next page

$n \in \mathbb{G}_{10,1}$	$L_{10,1}(n) \in \mathbb{P}$
182, 281, 380	463
116, 215, 314, 413, 512, 611, 710	727
146, 245, 344, 443, 542, 641, 740	787
166, 265, 364, 463, 562, 661, 760	827
196, 295, 394, 493, 592, 691, 790	887
118, 217, 316, 415, 514, 613, 712, 811, 910	929
158, 257, 356, 455, 554, 653, 752, 851, 950	1009
178, 277, 376, 475, 574, 673, 772, 871, 970	1049
188, 287, 386, 485, 584, 683, 782, 881, 980	1069
229, 328, 427, 526, 625, 724, 823, 922	1151
239, 338, 437, 536, 635, 734, 833, 932	1171
269, 368, 467, 566, 665, 764, 863, 962	1231
299, 398, 497, 596, 695, 794, 893, 992	1291
439, 538, 637, 736, 835, 934	1373
469, 568, 667, 766, 865, 964	1433
479, 578, 677, 776, 875, 974	1453
499, 598, 697, 796, 895, 994	1493
839, 938	1777
889, 988	1877

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,1}$ contains 139 integers with generates 27 primes using the Luhn function $L_{10,1}$.

5.6 The set $\mathbb{G}_{12,1}$

Sample numbers from the set $\mathbb{G}_{12,1}$:

- $n = 1 = 1_{(12)}$, $L_{12,1}(n) = 1_{(12)} + R(1_{(12)}) = 2_{(12)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{12,1}$,
- $n = 12 = 10_{(12)}$, $L_{12,1}(n) = 10_{(12)} + R(10_{(12)}) = 11_{(12)} = 13 \in \mathbb{P} \Rightarrow 12 \in \mathbb{G}_{12,1}$,
- $n = 168 = 120_{(12)}$, $L_{12,1}(n) = 120_{(12)} + R(120_{(12)}) = 141_{(12)} = 193 \in \mathbb{P} \Rightarrow 168 \in \mathbb{G}_{12,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,1}$ is: 1, 12, 162, 166, 168, 172, 174, 184, 186, 190, 192, 196, 228, 240, 250, 256, 258, 262, 276, 282, 305, 309, 315, 317, 323, 327, 329, 333, 339, 371, 383, 393, 399, 401, 405, 425, 448, 452, 458, 460, 466, 470, 472, 476, 482, 514, 526, 536, 542, 544, 548, 568, 591, 595, 601, 603, 609, 613, 615, 619, 625, 657, 669, 679, 685, 687, 691, 711, 734, 738, 744, 746, 752, 756, 758, 762, 768, 800, 812, 822, 828, 830, 834, 854, 877, 881, 889, 895, 901, 905, 943, 947, 955, 959, 965, 971, 973, 977, 997 .

Table 6: The set $\mathbb{G}_{12,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{12,1}$	$L_{12,1}(n) \in \mathbb{P}$
1	2
12	13
168	193
192	241
228	313
240	337
276	409
172, 315, 458, 601, 744	773
184, 327, 470, 613, 756	797
196, 339, 482, 625, 768	821
256, 399, 542, 685, 828	941
162, 305, 448, 591, 734, 877	1039
174, 317, 460, 603, 746, 889	1063
186, 329, 472, 615, 758, 901	1087
258, 401, 544, 687, 830, 973	1231
282, 425, 568, 711, 854, 997	1279
166, 309, 452, 595, 738, 881	1619
190, 333, 476, 619, 762, 905	1667
250, 393, 536, 679, 822, 965	1787
262, 405, 548, 691, 834, 977	1811
323, 466, 609, 752, 895	1933
371, 514, 657, 800, 943	2029
383, 526, 669, 812, 955	2053
947	2609
959	2633
971	2657

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,1}$ contains 99 integers with generates 26 primes using the Luhn function $L_{12,1}$.

5.7 The set $\mathbb{G}_{14,1}$

Sample numbers from the set $\mathbb{G}_{14,1}$:

- $n = 1 = 1_{(14)}$, $L_{14,1}(n) = 1_{(14)} + R(1_{(14)}) = 2_{(14)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{14,1}$,
- $n = 196 = 100_{(14)}$, $L_{14,1}(n) = 100_{(14)} + R(100_{(14)}) = 101_{(14)} = 197 \in \mathbb{P} \Rightarrow 196 \in \mathbb{G}_{14,1}$,

- $n = 238 = 130_{(14)}$, $L_{14,1}(n) = 130_{(14)} + R(130_{(14)}) = 161_{(14)} = 281 \in \mathbb{P} \Rightarrow 238 \in \mathbb{G}_{14,1}$.

For $1 \le n \le 10^3$ the set $\mathbb{G}_{14,1}$ is: 1, 196, 212, 214, 218, 226, 236, 238, 242, 248, 256, 266, 274, 284, 292, 296, 298, 308, 316, 322, 326, 344, 346, 358, 368, 374, 376, 386, 388, 407, 409, 413, 421, 431, 433, 437, 443, 451, 461, 469, 479, 487, 491, 493, 511, 521, 539, 541, 553, 563, 569, 571, 581, 583, 587, 602, 604, 608, 616, 626, 628, 632, 638, 646, 656, 664, 674, 682, 686, 688, 706, 716, 734, 736, 748, 758, 764, 766, 776, 778, 782, 799, 803, 821, 823, 827, 833, 839, 841, 851, 853, 859, 869, 877, 881, 883, 901, 911, 929, 931, 943, 953, 959, 961, 971, 973, 977, 994, 998 .

Table 7: The set $\mathbb{G}_{14,1}$ with $1 \le n \le 10^3$

$n \in \mathbb{G}_{14,1}$	$L_{14,1}(n) \in \mathbb{P}$
1	2
196	197
238	281
266	337
308	421
322	449
226, 421, 616	647
296, 491, 686	787
214, 409, 604, 799, 994	1013
242, 437, 632, 827	1069
256, 451, 646, 841	1097
284, 479, 674, 869	1153
298, 493, 688, 883	1181
326, 521, 716, 911	1237
368, 563, 758, 953	1321
218, 413, 608, 803, 998	1801
274, 469, 664, 859	1913
316, 511, 706, 901	1997
344, 539, 734, 929	2053
358, 553, 748, 943	2081
386, 581, 776, 971	2137
248, 443, 638, 833	2251
346, 541, 736, 931	2447
374, 569, 764, 959	2503
388, 583, 778, 973	2531
236, 431, 626, 821	2617
292, 487, 682, 877	2729

Continued on next page

$n \in \mathbb{G}_{14,1}$	$L_{14,1}(n) \in \mathbb{P}$
376, 571, 766, 961	2897
433, 628, 823	3011
461, 656, 851	3067
587, 782, 977	3319
839	3433
853	3461
881	3517

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{14,1}$ contains 109 integers with generates 35 primes using the Luhn function $L_{14,1}$.

5.8 The set $\mathbb{G}_{16,1}$

Sample numbers from the set $\mathbb{G}_{16,1}$:

- $n = 1 = 1_{(16)}$, $L_{16,1}(n) = 1_{(16)} + R(1_{(16)}) = 2_{(16)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{16,1}$,
- $n = 16 = 10_{(16)}$, $L_{16,1}(n) = 10_{(16)} + R(10_{(16)}) = 11_{(16)} = 17 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{16,1}$,
- $n = 256 = 100_{(16)}$, $L_{16,1}(n) = 100_{(16)} + R(100_{(16)}) = 101_{(16)} = 257 \in \mathbb{P} \Rightarrow 256 \in \mathbb{G}_{16,1}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,1}$ is: 1, 16, 256, 278, 284, 296, 302, 304, 308, 328, 332, 344, 352, 362, 364, 382, 394, 398, 406, 416, 418, 424, 434, 436, 448, 452, 464, 466, 472, 478, 484, 506, 508, 533, 539, 551, 557, 563, 583, 587, 599, 617, 619, 623, 637, 649, 653, 661, 671, 673, 679, 689, 691, 703, 707, 721, 727, 733, 739, 751, 761, 763, 788, 794, 806, 812, 818, 838, 842, 854, 872, 874, 878, 892, 904, 908, 916, 926, 928, 934, 944, 946, 958, 962, 976, 982, 988, 994 .

Table 8: The set $\mathbb{G}_{16,1}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{16,1}$	$L_{16,1}(n) \in \mathbb{P}$
1	2
16	17
256	257
304	353
352	449
416	577
448	641
464	673

Continued on next page

$n \in \mathbb{G}_{16,1}$	$L_{16,1}(n) \in \mathbb{P}$
418, 673, 928	1091
434, 689, 944	1123
466, 721, 976	1187
308, 563, 818	1381
436, 691, 946	1637
452, 707, 962	1669
484, 739, 994	1733
278, 533, 788	1831
406, 661, 916	2087
296, 551, 806	2377
328, 583, 838	2441
344, 599, 854	2473
424, 679, 934	2633
472, 727, 982	2729
362, 617, 872	3019
394, 649, 904	3083
506, 761	3307
284, 539, 794	3373
332, 587, 842	3469
364, 619, 874	3533
508, 763	3821
302, 557, 812	3919
382, 637, 892	4079
398, 653, 908	4111
478, 733, 988	4271
623, 878	4561
671, 926	4657
703, 958	4721
751	4817

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,1}$ contains 88 integers with generates 37 primes using the Luhn function $L_{16,1}$.

6 The sets $\mathbb{G}$ of order 2

6.1 The set $\mathbb{G}_{2,2}$

Examples:

- $n = 1 = 1_{(2)}$, $L_{2,2}(n) = (1_{(2)})^2 + \big(R(1_{(2)})\big)^2 = 10_{(2)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{2,2}$,
- $n = 2 = 10_{(2)}$, $L_{2,2}(n) = (10_{(2)})^2 + \big(R(10_{(2)})\big)^2 = 101_{(2)} = 5 \in \mathbb{P} \Rightarrow 2 \in \mathbb{G}_{2,2}$,
- $n = 4 = 100_{(2)}$, $L_{2,2}(n) = (100_{(2)})^2 + \big(R(100_{(2)})\big)^2 = 10001_{(2)} = 17 \in \mathbb{P} \Rightarrow 4 \in \mathbb{G}_{2,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,2}$ is: 1, 2, 4, 16, 22, 26, 38, 46, 50, 82, 104, 110, 122, 142, 200, 206, 256, 268, 334, 358, 386, 406, 412, 416, 422, 436, 446, 466, 472, 488, 506, 538, 548, 556, 562, 578, 586, 596, 614, 658, 664, 698, 736, 790, 820, 830, 838, 920, 944, 950 .

Table 9: The set $\mathbb{G}_{2,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{2,2}$	$L_{2,2}(n) \in \mathbb{P}$
1	2
2	5
4	17
16	257
22	653
26	797
38	2069
50	2861
46	2957
82	8093
104	10937
110	15581
122	17093
142	32933
200	40361
206	55661
256	65537
268	81233
334	163997
386	166157
358	170189
416	173177
412	182969
436	198377
406	209357
422	219293
472	225809

Continued on next page

$n \in \mathbb{G}_{2,2}$	$L_{2,2}(n) \in \mathbb{P}$
466	239957
488	240353
446	261917
506	292517
548	321329
556	352817
596	383777
578	404309
562	408869
538	414053
664	451097
586	451637
658	518813
736	542537
614	544277
698	626333
820	704441
790	799661
920	857009
838	858269
944	894161
830	937901
950	1095221

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,2}$ contains 50 integers with generates 50 primes using the Luhn function $L_{2,2}$.

6.2 The set $\mathbb{G}_{4,2}$

Examples:

- $n = 1 = 1_{(4)}$, $L_{4,2}(n) = (1_{(4)})^2 + \big(R(1_{(4)})\big)^2 = 2_{(4)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{4,2}$,
- $n = 4 = 10_{(4)}$, $L_{4,2}(n) = (10_{(4)})^2 + \big(R(10_{(4)})\big)^2 = 101_{(4)} = 17 \in \mathbb{P} \Rightarrow 4 \in \mathbb{G}_{4,2}$,
- $n = 16 = 100_{(4)}$, $L_{4,2}(n) = (100_{(4)})^2 + \big(R(100_{(4)})\big)^2 = 10001_{(4)} = 257 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{4,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,2}$ is: 1, 4, 11, 14, 16, 26, 28, 41, 47, 52, 56, 62, 74, 82, 92, 94, 104, 112, 133, 151, 161, 181, 208, 214, 232, 244, 256, 268, 284, 296, 302, 314, 326, 332, 376, 382, 392, 398, 406, 416, 418, 424, 452, 454, 466, 472, 478, 494, 496, 508, 523, 541, 551, 553, 581, 583, 589, 601, 631, 643, 647, 667, 671, 683, 689, 713, 719, 733, 737, 749, 751, 757, 763, 778, 808, 812, 824, 832, 838, 842, 848, 866, 886, 898, 922, 938, 958, 964, 974, 986, 988 .

Table 10: The set $\mathbb{G}_{4,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{4,2}$	$L_{4,2}(n) \in \mathbb{P}$
1	2
4	17
16	257
11, 14	317
28	953
26, 41	2357
52	2753
56	3257
47, 62	6053
92	11273
104	12497
112	12713
82, 133	24413
74, 161	31397
94, 181	41597
208	43313
232	55673
244	60497
256	65537
151, 214	68597
268	109073
296	113537
284	124337
332	149033
392	172433
376	174137
416	174737
424	208337
452	210233
472	247433
496	249737
508	322073
326, 581	443837

Continued on next page

$n \in \mathbb{G}_{4,2}$	$L_{4,2}(n) \in \mathbb{P}$
418, 553	480533
466, 541	509837
406, 601	526037
454, 589	553037
314, 689	573317
302, 737	634373
398, 713	666773
808	679433
832	692273
812	710873
824	711017
382, 757	718973
848	719633
478, 733	765773
494, 749	805037
964	935537
643, 778	1018733
988	1025873
583, 838	1042133
551, 866	1053557
523, 898	1079933
647, 842	1127573
631, 886	1183157
667, 922	1294973
683, 938	1346333
671, 986	1422437
719, 974	1465637
763, 958	1499933
751	1576037

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,2}$ contains 91 integers with generates 62 primes using the Luhn function $L_{4,2}$.

6.3 The set $\mathbb{G}_{6,2}$

Examples:

- $n = 1 = 1_{(6)}$, $L_{6,2}(n) = (1_{(6)})^2 + \big(R(1_{(6)})\big)^2 = 2_{(6)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{6,2}$,

- $n=6=10_{(6)}$, $L_{6,2}(n)=(10_{(6)})^2+\big(R(10_{(6)})\big)^2=101_{(6)}=37\in\mathbb{P}\Rightarrow 6\in\mathbb{G}_{6,2}$,
- $n=8=12_{(6)}$, $L_{6,2}(n)=(12_{(6)})^2+\big(R(12_{(6)})\big)^2=1025_{(6)}=233\in\mathbb{P}\Rightarrow 8\in\mathbb{G}_{6,2}$,
- $n=13=21_{(6)}$, $L_{6,2}(n)=(21_{(6)})^2+\big(R(21_{(6)})\big)^2=1025_{(6)}=233\in\mathbb{P}\Rightarrow 13\in\mathbb{G}_{6,2}$.

For $1\leq n\leq 10^3$ the set $\mathbb{G}_{6,2}$ is: 1, 6, 8, 13, 22, 27, 29, 34, 36, 46, 48, 68, 87, 89, 93, 103, 107, 122, 128, 136, 151, 171, 194, 198, 204, 212, 218, 248, 264, 268, 272, 274, 278, 282, 286, 312, 318, 324, 326, 348, 352, 358, 372, 384, 398, 412, 414, 418, 428, 433, 451, 463, 467, 473, 491, 507, 531, 537, 547, 551, 583, 611, 613, 643, 652, 662, 688, 724, 736, 766, 806, 842, 844, 862, 867, 873, 879, 887, 921, 923, 941, 943, 951, 967, 969, 971, 979, 989 .

Table 11: The set $\mathbb{G}_{6,2}$ with $1\leq n\leq 10^3$

$n\in\mathbb{G}_{6,2}$	$L_{6,2}(n)\in\mathbb{P}$
1	2
6	37
8, 13	233
22, 27	1213
36	1297
29, 34	1997
48	2473
68, 103	15233
87, 122	22453
46, 151	24917
93, 128	25033
198	39733
204	42457
89, 194	45557
136, 171	47737
107, 212	56393
264	75937
324	105337
282	114493
312	121993
318	138373
348	147673
372	147793
384	176017
414	190717

Continued on next page

$n \in \mathbb{G}_{6,2}$	$L_{6,2}(n) \in \mathbb{P}$
218, 433	235013
326, 451	309677
398, 463	372773
272, 547	373193
278, 583	417173
248, 613	437273
428, 643	596633
507, 662	695293
531, 806	931597
268, 943	961073
537, 842	997333
274, 979	1033517
412, 967	1104833
652, 867	1176793
352	1178633
418	1180733
286	1186397
688, 873	1235473
358	1258133
724, 879	1296817
473	1407473
766, 921	1434997
736, 951	1446097
551	1594097
844, 969	1651297
491	1671497
467	1810733
862	1902973
611	2027117
941	2086697
887	2207633
923	2287133
989	2356397
971	2571017

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,2}$ contains 88 integers with generates 59 primes using the Luhn function $L_{6,2}$.

6.4 The set $\mathbb{G}_{8,2}$

Examples:

- $n = 1 = 1_{(8)}$, $L_{8,2}(n) = (1_{(8)})^2 + \big(R(1_{(8)})\big)^2 = 2_{(8)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{8,2}$,
- $n = 10 = 12_{(8)}$, $L_{8,2}(n) = (12_{(8)})^2 + \big(R(12_{(8)})\big)^2 = 605_{(8)} = 389 \in \mathbb{P} \Rightarrow 10 \in \mathbb{G}_{8,2}$,
- $n = 17 = 21_{(8)}$, $L_{8,2}(n) = (21_{(8)})^2 + \big(R(21_{(8)})\big)^2 = 605_{(8)} = 389 \in \mathbb{P} \Rightarrow 17 \in \mathbb{G}_{8,2}$,
- $n = 80 = 120_{(8)}$, $L_{8,2}(n) = (120_{(8)})^2 + \big(R(120_{(8)})\big)^2 = 15041_{(8)} = 6689 \in \mathbb{P} \Rightarrow 80 \in \mathbb{G}_{8,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,2}$ is: 1, 10, 17, 55, 62, 76, 80, 82, 86, 88, 104, 118, 122, 143, 145, 151, 155, 167, 181, 185, 191, 206, 212, 218, 220, 230, 232, 263, 265, 275, 283, 311, 317, 370, 380, 395, 401, 407, 419, 431, 433, 452, 458, 466, 470, 482, 488, 494, 500, 506, 514, 530, 542, 548, 550, 556, 580, 604, 620, 632, 640, 646, 656, 664, 668, 722, 734, 736, 740, 764, 790, 806, 808, 860, 874, 914, 916, 928, 956, 976, 998, 1000 .

Table 12: The set $\mathbb{G}_{8,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{8,2}$	$L_{8,2}(n) \in \mathbb{P}$
1	2
10, 17	389
80	6689
55, 62	6869
88	8369
104	12497
82, 145	27749
122, 185	49109
232	55673
155, 218	71549
76, 265	76001
212, 275	120569
220, 283	128489
86, 401	168197
181, 370	169661
206, 395	198461
118, 433	201413
230, 419	228461
143, 458	230213
151, 466	239957

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$n \in \mathbb{G}_{8,2}$	$L_{8,2}(n) \in \mathbb{P}$
488	240353
317, 380	244889
167, 482	260213
263, 452	273473
191, 506	292517
311, 500	346721
407, 470	386549
640	409889
431, 494	429797
656	451361
664	484577
632	608273
736	620657
808	777473
928	954209
976	986801
1000	1142129
514	1314821
530	1610309
722	1906613
914	2277797
874	2682101
580	4567649
604	5422817
668	5540273
548	5613329
916	5789681
556	5921297
860	5942561
740	5971841
620	6034529
764	6939137
956	7390961
646	9959237
542	10953989
790	11076389
734	11356277
550	11384741
806	11945957

Continued on next page

$n \in \mathbb{G}_{8,2}$	$L_{8,2}(n) \in \mathbb{P}$
998	12454229

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,2}$ contains 82 integers with generates 60 primes using the Luhn function $L_{8,2}$.

6.5 The set $\mathbb{G}_{10,2}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,2}$ is: 1, 10, 14, 23, 25, 32, 41, 52, 58, 85, 104, 116, 124, 164, 170, 190, 194, 205, 227, 233, 283, 310, 320, 328, 332, 380, 382, 398, 401, 409, 419, 421, 425, 461, 491, 499, 502, 508, 518, 524, 598, 611, 689, 710, 722, 728, 758, 778, 805, 815, 823, 827, 857, 877, 893, 895, 904, 914, 980, 986, 994 . See figure 2.

Table 13: The set $\mathbb{G}_{10,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{10,2}$	$L_{10,2}(n) \in \mathbb{P}$
1	2
10	101
23, 32	1553
14, 41	1877
25, 52	3329
58, 85	10589
170	33941
190	44381
310	96269
320	102929
380	151289
233, 332	164513
104, 401	171617
124, 421	192617
283, 382	226013
164, 461	239417
194, 491	278717
205, 502	294029
116, 611	386777
425, 524	455201
710	504389
227, 722	572813

Continued on next page

$n \in \mathbb{G}_{10,2}$	$L_{10,2}(n) \in \mathbb{P}$
328, 823	784913
508, 805	906089
518, 815	932549
398, 893	955853
980	968321
409, 904	984497
419, 914	1010957
598, 895	1158629
728, 827	1213913
499, 994	1237037
758, 857	1309013
778, 877	1374413
689, 986	1446917

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,2}$ contains 61 integers with generates 35 primes using the Luhn function $L_{10,2}$.

6.6 The set $\mathbb{G}_{12,2}$

Examples:

- $n = 1 = 1_{(12)}$, $L_{12,2}(n) = (1_{(12)})^2 + \big(R(1_{(12)})\big)^2 = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{12,2}$,
- $n = 14 = 12_{(12)}$, $L_{12,2}(n) = (12_{(12)})^2 + \big(R(12_{(12)})\big)^2 = 585_{(12)} = 821 \in \mathbb{P} \Rightarrow 14 \in \mathbb{G}_{12,2}$,
- $n = 25 = 21_{(12)}$, $L_{12,2}(n) = (21_{(12)})^2 + \big(R(21_{(12)})\big)^2 = 585_{(12)} = 821 \in \mathbb{P} \Rightarrow 25 \in \mathbb{G}_{12,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,2}$ is: 1, 14, 16, 18, 25, 35, 40, 49, 51, 59, 73, 79, 83, 90, 92, 103, 105, 116, 134, 136, 138, 148, 164, 174, 180, 186, 204, 228, 230, 238, 246, 266, 268, 272, 307, 315, 323, 327, 343, 373, 379, 383, 401, 409, 411, 413, 427, 458, 470, 514, 554, 577, 597, 617, 621, 635, 641, 655, 697, 744, 760, 784, 786, 800, 802, 810, 816, 822, 830, 842, 862, 889, 899, 901, 907, 929, 953, 959, 961, 965, 967 .

Table 14: The set $\mathbb{G}_{12,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{12,2}$	$L_{12,2}(n) \in \mathbb{P}$
1	2
14, 25	821

Continued on next page

$n \in \mathbb{G}_{12,2}$	$L_{12,2}(n) \in \mathbb{P}$
16, 49	2657
40, 51	4201
18, 73	5653
79, 90	14341
92, 103	19073
35, 134	19181
59, 136	21977
105, 116	24481
83, 138	25933
180	33769
204	45337
228	59209
230, 373	192029
266, 409	238037
315, 458	308989
327, 470	327829
148, 577	354833
411, 554	475837
744	554377
268, 697	557633
816	676057
174, 889	820597
186, 901	846397
401, 830	849701
413, 842	879533
617, 760	958289
246, 961	984037
641, 784	1025537
307	1138733
343	1237013
379	1340477
164	1384121
786, 929	1480837
427	1486493
810, 953	1564309
655	1604081
822, 965	1606909
272	1694513
907	1925149

Continued on next page

$n \in \mathbb{G}_{12,2}$	$L_{12,2}(n) \in \mathbb{P}$
597	2077753
800	2150441
967	2167189
621	2170537
238	2382269
514	2559421
323	2696429
383	2935589
802	2944493
635	3079721
862	3229973
899	3413197
959	3721957

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,2}$ contains 81 integers with generates 54 primes using the Luhn function $L_{12,2}$.

6.7 The set $\mathbb{G}_{14,2}$

Examples:

- $n = 1 = 1_{(14)}$, $L_{14,2}(n) = (1_{(14)})^2 + \big(R(1_{(14)})\big)^2 = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{14,2}$,
- $n = 14 = 10_{(14)}$, $L_{14,2}(n) = (10_{(14)})^2 + \big(R(10_{(14)})\big)^2 = 101_{(14)} = 197 \in \mathbb{P} \Rightarrow 14 \in \mathbb{G}_{14,2}$,
- $n = 16 = 12_{(14)}$, $L_{14,2}(n) = (12_{(14)})^2 + \big(R(12_{(14)})\big)^2 = 585_{(14)} = 1097 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{14,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{14,2}$ is: 1, 14, 16, 29, 31, 41, 44, 67, 76, 82, 89, 97, 136, 149, 151, 158, 164, 173, 184, 188, 236, 248, 254, 266, 274, 284, 298, 314, 322, 326, 328, 344, 356, 386, 397, 419, 433, 449, 451, 457, 461, 487, 511, 521, 523, 551, 577, 602, 608, 616, 626, 634, 646, 652, 656, 682, 692, 694, 706, 716, 734, 736, 742, 748, 752, 764, 766, 772, 778, 782, 797, 811, 817, 821, 829, 869, 883, 911, 943, 959, 977, 982, 986, 992 .

Table 15: The set $\mathbb{G}_{14,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{14,2}$	$L_{14,2}(n) \in \mathbb{P}$
1	2
14	197

Continued on next page

$n \in \mathbb{G}_{14,2}$	$L_{14,2}(n) \in \mathbb{P}$
16, 29	1097
31, 44	2897
76, 89	13697
67, 158	29453
41, 184	35537
82, 173	36653
136, 149	40697
97, 188	44753
151, 164	49697
266	75797
322	119813
254, 449	266117
602	362693
616	380417
742	575213
451, 646	620717
521, 716	784097
284, 869	835817
298, 883	868493
577, 772	928913
326, 911	936197
634, 829	1089197
397, 982	1121933
523	1501193
551	1594097
817	1691633
314	1760117
608	1792913
328	1805393
356	1898297
692	2109593
706	2165117
943	2184293
734	2278517
748	2336393
986	2366957
511	2469317
274	2761397
344	3039017

Continued on next page

$n \in \mathbb{G}_{14,2}$	$L_{14,2}(n) \in \mathbb{P}$
652	3072233
386	3214997
694	3267197
959	3303617
736	3469217
457	3528533
764	3607817
778	3678293
821	3899657
248	4073513
626	4355957
682	4655333
752	5047193
766	5127917
487	5263733
236	5724857
656	6243257
977	6439493
992	6539513
419	6749657
433	6833573
461	7003757
782	7047893
797	7147913
811	7242077

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{14,2}$ contains 84 integers with generates 66 primes using the Luhn function $L_{14,2}$.

6.8 The set $\mathbb{G}_{16,2}$

Examples:

- $n = 1 = 1_{(16)}$, $L_{16,2}(n) = (1_{(16)})^2 + \big(R(1_{(16)})\big)^2 = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{16,2}$,
- $n = 16 = 10_{(16)}$, $L_{16,2}(n) = (10_{(16)})^2 + \big(R(10_{(16)})\big)^2 = 101_{(16)} = 257 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{16,2}$,
- $n = 52 = 34_{(16)}$, $L_{16,2}(n) = (34_{(16)})^2 + \big(R(34_{(16)})\big)^2 = 1c19_{(16)} = 7193 \in \mathbb{P} \Rightarrow 52 \in \mathbb{G}_{16,2}$,

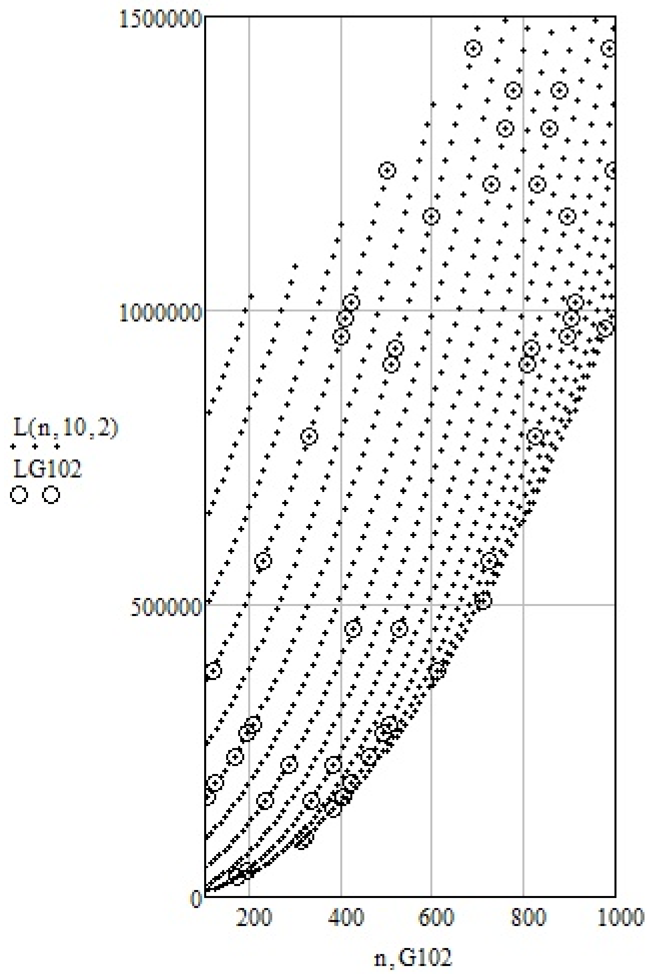

Figure 2: The function $L_{10,2}$ and the set $LG102 = L_{10,2}(\mathbb{G}_{10,2})$

- $n = 67 = 43_{(16)}$, $L_{16,2}(n) = (43_{(16)})^2 + \big(R(43_{(16)})\big)^2 = 1c19_{(16)} = 7193 \in \mathbb{P} \Rightarrow 67 \in \mathbb{G}_{16,2}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,2}$ is: 1, 16, 26, 37, 43, 47, 52, 56, 62, 67, 77, 79, 82, 86, 101, 103, 118, 122, 131, 143, 161, 167, 178, 212, 227, 239, 242, 244, 248, 254, 256, 266, 268, 298, 326, 328, 334, 338, 346, 356, 362, 364, 368, 382, 386, 398, 404, 416, 428, 454, 464, 484, 488, 506, 508, 517, 557, 571, 589, 593, 601, 613, 617, 619, 637, 641, 661, 671, 679, 683, 707, 713, 749, 751, 761, 776, 778, 836, 848, 868, 896, 902, 904, 916, 922, 944, 956, 962, 976 .

Table 16: The set $\mathbb{G}_{16,2}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{16,2}$	$L_{16,2}(n) \in \mathbb{P}$
1	2
16	257
52, 67	7193
37, 82	8093
86, 101	17597
56, 131	20297
103, 118	24533
26, 161	26597
43, 178	33533
122, 167	42773
77, 212	50873
62, 227	55373
47, 242	60773
256	65537
79, 244	65777
143, 248	81953
239, 254	121637
368	148193
416	198977
464	258977
338, 593	465893
386, 641	559877
848	725993
896	819977
944	923177
976	997097
356	1383377
707, 962	1425293
404	1529777
484	1794257
836	1889177
517	1910813
868	2014553
916	2210297
613	2274653
661	2470397
326	2669477

Continued on next page

$n \in \mathbb{G}_{16,2}$	$L_{16,2}(n) \in \mathbb{P}$
454	3195557
902	3592493
679	4279157
328	4572353
776	4808777
488	5404673
904	5565257
601	6054197
617	6150293
266	6629477
713	6748373
298	6812453
761	7061237
346	7094597
778	7174253
362	7190693
506	8101637
922	8177933
571	8539997
619	8874557
683	9334973
268	9515153
364	10175057
428	10635473
508	11234033
956	11482937
557	11613293
589	11866157
637	12253133
749	13191917
334	13426757
382	13813733
398	13944773
671	16338437
751	17096357

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,2}$ contains 89 integers with generates 72 primes using the Luhn function $L_{16,2}$.

7 The sets $\mathbb{G}$ of order 4

7.1 The set $\mathbb{G}_{2,4}$

Examples:

- $n = 1 = 1_{(2)}$, $L_{2,4}(n) = (1_{(2)})^4 + \big(R(1_{(2)})\big)^4 = 10_{(2)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{2,4}$,
- $n = 2 = 10_{(2)}$, $L_{2,4}(n) = (10_{(2)})^4 + \big(R(10_{(2)})\big)^4 = 10001_{(2)} = 257 \in \mathbb{P} \Rightarrow 2 \in \mathbb{G}_{2,4}$,
- $n = 16 = 10000_{(2)}$, $L_{2,4}(n) = (10000_{(4)})^4 + \big(R(10000_{(2)})\big)^4 = 10000000000000001_{(2)} = 65537 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{2,4}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,4}$ is: 1, 2, 4, 16, 26, 52, 82, 100, 122, 166, 172, 178, 188, 194, 226, 232, 286, 290, 302, 316, 346, 362, 382, 398, 412, 422, 436, 440, 446, 458, 484, 542, 562, 566, 568, 662, 688, 694, 698, 710, 782, 794, 808, 842, 844, 862, 916, 926, 928, 932, 974, 982, 986 .

Table 17: The set $\mathbb{G}_{2,4}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{2,4}$	$L_{2,4}(n) \in \mathbb{P}$
1	2
2	17
4	257
16	65537
26	471617
52	7326257
82	47086337
100	100130321
122	226413137
166	863393537
172	883103537
178	1039028897
188	1263044177
194	1436619617
226	2634169457
232	2897302817
290	7425085361
286	10063988177
316	10185579617
302	11265465137

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$n \in \mathbb{G}_{2,4}$	$L_{2,4}(n) \in \mathbb{P}$
346	15405203777
362	18068274977
382	25390965857
398	27747065057
412	28987926161
422	33412092737
436	36205064177
440	37493077361
446	43536701057
458	44778732017
484	54914823617
568	104249292737
562	108411082961
566	137780091857
542	139825200017
662	223472175617
688	224062432817
698	256724616257
694	268442395457
710	278957406881
794	413331850721
782	415333569377
808	426278860817
844	509120758577
842	514633566017
862	610234434497
916	704792767457
928	741638161697
932	755027538977
926	791514225137
974	945940221137
986	963306189137
982	964427926097

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,4}$ contains 53 integers with generates 53 primes using the Luhn function $L_{2,4}$.

7.2 The set $\mathbb{G}_{4,4}$

Examples:

- $n = 1 = 1_{(4)}$, $L_{4,4}(n) = (1_{(4)})^4 + \big(R(1_{(4)})\big)^4 = 2_{(4)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{4,4}$,
- $n = 4 = 10_{(4)}$, $L_{4,4}(n) = (10_{(4)})^4 + \big(R(10_{(4)})\big)^4 = 10001_{(4)} = 257 \in \mathbb{P} \Rightarrow 4 \in \mathbb{G}_{4,4}$,
- $n = 16 = 100_{(4)}$, $L_{4,4}(n) = (100_{(4)})^4 + \big(R(100_{(4)})\big)^4 = 100000001_{(4)} = 65537 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{4,4}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,4}$ is: 1, 4, 16, 28, 43, 47, 58, 62, 76, 98, 104, 112, 118, 137, 151, 157, 191, 196, 208, 212, 214, 254, 268, 284, 328, 346, 356, 388, 392, 398, 406, 416, 422, 424, 452, 494, 506, 523, 563, 601, 617, 661, 683, 691, 701, 703, 713, 731, 749, 784, 818, 826, 856, 898, 926, 938 .

Table 18: The set $\mathbb{G}_{4,4}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{4,4}$	$L_{4,4}(n) \in \mathbb{P}$
1	2
4	257
16	65537
28	643217
43, 58	14735297
47, 62	19656017
76	39126977
104	119811617
112	157380497
98, 137	444512177
118, 157	801450977
196	1475919377
208	1871776097
212	2020242977
151, 214	2617159217
191, 254	5493177617
268	6546174977
284	8413420097
328	11887217777
356	16166074097
388	22691894177
392	23964900257

Continued on next page

$n \in \mathbb{G}_{4,4}$	$L_{4,4}(n) \in \mathbb{P}$
416	29951204897
424	33135140897
452	41775277457
406, 601	157637069297
422, 617	176638025777
346, 661	205231880897
398, 713	283530867377
506, 701	307029376097
494, 749	374275691297
784	377802128657
856	537421931297
563, 818	548196274337
691, 826	693488645537
523, 898	725105525057
683, 938	991737451457
731, 926	1020806768897
703	1318209968657

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,4}$ contains 56 integers with generates 39 primes using the Luhn function $L_{4,4}$.

7.3 The set $\mathbb{G}_{6,4}$

Examples:

- $n = 1 = 1_{(6)}$, $L_{6,4}(n) = (1_{(6)})^4 + \big(R(1_{(6)})\big)^4 = 2_{(6)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{6,4}$,
- $n = 6 = 10_{(6)}$, $L_{6,4}(n) = (10_{(6)})^4 + \big(R(10_{(6)})\big)^4 = 10001_{(6)} = 1297 \in \mathbb{P} \Rightarrow 6 \in \mathbb{G}_{6,4}$,
- $n = 29 = 45_{(6)}$, $L_{6,4}(n) = (45_{(6)})^4 + \big(R(45_{(6)})\big)^4 = 501544111_{(6)} = 2043617 \in \mathbb{P} \Rightarrow 29 \in \mathbb{G}_{6,4}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,4}$ is: 1, 6, 29, 34, 46, 54, 58, 64, 68, 89, 103, 142, 149, 151, 163, 169, 177, 184, 186, 192, 194, 204, 226, 232, 234, 254, 298, 306, 314, 328, 342, 354, 372, 394, 396, 408, 422, 426, 439, 479, 489, 507, 509, 557, 561, 573, 579, 589, 593, 599, 607, 639, 652, 662, 674, 682, 694, 712, 718, 764, 766, 776, 824, 838, 848, 867, 883, 901, 909, 913, 921, 933, 937, 941, 953 .

Table 19: The set $\mathbb{G}_{6,4}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{6,4}$	$L_{6,4}(n) \in \mathbb{P}$
1	2
6	1297
29, 34	2043617
54	8633377
68, 103	133932257
46, 151	524363057
58, 163	717228257
64, 169	832507937
186	1196897857
192	1359038017
142, 177	1388093137
89, 194	1479210737
149, 184	1639113137
204	1732598737
234	3139377697
306	8982059377
342	13940721937
354	17272339057
372	19238660737
396	24592181377
408	27822814177
426	34915658017
254, 439	41303698097
314, 589	130075351457
422, 607	167468576657
507, 662	258131991937
579, 674	318753212257
489, 764	397879873057
561, 776	461665242817
328, 883	619489253777
573, 848	624910495057
639, 824	627734447617
226, 901	661629621377
298, 913	702723428177
652, 867	745749763537
232, 937	773726587937
694, 909	914713527457

Continued on next page

$n \in \mathbb{G}_{6,4}$	$L_{6,4}(n) \in \mathbb{P}$
766, 921	1063795397617
838, 933	1250897734657
712	1326745954657
394	1330000918817
682	1418014507057
718	1495222393057
509	1499539375457
479	1648762089857
557	1982987447057
593	2049323903057
941	2226996480017
599	2300812807217
953	2685950709857

For $1 \le n \le 10^3$ the set $\mathbb{G}_{6,4}$ contains 75 integers with generates 50 primes using the Luhn function $L_{6,4}$.

7.4 The set $\mathbb{G}_{8,4}$

Examples:

- $n = 1 = 1_{(8)}$, $L_{8,4}(n) = (1_{(8)})^4 + \big(R(1_{(8)})\big)^4 = 2_{(8)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{8,4}$,
- $n = 19 = 23_{(8)}$, $L_{8,4}(n) = (23_{(8)})^4 + \big(R(23_{(8)})\big)^4 = 2173041_{(8)} = 587297 \in \mathbb{P} \Rightarrow 19 \in \mathbb{G}_{8,4}$,
- $n = 26 = 32_{(8)}$, $L_{8,4}(n) = (32_{(8)})^4 + \big(R(32_{(8)})\big)^4 = 2173041_{(8)} = 587297 \in \mathbb{P} \Rightarrow 26 \in \mathbb{G}_{8,4}$.

For $1 \le n \le 10^3$ the set $\mathbb{G}_{8,4}$ is: 1, 19, 26, 55, 62, 76, 82, 104, 116, 145, 155, 173, 181, 212, 218, 236, 265, 271, 275, 299, 305, 326, 328, 344, 362, 370, 389, 439, 460, 502, 530, 538, 548, 554, 604, 610, 620, 634, 656, 662, 664, 688, 734, 746, 800, 802, 814, 832, 844, 920, 922, 934, 940, 958, 1000 .

Table 20: The set $\mathbb{G}_{8,4}$ with $1 \le n \le 10^3$

$n \in \mathbb{G}_{8,4}$	$L_{8,4}(n) \in \mathbb{P}$
1	2

Continued on next page

$n \in \mathbb{G}_{8,4}$	$L_{8,4}(n) \in \mathbb{P}$
19, 26	587297
55, 62	23926961
104	119811617
82, 145	487262801
155, 218	2835731201
76, 265	4964912801
212, 275	7739103761
116, 305	8834714561
236, 299	11094583217
328	11574345617
344	14004116177
173, 362	18068274977
181, 370	19814893121
326, 389	34192633217
271, 460	50168140481
439, 502	100647399857
656	185631123521
664	196297312577
688	249911503937
800	416575757441
832	479176891937
920	719766362561
1000	1020200652641
530	1846233099281
538	2277402455057
610	2899110443441
922	3283362857681
554	3366768881681
802	3385780498577
746	3822189512177
634	4972401102257
844	21994439174177
604	25716464829857
548	28318417143041
620	32071721076641
940	34908416048321
662	107295700933457
734	117309018612977
934	130815163239377

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$n \in \mathbb{G}_{8,4}$	$L_{8,4}(n) \in \mathbb{P}$
814	138046596349841
958	163092846052817

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,4}$ contains 55 integers with generates 42 primes using the Luhn function $L_{8,4}$.

7.5 The set $\mathbb{G}_{10,4}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,4}$ is: 1, 23, 32, 34, 43, 47, 56, 58, 65, 74, 85, 106, 112, 128, 134, 140, 158, 160, 170, 194, 211, 233, 239, 245, 263, 304, 314, 332, 340, 350, 362, 380, 386, 398, 403, 413, 419, 431, 437, 487, 491, 526, 542, 556, 568, 601, 625, 637, 655, 683, 697, 734, 736, 758, 784, 796, 821, 851, 857, 865, 893, 910, 914, 920, 932 .

Table 21: The set $\mathbb{G}_{10,4}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{10,4}$	$L_{10,4}(n) \in \mathbb{P}$
1	2
23, 32	1328417
34, 43	4755137
56, 65	27685121
47, 74	34866257
58, 85	63517121
140	386985761
160	669205841
170	860621681
112, 211	2139471377
340	13366778801
350	15014140481
233, 332	15096625697
380	20898818321
263, 362	21956880497
134, 431	34829567057
304, 403	34917400337
314, 413	38814954977
194, 491	59536517057
245, 542	89900288321
106, 601	130592410097

Continued on next page

$n \in \mathbb{G}_{10,4}$	$L_{10,4}(n) \in \mathbb{P}$
526, 625	229137499601
386, 683	239811795137
556, 655	279627517121
437, 734	326727186497
487, 784	434051132897
128, 821	454599705137
637, 736	458083037777
158, 851	525090289697
697, 796	637479619937
398, 893	661016734817
568, 865	663926896001
910	685749740321
920	716393667281
419, 914	728708141537
239, 932	757770462017
758, 857	869539123697

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,4}$ contains 65 integers with generates 37 primes using the Luhn function $L_{10,4}$.

7.6 The set $\mathbb{G}_{12,4}$

Examples:

- $n = 1 = 1_{(12)}$, $L_{12,4}(n) = (1_{(12)})^4 + \big(R(1_{(12)})\big)^4 = 2_{(12)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{12,4}$,
- $n = 27 = 23_{(12)}$, $L_{12,4}(n) = (23_{(12)})^4 + \big(R(23_{(12)})\big)^4 = a62281_{(12)} = 2616577 \in \mathbb{P} \Rightarrow 27 \in \mathbb{G}_{12,4}$,
- $n = 38 = 32_{(12)}$, $L_{12,4}(n) = (32_{(12)})^4 + \big(R(32_{(12)})\big)^4 = a62281_{(12)} = 2616577 \in \mathbb{P} \Rightarrow 38 \in \mathbb{G}_{12,4}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,4}$ is: 1, 20, 27, 38, 57, 92, 97, 103, 112, 118, 129, 172, 174, 178, 184, 204, 212, 218, 266, 268, 272, 274, 280, 282, 291, 303, 305, 309, 327, 331, 345, 359, 361, 371, 391, 393, 409, 423, 434, 446, 452, 470, 502, 508, 548, 566, 601, 613, 619, 629, 631, 647, 651, 659, 665, 669, 697, 709, 734, 744, 768, 772, 776, 798, 802, 808, 816, 822, 826, 883, 887, 889, 907, 931, 941, 947, 965, 967, 991, 997 .

Table 22: The set $\mathbb{G}_{12,4}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{12,4}$	$L_{12,4}(n) \in \mathbb{P}$
1	2
27, 38	2616577
20, 97	88689281
57, 112	167907937
92, 103	184190177
118, 129	470800657
204	1745737297
218, 361	19242093617
266, 409	32989344497
291, 434	42648854497
303, 446	47996467537
327, 470	60230621041
172, 601	131341375457
423, 566	134643553777
184, 613	142348570097
268, 697	241169071457
508, 651	246204315697
280, 709	258834747761
305, 734	298911678161
744	306402810577
768	347900241457
816	443468273137
629, 772	511728729137
665, 808	621794353121
174, 889	625523919217
282, 997	994377958657
798, 941	1189595935777
331	1209093434177
822, 965	1323728868481
619	1353084028337
631	1421009141921
391	1519678911857
883	1716041697697
907	1892257627201
452	1896481772737
931	2081781513697
212	2166946697297

Continued on next page

$n \in \mathbb{G}_{12,4}$	$L_{12,4}(n) \in \mathbb{P}$
967	2392461847921
776	2470992535601
991	2618165924497
272	2631587872097
548	2634746630977
309	2954115831361
345	3296481815281
393	3800020645297
669	3869282536657
178	4607285626481
502	5166627678097
802	5709642447137
274	5943245637617
826	6104610391937
887	7205434950577
359	7357003500017
647	7551368032097
371	7575748118177
659	7781931346961
947	8434258298017

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,4}$ contains 80 integers with generates 57 primes using the Luhn function $L_{12,4}$.

7.7 The set $\mathbb{G}_{14,4}$

Examples:

- $n = 1 = 1_{(14)}$, $L_{14,4}(n) = (1_{(14)})^4 + \big(R(1_{(14)})\big)^4 = 2_{(14)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{14,4}$,
- $n = 61 = 45_{(14)}$, $L_{14,4}(n) = (45_{(14)})^4 + \big(R(45_{(14)})\big)^4 = 5b6dcad_{(14)} = 43832417 \in \mathbb{P} \Rightarrow 61 \in \mathbb{G}_{14,4}$,
- $n = 74 = 54_{(14)}$, $L_{14,4}(n) = (54_{(14)})^4 + \big(R(54_{(14)})\big)^4 = 5b6dcad_{(14)} = 43832417 \in \mathbb{P} \Rightarrow 74 \in \mathbb{G}_{14,4}$.

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{14,4}$ is: 1, 61, 67, 74, 121, 134, 158, 181, 194, 214, 218, 224, 238, 302, 314, 346, 352, 368, 374, 386, 388, 467, 473, 497, 509, 521, 523, 527, 547, 553, 557, 563, 577, 628, 716, 742, 746, 758, 772, 782, 799, 803, 811, 881, 929, 941, 953, 994, 998 .

Table 23: The set $\mathbb{G}_{14,4}$ with $1 \le n \le 10^3$

$n \in \mathbb{G}_{14,4}$	$L_{14,4}(n) \in \mathbb{P}$
1	2
61, 74	43832417
121, 134	536776817
67, 158	643352417
181, 194	2489751617
224	2518338257
238	3211961537
352, 547	104878226897
742	303728291297
521, 716	336496390817
214, 799	409653110417
563, 758	430593137057
577, 772	466038647297
368, 953	843183247457
994	976215267617
746, 941	1093786659617
467	1272356555537
803, 998	1407802614497
509	1499539375457
523	1581977010737
929	2340958685057
314	2770373204657
497	4755966348737
553	5544735471137
218	6281738210897
302	7730542243937
386	9422561940017
527	12891127750337
346	19499502389057
374	20564399730257
388	21113251313537
473	24691191817697
557	28668956920337
628	32403047498177
782	41800807506737
811	43786340848577
881	48884022559937

For $1 \le n \le 10^3$ the set $\mathbb{G}_{14,4}$ contains 49 integers with generates 37 primes using the Luhn function $L_{14,4}$.

7.8 The set $\mathbb{G}_{16,4}$

Examples:

- $n = 1 = 1_{(16)}$, $L_{16,4}(n) = (1_{(16)})^4 + \big(R(1_{(16)})\big)^4 = 2_{(16)} = 2 \in \mathbb{P} \Rightarrow 1 \in \mathbb{G}_{16,4}$,
- $n = 16 = 10_{(16)}$, $L_{16,4}(n) = (10_{(16)})^4 + \big(R(10_{(16)})\big)^4 = 10001_{(16)} = 65537 \in \mathbb{P} \Rightarrow 16 \in \mathbb{G}_{16,4}$,
- $n = 37 = 23_{(16)}$, $L_{16,4}(n) = (23_{(16)})^4 + \big(R(23_{(16)})\big)^4 = 2ce7b01_{(16)} = 47086337 \in \mathbb{P} \Rightarrow 37 \in \mathbb{G}_{16,4}$.

For $1 \le n \le 10^3$ the set $\mathbb{G}_{16,4}$ is: 1, 16, 22, 28, 37, 47, 56, 58, 73, 82, 92, 97, 124, 131, 139, 143, 148, 158, 163, 184, 193, 197, 199, 233, 239, 242, 248, 254, 262, 278, 296, 302, 322, 332, 346, 352, 368, 376, 382, 386, 388, 398, 406, 422, 448, 458, 482, 484, 488, 539, 551, 563, 577, 601, 607, 617, 631, 641, 647, 679, 709, 727, 737, 749, 757, 763, 784, 818, 832, 896, 908, 958, 988 .

Table 24: The set $\mathbb{G}_{16,4}$ with $1 \le n \le 10^3$

$n \in \mathbb{G}_{16,4}$	$L_{16,4}(n) \in \mathbb{P}$
1	2
16	65537
37, 82	47086337
22, 97	88763537
56, 131	304334417
73, 148	508183457
58, 163	717228257
28, 193	1388102657
139, 184	1519529777
92, 197	1577777777
124, 199	1804660577
47, 242	3434621777
158, 233	3570496817
143, 248	4200903617
239, 254	7425122897

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$n \in \mathbb{G}_{16,4}$	$L_{16,4}(n) \in \mathbb{P}$
352	15440730497
368	18502707137
448	41669583617
322, 577	121592090897
386, 641	191023004177
482, 737	349007003537
784	377802128657
832	479194217297
563, 818	548196274337
896	644808029777
388	1789991785217
484	2488478993537
709	4973209188737
262	5585499290897
757	5694483278657
278	5822795469137
406	8012096136017
422	8325013787537
551	11209570011377
631	13356029584817
647	13821490093937
679	14790568593137
727	16343438967857
296	18761435138177
376	21828149319617
488	26749734472577
601	32540669618417
617	33433254431537
346	48663296884817
458	57485378357777
539	64590294232577
763	87786929405057
332	96852883841537
908	105931110192977
988	117120059600657
749	159854761121057
302	171165069640337
382	186830296674257
398	190089062031377

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$n \in \mathbb{G}_{16,4}$	$L_{16,4}(n) \in \mathbb{P}$
958	201352676868257
607	236744262848657

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,4}$ contains 73 integers with generates 56 primes using the Luhn function $L_{16,4}$.

8 The sets $\mathbb{G}$ of order 8

8.1 Very Probably Primes

The *IsPrime* command is a probabilistic primality testing routine . It returns false if n is shown to be composite within one *strong pseudo-primality* Miller–Rabin test and one Lucas test . It returns true otherwise . If *IsPrime* returns true, n is very probably prime, [3, 6, 8, 9, 7, 1] . No counterexample is known and it has been conjectured that such a counterexample must be hundreds of digits long .

We call very probably primes those natural numbers $n \in \mathbb{N}$ that *IsPrime* Mathcad function returns 1 to .

We denote this set of numbers by $\mathrm{VP}\mathbb{P}$. Is is proven that up to 10^{16} Miller–Rabin and Lucas tests are equivalent to a deterministic algorithm, [5] . Even more so, there are no known counter example for the two tests . As a consequence we define:

Definition 11. For any $n \in \mathbb{N}$

- If $n < 10^{16}$ and $IsPrime(n) = 1 \Rightarrow n \in \mathbb{P}$;
- If $n \geq 10^{16}$ and $IsPrime(n) = 1 \Rightarrow n \in \mathrm{VP}\mathbb{P}$;
- If $IsPrime(n) = 0 \Rightarrow n \notin \mathrm{VP}\mathbb{P}$.

We observe that $\mathbb{P} \subset \mathrm{VP}\mathbb{P}$. In accordance with definition 11 we can say that the sets $\mathbb{G}_{b,\omega}$ are $\mathrm{VP}\mathbb{G}_{b,\omega}$.

Definition 12. For any $n \in \mathbb{N}$

- If $L_{b,\omega}(n) < 10^{16}$ and $IsPrime(L_{b,\omega}(n)) = 1 \Rightarrow n \in \mathbb{G}_{b,\omega}$;
- If $L_{b,\omega}(n) \geq 10^{16}$ and $IsPrime(L_{b,\omega}(n)) = 1 \Rightarrow n \in \mathrm{VP}\mathbb{G}_{b,\omega}$;
- If $IsPrime(L_{b,\omega}(n)) = 0 \Rightarrow n \notin \mathrm{VP}\mathbb{G}_{b,\omega}$.

8.2 The sets $\mathbb{G}_{2,8}$ and $\mathrm{VP}\mathbb{G}_{2,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,8}$ is: 1, 2, 4, 38, 50, 74, 92 .

Table 25: The set $\mathbb{G}_{2,8}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{2,8}$	$L_{2,8}(n) \in \mathbb{P}$
1	2
2	257
4	65537
38	4500380029121
50	39079483563041
74	907179665432897
92	5132688977788577

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,8}$ contains 7 integers with generates 7 primes using the Luhn function $L_{2,8}$.

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{2,8}$ is: 142, 148, 164, 166, 184, 220, 226, 278, 298, 314, 424, 478, 494, 568, 610, 614, 688, 692, 698, 734, 802, 872, 890 .

Table 26: The set $\mathrm{VP}\mathbb{G}_{2,8}$ with $1 \leq n \leq 10^3$

$n \in \mathrm{VP}\mathbb{G}_{2,8}$	$L_{2,8}(n) \in \mathrm{VP}\mathbb{P}$
142	191897345927979137
148	230201838417395777
164	523303572295127777
166	587415378483875297
184	1313840815478570657
220	5487734184037604321
226	6806262887371711937
278	39315116375052739457
298	62856784992960152897
314	95873232097474809281
424	1044544285812732628577
478	2739209592437356204097
494	3557273536121258202497
568	10833973060914810445697
610	19209604523580723509441
614	20982839869616107715777
688	50200438003654106748257
692	52584563057266967906657
698	56718131611660623370337

Continued on next page

$n \in \mathrm{VP}\mathbb{G}_{2,8}$	$L_{2,8}(n) \in \mathrm{VP}\mathbb{P}$
734	87739320186635588824097
802	171189819157799177090081
872	334296543034194855583457
890	394084590401408677608161

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{2,8}$ contains 23 integers with generates 23 primes using the Luhn function $L_{2,8}$.

8.3 The sets $\mathbb{G}_{4,8}$ and $\mathrm{VP}\mathbb{G}_{4,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,8}$ is: 1, 4, 56, 88 .

Table 27: The set $\mathbb{G}_{4,8}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{4,8}$	$L_{4,8}(n) \in \mathbb{P}$
1	2
4	65537
56	96717525932897
88	3599857727509217

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,8}$ contains 4 integers with generates 4 primes using the Luhn function $L_{4,8}$.

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{4,8}$ is: 191, 224, 244, 254, 292, 316, 424, 446, 496, 667, 671, 761, 796, 824, 892, 904, 922, 956, 986 .

Table 28: The set $\mathrm{VP}\mathbb{G}_{4,8}$ with $1 \leq n \leq 10^3$

$n \in \mathrm{VP}\mathbb{G}_{4,8}$	$L_{4,8}(n) \in \mathrm{VP}\mathbb{P}$
224	6338465731529071457
244	12563731317480845057
191, 254	19096057251353049857
292	52860006015964869377
316	110805087804597740417
424	1045209690733715530817
496	3663139304567919526817

Continued on next page

$n \in \mathrm{VP}\mathbb{G}_{4,8}$	$L_{4,8}(n) \in \mathrm{VP}\mathbb{P}$
446, 761	1140460971273722229038017
796	1611814758151035576804417
824	2125298067380650390615371
892	4008056609420460950853777
904	4460130638370360029940177
667, 922	5613874160720371620141777
956	6977067820092815285652177
671, 986	9344311603054550869596177

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{4,8}$ contains 19 integers with generates 15 very probable primes using the Luhn function $L_{4,8}$.

8.4 The sets $\mathbb{G}_{6,8}$ and $\mathrm{VP}\mathbb{G}_{6,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,8}$ is: 1, 8, 13, 17, 32 .

Table 29: The set $\mathbb{G}_{6,8}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{6,8}$	$L_{6,8}(n) \in \mathbb{P}$
1	2
8, 13	832507937
17, 32	1106487385217

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,8}$ contains 5 integers with generates 3 primes using the Luhn function $L_{6,8}$.

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{6,8}$ is: 64, 68, 83, 103, 118, 153, 169, 188, 272, 284, 324, 396, 547, 557, 619, 621, 716, 772, 957 .

Table 30: The set $\mathrm{VP}\mathbb{G}_{6,8}$ with $1 \leq n \leq 10^3$

$n \in \mathrm{VP}\mathbb{G}_{6,8}$	$L_{6,8}(n) \in \mathrm{VP}\mathbb{P}$
68, 103	131248640535295377
118, 153	3378720763531071377
64, 169	6656980841598904977
83, 188	15627487748973079377

Continued on next page

$n \in \text{VP}\mathbb{G}_{6,8}$	$L_{6,8}(n) \in \text{VP}\mathbb{P}$
324	1214395311135778148 17
396	6047299637931727541 77
272, 547	8044869924308995163297
284, 619	2159618677999927458 3137
621, 716	9118967676689398857 7057
772, 957	8297157242749946946 23137
557	356902634997256799 4447137

For $1 \leq n \leq 10^3$ the set $\text{VP}\mathbb{G}_{6,8}$ contains 19 integers with generates 11 very probable primes using the Luhn function $L_{6,8}$.

8.5 The sets $\mathbb{G}_{8,8}$ and $\text{VP}\mathbb{G}_{8,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,8}$ is: 1, 19, 26 .

Table 31: The set $\mathbb{G}_{8,8}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{8,8}$	$L_{8,8}(n) \in \mathbb{P}$
1	2
19, 26	225810627617

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,8}$ contains 3 integers with generates 2 primes using the Luhn function $L_{8,8}$.

For $1 \leq n \leq 10^3$ the set $\text{VP}\mathbb{G}_{8,8}$ is: 100, 118, 163, 167, 214, 226, 289, 317, 334, 380, 397, 403, 433, 482, 538, 544, 586, 596, 694, 766, 848, 962, 1000 .

Table 32: The set $\text{VP}\mathbb{G}_{8,8}$ with $1 \leq n \leq 10^3$

$n \in \text{VP}\mathbb{G}_{8,8}$	$L_{8,8}(n) \in \text{VP}\mathbb{P}$
163, 226	73039285481585872 97
100, 289	48671191875666868481
317, 380	5367496079388464526 41
214, 403	7001279775265546404 17
334, 397	7719268353463974986 57
118, 433	12357094891144659708 17

Continued on next page

$n \in VP\mathbb{G}_{8,8}$	$L_{8,8}(n) \in VP\mathbb{P}$
167, 482	2913845245788668967617
544	7688957566924055382017
848	26740399959248946502969 7
1000	100040806636712234027488 1
586	211116892425942554113081 7
962	259818609303667117994233 7
538	481900772410178799560473 7
596	5195469972882306998429929 61
694	2116602821008962171741321 0497
766	2494204049689328119574759 3921

For $1 \leq n \leq 10^3$ the set $VP\mathbb{G}_{8,8}$ contains 23 integers with generates 16 very probable primes using the Luhn function $L_{8,8}$.

8.6 The sets $\mathbb{G}_{10,8}$ and $VP\mathbb{G}_{10,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,8}$ is: 1, 29, 56, 65 .

Table 33: The set $\mathbb{G}_{10,8}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{10,8}$	$L_{10,8}(n) \in \mathbb{P}$
1	2
56, 65	415362124464641

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,8}$ contains 3 integers with generates 2 primes using the Luhn function $L_{10,8}$.

For $1 \leq n \leq 10^3$ the set $VP\mathbb{G}_{10,8}$ is: 67, 76, 92, 152, 190, 251, 277, 326, 328, 467, 475, 526, 574, 623, 625, 677, 758, 764, 772, 776, 823, 857 .

Table 34: The set $VP\mathbb{G}_{10,8}$ with $1 \leq n \leq 10^3$

$n \in VP\mathbb{G}_{10,8}$	$L_{10,8}(n) \in VP\mathbb{P}$
67, 76	1519102465011617
29, 92	5132688977788577
190	1703058829376151521

Continued on next page

$n \in VP\mathbb{G}_{10,8}$	$L_{10,8}(n) \in VP\mathbb{P}$
152, 251	16038898117402725857
475, 574	143755513258379646904O1
326, 623	228212188057656O1257857
526, 625	29142906999765462659201
467, 764	118339406390336068523297
277, 772	126199518608191565394017
677, 776	175617818926353001682657
328, 823	210608074785323633233217
758, 857	399950618911225987316417

For $1 \leq n \leq 10^3$ the set $VP\mathbb{G}_{10,8}$ contains 23 integers with generates 12 very probable primes using the Luhn function $L_{10,8}$.

8.7 The sets $\mathbb{G}_{12,8}$ and $VP\mathbb{G}_{12,8}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{12,8}$ contains 1 that generates $2 \in \mathbb{P}$ using the Luhn function $L_{12,8}$ and the set $VP\mathbb{G}_{12,8}$ is: 59, 136, 186, 228, 236, 321, 335, 369, 379, 454, 568, 665, 667, 669, 681, 711, 740, 808, 901 .

Table 35: The set $VP\mathbb{G}_{12,8}$ with $1 \leq n \leq 10^3$

$n \in VP\mathbb{G}_{12,8}$	$L_{12,8}(n) \in VP\mathbb{P}$
59, 136	117180619788868577
228	7305346145742488161
568, 711	76140506143162648817857
665, 808	219918076130863342920641
186, 901	434309931190840589189857
379	2052242484035378885130977
667	2121192565702907682993377
740	3577434514845045845491841
236	5482243979911769123591777
321	9329468204119196251466497
369	12410136721616575932762241
669	13501477682991268315275937
681	14470178311917522866189377
454	20088289956814789318258561
335	47908021625564485370450561

For $1 \le n \le 10^3$ the set $\mathrm{VP}\mathbb{G}_{12,8}$ contains 19 integers with generates 15 very probable primes using the Luhn function $L_{12,8}$.

8.8 The sets $\mathbb{G}_{14,8}$ and $\mathrm{VP}\mathbb{G}_{14,8}$

For $1 \le n \le 10^3$ the set $\mathbb{G}_{14,8}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{14,8}$ and the set $\mathrm{VP}\mathbb{G}_{14,8}$ is: 181, 194, 214, 244, 386, 431, 451, 493, 646, 658, 688, 712, 799, 994 .

Table 36: The set $\mathrm{VP}\mathbb{G}_{14,8}$ with $1 \le n \le 10^3$

$n \in \mathrm{VP}\mathbb{G}_{14,8}$	$L_{14,8}(n) \in \mathrm{VP}\mathbb{P}$
181, 194	3158319657984002657
451, 646	32040782999017820722337
658	3514043707116574938l697
493, 688	53690035578940670929697
214, 799	166106158667183715629057
994	952995994285865113754657
244	4875630263134172695985057
386	8836730104423433340612025 7
431	52143749006838600253591129 7
712	1372063152953867877968664737

For $1 \le n \le 10^3$ the set $\mathrm{VP}\mathbb{G}_{14,8}$ contains 14 integers with generates 10 very probable primes using the Luhn function $L_{14,8}$.

8.9 The sets $\mathbb{G}_{16,8}$ and $\mathrm{VP}\mathbb{G}_{16,8}$

For $1 \le n \le 10^3$ the set $\mathbb{G}_{16,8}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{16,8}$ and the set $\mathrm{VP}\mathbb{G}_{16,8}$ is: 71, 77, 116, 139, 184, 212, 266, 268, 316, 338, 398, 406, 412, 418, 448, 452, 482, 593, 623, 643, 649, 659, 671, 673, 737, 743, 772, 838, 848, 892, 896, 898, 914, 932, 934, 974, 998 .

Table 37: The set $\mathrm{VP}\mathbb{G}_{16,8}$ with $1 \le n \le 10^3$

$n \in \mathrm{VP}\mathbb{G}_{16,8}$	$L_{16,8}(n) \in \mathrm{VP}\mathbb{P}$
71, 116	33429902451057857

Continued on next page

$n \in \mathrm{VP}\mathbb{G}_{16,8}$	$L_{16,8}(n) \in \mathrm{VP}\mathbb{P}$
139, 184	1453193982443841377
77, 212	4081486807090502177
448	1624572350169485395457
338, 593	15461405049627376717697
418, 673	43016327196463504262657
482, 737	89957453249982086711297
848	267403336428172513988897
896	415397776897644472386977
643, 898	452094226632482678109857
659, 914	522615489604338675513377
772	1363717121183028958362017
932	4510275599055724509044897
452	4813731237433310344439297
838	43841355408656299068727457
406	63759769172834605446787457
934	70009107124921551398728097
998	94313516832753510520194977
743	275116054866006633227229377
649	1231900658266745681955474497
266	1850446927417766295002296577
268	7952441508860938981730246657
316	9002231025798406347589737857
892	10643215663120828345297378337
412	11471191201891914359473553537
398	36124313399367993437620024577
974	41593320514820912570815589537
623	57837014218674166026501179777
671	63723355276719622258282863617

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{16,8}$ contains 37 integers with generates 29 very probable primes using the Luhn function $L_{16,8}$.

9 The sets $\mathbb{G}$ of order 16

9.1 The sets $\mathbb{G}_{2,16}$ and $\mathrm{VP}\mathbb{G}_{2,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{2,16}$ contains 2 integers with generates 2 primes using the Luhn function $L_{2,16}$.

Table 38: The set $\mathbb{G}_{2,16}$ with $1 \leq n \leq 10^3$

$n \in \mathbb{G}_{2,16}$	$L_{2,16}(n) \in \mathbb{P}$
1	2
2	65537

For $1 \leq n \leq 10^3$ the set $\text{VP}\mathbb{G}_{2,16}$ is: 38, 88, 122, 142, 158, 200, 290, 298, 332, 346, 358, 364, 376, 464, 550, 566, 634, 638, 728, 752, 758, 764, 898, 920, 928, 974 .

Table 39: The set $\text{VP}\mathbb{G}_{2,16}$ with $1 \leq n \leq 10^3$

n	$L_{2,16}(n) \in \text{VP}\mathbb{P}$
38	189265795439330078080032641
88	12933699143210573934279056827457
122	2408559575565965370674383340713217
142	28035088781234219279992140179504897
158	152950289705409385459566483544917377
200	6553600000000000288441413567621167681
290	250248013709969575116468671562165066 3041
298	386820908071043091048663057721789876441 7
332	2178759739974965232210423173632476103417 7
346	4219210950626617413481973726665963136377 7
358	7280923384980747217813820451703862086048 1
364	9497780755251241753862424508377756651577 7
376	15958994271672235164135230094666707675833 7
464	46162326553296607426317912823022151648328 97
550	70560726056856617770390325591542947859206 401
566	11246043195058066517907590965238837119259 9297
634	68160730799763513531419216692617179965576 5377
638	77147472563770807663410572732327276168449 9841
728	62224465569742054204288521147289371424895 017537
752	10458886485880707514910769658596121098226 674497
758	11879379207477450243263906267697946350900 585281
764	13473913231865104769625661586066208207967 233857
898	17882218121066367294931154778732289083031 1714177
920	26339361174458870812972070367331179380544 4097921
928	30252942329968464642871517285336631879602 9078657
974	65606408260934463116464528109228410303151 2374017

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{2,16}$ contains 26 integers with generates 26 very probable primes using the Luhn function $L_{2,16}$.

9.2 The sets $\mathbb{G}_{4,16}$ and $\mathrm{VP}\mathbb{G}_{4,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{4,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{4,16}$ and the set $\mathrm{VP}\mathbb{G}_{4,16}$ is: 28, 43, 58, 278, 404, 412, 428, 482, 557, 593, 599, 739, 751, 796, 814, 854, 992 .

Table 40: The set $\mathrm{VP}\mathbb{G}_{4,16}$ with $1 \leq n \leq 10^3$

n	$L_{4,16}(n) \in \mathrm{VP}\mathbb{P}$
28	142735015363284129948737
43, 58	16536766924844555943601291457
404	503618693209024887524287132682774085571457
412	689240341963090556902835879701868213197697
428	1268020131418850783962531084341927946370177
482, 557	94325667147303523020799929617600272848563777
278, 593	233817739593871516318608064906601755642813697
796	25978201678054221161749024087300514403770875457
739, 814	45065158988743197446813715926849661424382898497
599, 854	80318155645748105387320802049391653642542164097
992	879400593664848129000807111778239212881255019777
751	1110681915711859894729891623370615267403075635457

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{4,16}$ contains 17 integers with generates 12 very probable primes using the Luhn function $L_{4,16}$.

9.3 The sets $\mathbb{G}_{6,16}$ and $\mathrm{VP}\mathbb{G}_{6,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{6,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{6,16}$ and the set $\mathrm{VP}\mathbb{G}_{6,16}$ is: 8, 13, 22, 27, 29, 34, 44, 54, 79, 179, 214, 228, 338, 386, 471, 523, 601, 611, 656, 688, 856, 873 .

Table 41: The set $\mathrm{VP}\mathbb{G}_{6,16}$ with $1 \leq n \leq 10^3$

n	$L_{6,16}(n) \in \mathrm{VP}\mathbb{P}$
8, 13	665698084159890497
22, 27	8277780457321157500665 7
29, 34	3439306344444051241558337
54	52275739019273303740263941 77
44, 79	230181649368312591678678189 3377
179, 214	2045813263315263410092298544339605 3057
228	5332827763246422125514226030706010 5857
338, 523	313638248213348031253449982243845280256 80577
386, 601	289971605735919812344055502306578385761 547137
471, 656	118201242376682419116018304058597319350 7893377
688, 873	116342526063743708213803080469881266383 255118977
856	198510110058814332892055298357448561964 6467389697
611	559575157367717045397122432207972492763 24904933697

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{6,16}$ contains 22 integers with generates 13 very probable primes with the Luhn function $L_{6,16}$.

9.4 The sets $\mathbb{G}_{8,16}$ and $\mathrm{VP}\mathbb{G}_{8,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{8,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{8,16}$ and the set $\mathrm{VP}\mathbb{G}_{8,16}$ is: 10, 17, 76, 88, 163, 226, 230, 265, 271, 328, 376, 391, 419, 454, 460, 464, 496, 638, 682, 754, 814, 830, 934 .

Table 42: The set $\mathrm{VP}\mathbb{G}_{8,16}$ with $1 \leq n \leq 10^3$

n	$L_{8,16}(n) \in \mathrm{VP}\mathbb{P}$
10, 17	48671191875666868481
88	1293369916649297288305683653 8241
163, 226	46564738838062647871250321444626359617
76, 265	59147171909140332977773417603169005120 1
328	17946571741795964394173169717319234102337
376	15958994271672235164135230094666707675833 7
230, 419	90251385745844679967373401133252801348128 1
391, 454	35559892990252353097940162677513094111983 37
271, 460	40199138314525682573348602682076319470323 21

Continued on next page

n	$L_{8,16}(n) \in \mathrm{VP}\mathbb{P}$
464	46162326553296607426317912823022151648328897
496	13418588160169198432884807893789072503265921
682	138591586924366035655336725143669057642427364347137
754	316194299198492513527854986262556564142016669026177
934	286086240525010740419455466235010623190430744474986744577
814	358566088226134566344743307679795336226980491866327528961
638	578664091938444371477769958731992311448140972132833540737
830	644945949936632469986299860354019751108477975153164664321

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{8,16}$ contains 23 integers with generates 17 very probable primes with the Luhn function $L_{8,16}$.

9.5 The sets $\mathbb{G}_{10,16}$ and $\mathrm{VP}\mathbb{G}_{10,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{10,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{10,16}$ and the set $\mathrm{VP}\mathbb{G}_{10,16}$ is: 58, 85, 106, 146, 148, 170, 229, 601, 607, 629, 641, 706, 841, 922, 926, 940 .

Table 43: The set $\mathrm{VP}\mathbb{G}_{10,16}$ with $1 \leq n \leq 10^3$

n	$L_{10,16}(n) \in \mathrm{VP}\mathbb{P}$
58, 85	7441508776505509970565226866881
170	486612335754291801180028124580469121
106, 601	289728722872489494913618807065119667963638657
146, 641	812322480694375776817863098042516994477991937
607, 706	4149203674104799357784379201594538760284781057
148, 841	62623297589501767571014001416042511462083547777
229, 922	272706027426988304839179129660520178298958906817
629, 926	292864669649605284316490636270026945009107772097
940	371574290834100916860555325532803920646305299201

For $1 \leq n \leq 10^3$ the set $\mathrm{VP}\mathbb{G}_{10,16}$ contains 16 integers with generates 9 very probable primes with the Luhn function $L_{10,16}$.

9.6 The sets $\mathbb{G}_{12,16}$ and $\mathrm{VP}\mathbb{G}_{12,16}$

For $1 \le n \le 10^3$ the set $\mathbb{G}_{12,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{12,16}$ and the set $\mathrm{VP}\mathbb{G}_{12,16}$ is: 83, 92, 103, 138, 218, 246, 248, 254, 256, 258, 303, 309, 361, 397, 446, 526, 544, 641, 685, 687, 719, 742, 784, 828, 931, 947, 961, 973 .

Table 44: The set $\mathrm{VP}\mathbb{G}_{12,16}$ with $1 \le n \le 10^3$

n	$L_{12,16}(n) \in \mathrm{VP}\mathbb{P}$
92, 103	186810005084337606559713123477377
83, 138	17305840822544477653540506096952897
218, 361	83224468857509583494747144443019935351937
254, 397	381057336526733944762779240519362077105217
303, 446	2456128995114057482379611193940027757634817
256, 685	2349899055837678425564260224034861490606102081
544, 687	2520939703877870992282224495027047826530340097
641, 784	21185417135351618991615625987294775582825261057
828	48807368017733302351351829130125663602558369537
246, 961	529144398232291548520051055082451083852634438657
258, 973	645365053325718515569777641868242916435023614017
931	3452342659947122898891803127720270099408638965057
248	35075107761521665506976210970651842412583561473537
309	75221281944659957040685900741797002480364301967041
742	412440892588758983636987725971523510194828179673857
526	873825290377091836151228337688334811572333264363137
947	3389611489016506674900928548814745966718548191311937
719	5867532827904289793774734195002877625373186396916481

For $1 \le n \le 10^3$ the set $\mathrm{VP}\mathbb{G}_{12,16}$ contains 28 integers with generates 18 very probable primes with the Luhn function $L_{12,16}$.

9.7 The sets $\mathbb{G}_{14,16}$ and $\mathrm{VP}\mathbb{G}_{14,16}$

For $1 \le n \le 10^3$ the set $\mathbb{G}_{14,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{14,16}$ and the set $\mathrm{VP}\mathbb{G}_{14,16}$ is: 326, 356, 376, 437, 487, 581, 632, 634, 658, 682, 829, 901, 911, 947 .

Table 45: The set $VP\mathbb{G}_{14,16}$ with $1 \leq n \leq 10^3$

n	$L_{14,16}(n) \in VP\mathbb{P}$
437, 632	649617092953983875482505331197932370460788417
658	1234850260873841152009946604912647567316198017
634, 829	50440537604361115399781194030183972686629935937
326, 911	225057796766475265972725346014814847118890811137
901	4523421892508289084626047833237872652165947279297
356	97017233851430117068242718631325447328380220342337
581	1180738010845875364703342373578377306668994468889537
947	34678425678459326614078256049556517201099341053130817
682	95035418737597372360609211453636499169405879061610497
487	407539522391571821105438845633644579513281010585882497
376	2661739385270214101383652217757857540607999355190733697

For $1 \leq n \leq 10^3$ the set $VP\mathbb{G}_{14,16}$ contains 14 integers with generates 11 very probable primes with the Luhn function $L_{14,16}$.

9.8 The sets $\mathbb{G}_{16,16}$ and $VP\mathbb{G}_{16,16}$

For $1 \leq n \leq 10^3$ the set $\mathbb{G}_{16,16}$ contains 1 generates $2 \in \mathbb{P}$ with the Luhn function $L_{16,16}$ and the set $VP\mathbb{G}_{16,16}$ is: 22, 52, 67, 71, 86, 97, 101, 116, 239, 254, 478, 749, 751, 761, 838, 892 .

Table 46: The set $VP\mathbb{G}_{16,16}$ with $1 \leq n \leq 10^3$

n	$L_{16,16}(n) \in VP\mathbb{P}$
52, 67	167748901330901135586144682817
22, 97	61425365349279931942536832738817
86, 101	126211001282566049409008850214337
71, 116	1075217418019532990990832528382337
239, 254	413486390327489703798666018639815731457
838	1900858855861837207941561729061942589767303441097537
761	3116982692821303896593766817588141869759214926209232257
892	113269508545694387338216268857773130973116142106905951297
749	647856432985105681224302514727674370971729884850411858497
478	1835377837603568251638014926187922825429269032350602634497
751	5580562659546599999646654983855216504553276696848034908417

For $1 \leq n \leq 10^3$ the set $VP\mathbb{G}_{16,16}$ contains 16 integers with generates 11 very probable primes with the Luhn function $L_{16,16}$.

10 Intersections and reunions of sets $\mathbb{G}$

10.1 The sets $\mathbb{G}$ of order 1

For n, $1 \leq n \leq 10^3$, we have 450 integers generating primes using the Luhn functions of order 1: 1, 2, 4, 6, 10, 12, 16, 22, 26, 28, 36, 37, 41, 43, 46, 47, 48, 52, 54, 56, 58, 62, 64, 66, 74, 76, 80, 81, 82, 83, 86, 88, 89, 92, 94, 99, 100, 101, 106, 116, 118, 122, 124, 134, 136, 137, 140, 142, 143, 145, 146, 149, 151, 153, 155, 157, 158, 159, 162, 163, 166, 168, 169, 170, 171, 172, 174, 178, 179, 182, 184, 185, 186, 188, 190, 192, 194, 196, 200, 204, 206, 208, 212, 214, 215, 217, 218, 220, 226, 228, 229, 236, 238, 239, 240, 241, 242, 245, 248, 250, 256, 257, 258, 262, 265, 266, 268, 269, 271, 274, 275, 276, 277, 278, 281, 282, 283, 284, 287, 289, 292, 295, 296, 298, 299, 302, 304, 305, 308, 309, 311, 314, 315, 316, 317, 319, 322, 323, 326, 327, 328, 329, 332, 333, 334, 338, 339, 340, 344, 346, 352, 356, 358, 362, 364, 368, 370, 371, 374, 376, 380, 382, 383, 386, 388, 393, 394, 395, 398, 399, 400, 401, 403, 404, 405, 406, 407, 409, 413, 415, 416, 418, 421, 422, 424, 425, 427, 428, 431, 433, 434, 436, 437, 439, 440, 443, 445, 448, 451, 452, 454, 455, 458, 460, 461, 463, 464, 466, 467, 469, 470, 472, 475, 476, 478, 479, 482, 484, 485, 487, 488, 491, 493, 494, 496, 497, 499, 500, 502, 506, 508, 511, 512, 514, 517, 521, 526, 527, 533, 536, 538, 539, 541, 542, 544, 548, 551, 553, 554, 557, 559, 562, 563, 566, 568, 569, 571, 574, 577, 578, 581, 583, 584, 587, 591, 592, 595, 596, 598, 599, 601, 602, 603, 604, 608, 609, 611, 613, 615, 616, 617, 619, 623, 625, 626, 628, 632, 635, 637, 638, 641, 646, 647, 649, 653, 656, 657, 659, 661, 664, 665, 667, 669, 671, 673, 674, 677, 679, 682, 683, 685, 686, 687, 688, 689, 691, 695, 697, 703, 706, 707, 710, 711, 712, 716, 719, 721, 724, 727, 731, 733, 734, 736, 737, 738, 739, 740, 743, 744, 746, 748, 751, 752, 756, 758, 760, 761, 762, 763, 764, 766, 767, 768, 772, 776, 778, 782, 788, 790, 794, 796, 799, 800, 803, 806, 808, 811, 812, 814, 818, 821, 822, 823, 827, 828, 830, 832, 833, 834, 835, 836, 838, 839, 841, 842, 851, 853, 854, 859, 863, 865, 868, 869, 871, 872, 874, 875, 877, 878, 881, 883, 889, 892, 893, 895, 896, 901, 902, 904, 905, 908, 910, 911, 914, 916, 922, 926, 928, 929, 931, 932, 934, 938, 943, 944, 946, 947, 950, 953, 955, 958, 959, 961, 962, 964, 965, 970, 971, 973, 974, 976, 977, 980, 982, 986, 988, 992, 994, 997, 998 .

These numbers generated the following 166 primes: 2, 3, 5, 7, 11, 13, 17, 37, 41, 59, 61, 67, 73, 97, 101, 109, 113, 139, 149, 173, 181, 193, 197, 211, 227, 233, 241, 257, 271, 281, 283, 307, 313, 337, 353, 373, 383, 389, 397, 409, 419, 421, 433, 443, 449, 457, 461, 463, 467, 487, 499, 503, 523, 557, 563, 577, 593, 601, 617, 619, 641, 647, 653, 661, 673, 727, 733, 761, 773, 787, 797, 811, 821, 827, 839, 877, 887, 907, 929, 941, 971, 1009, 1013, 1039, 1049, 1063, 1069, 1087, 1091, 1097, 1123, 1151, 1153, 1171, 1181, 1187, 1231, 1237, 1279, 1291, 1321, 1373, 1381, 1433, 1453, 1489, 1493, 1553, 1619, 1621, 1637, 1657, 1667, 1669, 1693, 1721, 1733, 1777, 1787, 1789, 1801, 1811, 1831, 1877, 1913, 1933, 1997, 2029, 2053, 2081, 2087, 2137, 2251, 2377, 2441, 2447, 2473, 2503, 2531, 2609, 2617, 2633, 2657, 2729, 2897, 3011, 3019, 3067, 3083, 3307, 3319, 3373, 3433, 3461, 3469, 3517, 3533, 3821, 3919, 4079, 4111, 4271, 4561, 4657, 4721, 4817 .

For n, $1 \leq n \leq 10^3$ we have the following common elements:

- $\mathbb{G}_{2,1} \cap \mathbb{G}_{4,1} =$ 1, 4, 16, 26, 256, 296, 298, 308, 326, 356, 358, 368, 418, 424, 484 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{6,1} =$ 1, 116, 118 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{8,1} =$ 1, 86, 88, 116, 332, 338, 344, 368, 464, 466 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{10,1} =$ 1, 116, 118, 314, 338, 344, 356, 368, 394 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{12,1} =$ 1, 256, 452, 460, 466 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{14,1} =$ 1, 256, 274, 284, 296, 298, 308, 326, 344, 358, 368 ;
- $\mathbb{G}_{2,1} \cap \mathbb{G}_{16,1} =$ 1, 16, 256, 278, 284, 296, 308, 332, 344, 394, 406, 418, 424, 452, 464, 466, 484 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{6,1} =$ 1 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{8,1} =$ 1, 352, 368, 508 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{10,1} =$ 1, 316, 328, 356, 364, 368, 376, 386, 613, 641, 673, 677, 683, 772, 776, 782, 794, 932, 934, 938, 962, 974, 992, 994 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{12,1} =$ 1, 256, 262, 448, 482, 613, 619, 679, 812, 854 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{14,1} =$ 1, 256, 266, 292, 296, 298, 308, 316, 322, 326, 358, 368, 376, 386, 388, 521, 539, 553, 563, 581, 587, 776, 782, 994, 998 ;
- $\mathbb{G}_{4,1} \cap \mathbb{G}_{16,1} =$ 1, 16, 256, 296, 302, 304, 308, 328, 352, 362, 364, 416, 418, 424, 436, 448, 484, 508, 539, 557, 563, 587, 599, 617, 619, 623, 671, 673, 679, 703, 707, 739, 763, 794, 812, 818, 842, 854, 872, 874, 878, 926, 928, 934, 958, 962, 994 ;
- $\mathbb{G}_{6,1} \cap \mathbb{G}_{8,1} =$ 1, 116, 151, 206 ;
- $\mathbb{G}_{6,1} \cap \mathbb{G}_{10,1} =$ 1, 116, 118, 188 ;
- $\mathbb{G}_{6,1} \cap \mathbb{G}_{12,1} =$ 1, 186 ;
- $\mathbb{G}_{6,1} \cap \mathbb{G}_{14,1} =$ 1 ;
- $\mathbb{G}_{6,1} \cap \mathbb{G}_{16,1} =$ 1 ;
- $\mathbb{G}_{8,1} \cap \mathbb{G}_{10,1} =$ 1, 100, 116, 269, 277, 281, 338, 340, 344, 368, 437, 439 ;
- $\mathbb{G}_{8,1} \cap \mathbb{G}_{12,1} =$ 1, 305, 401, 458, 466, 470, 472 ;

- $\mathbb{G}_{8,1} \cap \mathbb{G}_{14,1} =$ 1, 212, 214, 218, 226, 242, 248, 344, 346, 368, 374, 407, 409, 437 ;
- $\mathbb{G}_{8,1} \cap \mathbb{G}_{16,1} =$ 1, 332, 344, 352, 382, 464, 466, 472, 508 ;
- $\mathbb{G}_{10,1} \cap \mathbb{G}_{12,1} =$ 1, 166, 172, 190, 196, 514, 526, 536, 542, 568, 613, 625, 691, 734, 752, 881, 889, 895 ;
- $\mathbb{G}_{10,1} \cap \mathbb{G}_{14,1} =$ 1, 196, 316, 344, 368, 376, 386, 413, 437, 443, 469, 479, 493, 734, 736, 764, 766, 776, 782, 823, 833, 839, 851, 881, 994 ;
- $\mathbb{G}_{10,1} \cap \mathbb{G}_{16,1} =$ 1, 328, 344, 364, 394, 398, 637, 653, 661, 673, 691, 794, 934, 962, 988, 994 ;
- $\mathbb{G}_{12,1} \cap \mathbb{G}_{14,1} =$ 1, 196, 256, 734, 758, 877, 881, 901, 943, 959, 971, 973, 977 ;
- $\mathbb{G}_{12,1} \cap \mathbb{G}_{16,1} =$ 1, 256, 448, 452, 466, 472, 619, 679, 691, 812, 854 ;
- $\mathbb{G}_{14,1} \cap \mathbb{G}_{16,1} =$ 1, 256, 284, 296, 308, 344, 539, 563, 583, 587, 994 .

10.2 The sets $\mathbb{G}$ of order 2

The n, $1 \leq n \leq 10^3$, we have 435 generating primes using the Luhn functions of order 2: 1, 2, 4, 6, 8, 10, 11, 13, 14, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 31, 32, 34, 35, 36, 37, 38, 40, 41, 43, 44, 46, 47, 48, 49, 50, 51, 52, 55, 56, 58, 59, 62, 67, 68, 73, 74, 76, 77, 79, 80, 82, 83, 85, 86, 87, 88, 89, 90, 92, 93, 94, 97, 101, 103, 104, 105, 107, 110, 112, 116, 118, 122, 124, 128, 131, 133, 134, 136, 138, 142, 143, 145, 148, 149, 151, 155, 158, 161, 164, 167, 170, 171, 173, 174, 178, 180, 181, 184, 185, 186, 188, 190, 191, 194, 198, 200, 204, 205, 206, 208, 212, 214, 218, 220, 227, 228, 230, 232, 233, 236, 238, 239, 242, 244, 246, 248, 254, 256, 263, 264, 265, 266, 268, 272, 274, 275, 278, 282, 283, 284, 286, 296, 298, 302, 307, 310, 311, 312, 314, 315, 317, 318, 320, 322, 323, 324, 326, 327, 328, 332, 334, 338, 343, 344, 346, 348, 352, 356, 358, 362, 364, 368, 370, 372, 373, 376, 379, 380, 382, 383, 384, 386, 392, 395, 397, 398, 401, 404, 406, 407, 409, 411, 412, 413, 414, 416, 418, 419, 421, 422, 424, 425, 427, 428, 431, 433, 436, 446, 449, 451, 452, 454, 457, 458, 461, 463, 464, 466, 467, 470, 472, 473, 478, 482, 484, 487, 488, 491, 494, 496, 499, 500, 502, 506, 507, 508, 511, 514, 517, 518, 521, 523, 524, 530, 531, 537, 538, 541, 542, 547, 548, 550, 551, 553, 554, 556, 557, 562, 571, 577, 578, 580, 581, 583, 586, 589, 593, 596, 597, 598, 601, 602, 604, 608, 611, 613, 614, 616, 617, 619, 620, 621, 626, 631, 632, 634, 635, 637, 640, 641, 643, 646, 647, 652, 655, 656, 658, 661, 662, 664, 667, 668, 671, 679, 682, 683, 688, 689, 692, 694, 697, 698, 706, 707, 710, 713, 716, 719, 722, 724, 728, 733, 734, 736, 737, 740, 742, 744, 748, 749, 751, 752, 757, 758, 760, 761, 763, 764, 766, 772, 776, 778, 782, 784, 786, 790, 797, 800, 802, 805, 806, 808, 810, 811, 812, 815, 816, 817, 820, 821, 822, 823, 824, 827, 829, 830, 832, 836, 838, 842, 844, 848, 857, 860, 862, 866, 867, 868, 869, 873, 874, 877, 879, 883, 886, 887, 889, 893, 895, 896, 898, 899, 901, 902, 904, 907, 911, 914, 916, 920, 921, 922, 923, 928, 929, 938, 941, 943, 944, 950, 951, 953, 956, 958, 959, 961, 962, 964, 965, 967, 969, 971, 974, 976, 977, 979, 980, 982, 986, 988, 989, 992, 994, 998, 1000 .

Table 47: Primes generated by numbers $\mathbb{G}_{2\beta,2}$, $\beta = \overline{1,8}$

2	5	17	37	101	197	233	257
317	389	653	797	821	953	1097	1213
1297	1553	1877	1997	2069	2357	2473	2657
2753	2861	2897	2957	3257	3329	4201	5653
6053	6689	6869	7193	8093	8369	10589	10937
11273	12497	12713	13697	14341	15233	15581	17093
17597	19073	19181	20297	21977	22453	24413	24481
24533	24917	25033	25933	26597	27749	29453	31397
32933	33533	33769	33941	35537	36653	39733	40361
40697	41597	42457	42773	43313	44381	44753	45337
45557	47737	49109	49697	50873	55373	55661	55673
56393	59209	60497	60773	65537	65777	68597	71549
75797	75937	76001	81233	81953	96269	102929	105337
109073	113537	114493	119813	120569	121637	121993	124337
128489	138373	147673	147793	148193	149033	151289	163997
164513	166157	168197	169661	170189	171617	172433	173177
174137	174737	176017	182969	190717	192029	192617	198377
198461	198977	201413	208337	209357	210233	219293	225809
226013	228461	230213	235013	238037	239417	239957	240353
244889	247433	249737	258977	260213	261917	266117	273473
278717	292517	294029	308989	309677	321329	322073	327829
346721	352817	354833	362693	372773	373193	380417	383777
386549	386777	404309	408869	409889	414053	417173	429797
437273	443837	451097	451361	451637	455201	465893	475837
480533	484577	504389	509837	518813	526037	542537	544277
553037	554377	557633	559877	572813	573317	575213	596633
608273	620657	620717	626333	634373	666773	676057	679433
692273	695293	704441	710873	711017	718973	719633	725993
765773	777473	784097	784913	799661	805037	819977	820597
835817	846397	849701	857009	858269	868493	879533	894161
906089	923177	928913	931597	932549	935537	936197	937901
954209	955853	958289	961073	968321	984037	984497	986801
997097	997333	1010957	1018733	1025537	1025873	1033517	1042133
1053557	1079933	1089197	1095221	1104833	1121933	1127573	1138733
1142129	1158629	1176793	1178633	1180733	1183157	1186397	1213913
1235473	1237013	1237037	1258133	1294973	1296817	1309013	1314821
1340477	1346333	1374413	1383377	1384121	1407473	1422437	1425293
1434997	1446097	1446917	1465637	1480837	1486493	1499933	1501193
1529777	1564309	1576037	1594097	1604081	1606909	1610309	1651297

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1671497	1691633	1694513	1760117	1792913	1794257	1805393	1810733
1889177	1898297	1902973	1906613	1910813	1925149	2014553	2027117
2077753	2086697	2109593	2150441	2165117	2167189	2170537	2184293
2207633	2210297	2274653	2277797	2278517	2287133	2336393	2356397
2366957	2382269	2469317	2470397	2559421	2571017	2669477	2682101
2696429	2761397	2935589	2944493	3039017	3072233	3079721	3195557
3214997	3229973	3267197	3303617	3413197	3469217	3528533	3592493
3607817	3678293	3721957	3899657	4073513	4279157	4355957	4567649
4572353	4655333	4808777	5047193	5127917	5263733	5404673	5422817
5540273	5565257	5613329	5724857	5789681	5921297	5942561	5971841
6034529	6054197	6150293	6243257	6439493	6539513	6629477	6748373
6749657	6812453	6833573	6939137	7003757	7047893	7061237	7094597
7147913	7174253	7190693	7242077	7390961	8101637	8177933	8539997
8874557	9334973	9515153	9959237	10175057	10635473	10953989	11076389
11234033	11356277	11384741	11482937	11613293	11866157	11945957	12253133
12454229	13191917	13426757	13813733	13944773	16338437	17096357	

For n, $1 \leq n \leq 10^3$ we have the following common elements:

- $\mathbb{G}_{2,2} \cap \mathbb{G}_{4,2} =$ 1, 4, 16, 26, 82, 104, 256, 268, 406, 416, 466, 472, 838 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{6,2} =$ 1, 22, 46, 122, 268, 358, 412, 736 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{8,2} =$ 1, 82, 104, 122, 206, 466, 488, 506, 548, 556, 664, 736, 790 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{10,2} =$ 1, 104 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{12,2} =$ 1, 16, 268, 830 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{14,2} =$ 1, 16, 82, 386, 736 ;
- $\mathbb{G}_{2,2} \cap \mathbb{G}_{16,2} =$ 1, 16, 26, 82, 122, 256, 268, 334, 386, 416, 488, 506, 944 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{6,2} =$ 1, 151, 268, 326, 398, 418, 551, 583, 643, 842 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{8,2} =$ 1, 62, 82, 104, 151, 181, 232, 452, 466, 494, 808 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{10,2} =$ 1, 14, 41, 52, 104, 332, 382, 398, 508, 689, 778, 986 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{12,2} =$ 1, 14, 16, 92, 268, 842 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{14,2} =$ 1, 14, 16, 41, 82, 151, 284, 314, 326, 523, 551, 778, 986 ;
- $\mathbb{G}_{4,2} \cap \mathbb{G}_{16,2} =$ 1, 16, 26, 47, 52, 56, 62, 82, 161, 244, 256, 268, 326, 382, 398, 416, 454, 508, 589, 601, 671, 683, 713, 749, 751, 778, 848, 922 ;

- $\mathbb{G}_{6,2} \cap \mathbb{G}_{8,2} = 1$, 122, 151, 212, 218, 433, 736, 806 ;
- $\mathbb{G}_{6,2} \cap \mathbb{G}_{10,2} = 1$, 194, 398, 491, 611 ;
- $\mathbb{G}_{6,2} \cap \mathbb{G}_{12,2} = 1$, 103, 136, 204, 268, 272, 842, 862, 967 ;
- $\mathbb{G}_{6,2} \cap \mathbb{G}_{14,2} = 1$, 29, 89, 136, 151, 248, 274, 326, 433, 451, 551, 652, 736, 766, 943 ;
- $\mathbb{G}_{6,2} \cap \mathbb{G}_{16,2} = 1$, 103, 122, 212, 248, 268, 326, 398, 428, 613 ;
- $\mathbb{G}_{8,2} \cap \mathbb{G}_{10,2} = 1$, 10, 104, 283, 380, 401, 419, 722, 914 ;
- $\mathbb{G}_{8,2} \cap \mathbb{G}_{12,2} = 1$, 230, 401, 458, 470, 514 ;
- $\mathbb{G}_{8,2} \cap \mathbb{G}_{14,2} = 1$, 76, 82, 151, 419, 433, 646, 656, 734, 736, 764 ;
- $\mathbb{G}_{8,2} \cap \mathbb{G}_{16,2} = 1$, 62, 82, 86, 118, 122, 143, 167, 212, 488, 506, 916, 956, 976 ;
- $\mathbb{G}_{10,2} \cap \mathbb{G}_{12,2} = 1$, 14, 25, 116, 164, 401, 409 ;
- $\mathbb{G}_{10,2} \cap \mathbb{G}_{14,2} = 1$, 14, 41, 164, 328, 419, 461, 778, 986 ;
- $\mathbb{G}_{10,2} \cap \mathbb{G}_{16,2} = 1$, 52, 227, 328, 382, 398, 508, 778, 904 ;
- $\mathbb{G}_{12,2} \cap \mathbb{G}_{14,2} = 1$, 14, 16, 136, 164, 266, 577, 959 ;
- $\mathbb{G}_{12,2} \cap \mathbb{G}_{16,2} = 1$, 16, 79, 103, 266, 268, 617, 641 ;
- $\mathbb{G}_{14,2} \cap \mathbb{G}_{16,2} = 1$, 16, 67, 82, 248, 254, 266, 298, 326, 328, 356, 386, 778 .

11 Numbers of higher order sets $\mathbb{G}$

11.1 The set $\mathbb{G}_{b,32}$

For n, $1 \leq n \leq 10^3$ we have 70 numbers from sets $\mathbb{G}_{b,32}$ with $b = 2, 4, \ldots 16$: 20, 47, 61, 68, 74, 97, 103, 122, 134, 143, 152, 173, 181, 194, 202, 218, 242, 244, 292, 331, 346, 368, 382, 388, 407, 412, 413, 414, 416, 452, 458, 464, 470, 472, 477, 484, 488, 494, 521, 583, 587, 599, 607, 631, 634, 638, 673, 679, 683, 692, 698, 736, 763, 782, 794, 796, 814, 830, 832, 834, 842, 850, 863, 878, 881, 886, 902, 967, 976, 977 .

Table 48: The sets $\text{VP}\mathbb{G}_{b,32}$ with $1 \leq n \leq 10^3$

b	$n \in \text{VP}\mathbb{G}_{b,32}$	$\text{VP}\mathbb{P}$	[1]$nrd(L_{b,32}(n), 10)$
2	122	$L_{b,32}(n)$	67
2	134	$L_{b,32}(n)$	69

Continued on next page

b	$n \in \mathrm{VP}\mathbb{G}_{b,32}$	$\mathrm{VP}\mathbb{P}$	$nrd(L_{b,32}(n), 10)$
2	152	$L_{b,32}(n)$	70
2	346	$L_{b,32}(n)$	82
2	388	$L_{b,32}(n)$	83
2	416	$L_{b,32}(n)$	84
2	452	$L_{b,32}(n)$	85
2	464	$L_{b,32}(n)$	86
2	472	$L_{b,32}(n)$	86
2	494	$L_{b,32}(n)$	87
2	638	$L_{b,32}(n)$	90
2	736	$L_{b,32}(n)$	92
2	814	$L_{b,32}(n)$	94
2	850	$L_{b,32}(n)$	94
4	412	$L_{b,32}(n)$	84
4	587	$L_{b,32}(n)$	95
4	484	$L_{b,32}(n)$	86
4	796	$L_{b,32}(n)$	93
4	902	$L_{b,32}(n)$	95
6	68	$L_{b,32}(n)$	65
6	103	$L_{b,32}(n)$	65
6	414	$L_{b,32}(n)$	84
6	477	$L_{b,32}(n)$	91
6	692	$L_{b,32}(n)$	91
6	881	$L_{b,32}(n)$	99
8	143	$L_{b,32}(n)$	86
8	407	$L_{b,32}(n)$	86
8	458	$L_{b,32}(n)$	86
8	464	$L_{b,32}(n)$	86
8	470	$L_{b,32}(n)$	86
8	634	$L_{b,32}(n)$	102
8	782	$L_{b,32}(n)$	113
8	830	$L_{b,32}(n)$	114
8	832	$L_{b,32}(n)$	94
10	368	$L_{b,32}(n)$	94
10	679	$L_{b,32}(n)$	96
10	863	$L_{b,32}(n)$	94
10	976	$L_{b,32}(n)$	96
12	20	$L_{b,32}(n)$	64
12	97	$L_{b,32}(n)$	64
12	244	$L_{b,32}(n)$	91
12	331	$L_{b,32}(n)$	97

Continued on next page

b	$n \in \mathrm{VP}\mathbb{G}_{b,32}$	$\mathrm{VP}\mathbb{P}$	$nrd(L_{b,32}(n),10)$
12	413	$L_{b,32}(n)$	94
12	599	$L_{b,32}(n)$	103
12	607	$L_{b,32}(n)$	97
12	673	$L_{b,32}(n)$	91
12	683	$L_{b,32}(n)$	104
12	834	$L_{b,32}(n)$	96
12	842	$L_{b,32}(n)$	94
12	967	$L_{b,32}(n)$	98
12	977	$L_{b,32}(n)$	96
14	61	$L_{b,32}(n)$	60
14	74	$L_{b,32}(n)$	60
14	181	$L_{b,32}(n)$	74
14	194	$L_{b,32}(n)$	74
14	202	$L_{b,32}(n)$	99
14	382	$L_{b,32}(n)$	96
14	583	$L_{b,32}(n)$	106
14	698	$L_{b,32}(n)$	109
14	967	$L_{b,32}(n)$	96
16	47	$L_{b,32}(n)$	77
16	173	$L_{b,32}(n)$	75
16	218	$L_{b,32}(n)$	75
16	242	$L_{b,32}(n)$	77
16	292	$L_{b,32}(n)$	97
16	488	$L_{b,32}(n)$	108
16	521	$L_{b,32}(n)$	108
16	631	$L_{b,32}(n)$	105
16	763	$L_{b,32}(n)$	112
16	794	$L_{b,32}(n)$	110
16	878	$L_{b,32}(n)$	115
16	886	$L_{b,32}(n)$	103

11.2 The set $\mathbb{G}_{b,64}$

For n, $1 \le n \le 10^3$ we have 40 numbers from sets $\mathbb{G}_{b,64}$ with $b = 2, 4, \ldots, 16$: 35, 92, 94, 106, 118, 119, 127, 134, 136, 139, 140, 197, 208, 226, 265, 309, 334, 397, 427, 556, 562, 598, 610, 631, 640, 655, 744, 766, 768, 776, 796, 808, 809, 811, 880, 886, 895, 908, 921, 929 .

[1] nrd is program 2

Table 49: The sets $\mathrm{VP}\mathbb{G}_{b,64}$ with $1 \leq n \leq 10^3$

b	$n \in \mathrm{VP}\mathbb{G}_{b,64}$	$\mathrm{VP}\mathbb{P}$	${}^{2}nrd(L_{b,64}(n), 10)$
2	334	$L_{b,64}(n)$	162
2	610	$L_{b,64}(n)$	179
2	880	$L_{b,64}(n)$	189
4	92	$L_{b,64}(n)$	126
4	208	$L_{b,64}(n)$	149
4	139	$L_{b,64}(n)$	151
4	226	$L_{b,64}(n)$	151
4	631	$L_{b,64}(n)$	189
4	776	$L_{b,64}(n)$	185
4	886	$L_{b,64}(n)$	189
6	766	$L_{b,64}(n)$	190
6	921	$L_{b,64}(n)$	190
8	334	$L_{b,64}(n)$	167
8	397	$L_{b,64}(n)$	167
8	640	$L_{b,64}(n)$	180
8	796	$L_{b,64}(n)$	215
10	118	$L_{b,64}(n)$	187
10	136	$L_{b,64}(n)$	180
10	140	$L_{b,64}(n)$	138
10	265	$L_{b,64}(n)$	176
10	556	$L_{b,64}(n)$	181
10	562	$L_{b,64}(n)$	176
10	598	$L_{b,64}(n)$	189
10	631	$L_{b,64}(n)$	180
10	655	$L_{b,64}(n)$	181
10	809	$L_{b,64}(n)$	190
10	811	$L_{b,64}(n)$	187
10	895	$L_{b,64}(n)$	189
10	908	$L_{b,64}(n)$	190
12	35	$L_{b,64}(n)$	137
12	94	$L_{b,64}(n)$	135
12	127	$L_{b,64}(n)$	135
12	134	$L_{b,64}(n)$	137
12	309	$L_{b,64}(n)$	200
12	744	$L_{b,64}(n)$	184
12	768	$L_{b,64}(n)$	185
14	106	$L_{b,64}(n)$	133

Continued on next page

b	$n \in \mathrm{VP}\mathbb{G}_{b,64}$	VPℙ	$nrd(L_{b,64}(n), 10)$
14	119	$L_{b,64}(n)$	133
14	427	$L_{b,64}(n)$	202
14	929	$L_{b,64}(n)$	196
16	92	$L_{b,64}(n)$	147
16	197	$L_{b,64}(n)$	147
16	808	$L_{b,64}(n)$	213

11.3 The set $\mathbb{G}_{b,128}$

For n, $1 \leq n \leq 10^3$ we have 20 numbers from sets $\mathbb{G}_{b,128}$ with $b = 2, 4, \ldots, 16$: 56, 86, 131, 149, 182, 281, 302, 448, 536, 550, 556, 587, 591, 634, 655, 710, 808, 829, 862, 970 .

Table 50: The sets $\mathrm{VP}\mathbb{G}_{b,128}$ with $1 \leq n \leq 10^3$

b	$n \in \mathrm{VP}\mathbb{G}_{b,128}$	VPℙ	[3] $nrd(L_{b,128}(n), 10)$
2	302	$L_{b,128}(n)$	318
2	550	$L_{b,128}(n)$	351
4	86	$L_{b,128}(n$	279
4	149	$L_{b,128}(n)$	279
4	808	$L_{b,128}(n)$	373
8	536	$L_{b,128}(n)$	350
8	710	$L_{b,128}(n)$	447
10	182	$L_{b,128}(n)$	314
10	281	$L_{b,128}(n)$	314
10	556	$L_{b,128}(n)$	361
10	655	$L_{b,128}(n)$	361
10	970	$L_{b,128}(n)$	383
12	448	$L_{b,128}(n)$	355
12	591	$L_{b,128}(n)$	355
14	587	$L_{b,128}(n)$	440
14	634	$L_{b,128}(n)$	374
14	829	$L_{b,128}(n)$	374
16	56	$L_{b,128}(n)$	272
16	131	$L_{b,128}(n)$	272
16	862	$L_{b,128}(n)$	457

[2] nrd is program 2

The Mersenne numbers $2^n - 1$, the primorial primes $p\# - 1$, the factorial primes $p! - 1$, $\ldots$ all have a high probability of being large primes, [3] . The $L_{b,\omega}(n)$ numbers could also have a high probability of being large primes if $b = 2\beta$, $\beta \in \mathbb{N}$ and $\omega = 2^m$ with $m \in \mathbb{N}^*$. For example, $862 \in \mathrm{VP}\mathbb{L}_{16,128}$, i.e.

$$
\begin{aligned}
L_{16,128}(862) = 35d_{(16)}^{128} + d53_{(16)}^{128} = 862^{128} + 3667^{128} = \\
1705283048563528561047599128596029476529385980344270052642 83 \iddots \\
3424963472191728768191556879364459480322676843164952454046 34 \iddots \\
8551403814843180105104929377347720148155206380412689201290 89 \iddots \\
1650311475262343758896395350923294439899562729715665356742 38 \iddots \\
0725578012623223180387582690811244151935739135478828473539 34 \iddots \\
0726569470555613343836936835724213663902011181605849409754 16 \iddots \\
6584060138657305398012752971837048439244526620769982398295 33 \iddots \\
10150278919745289260005157620396815 37 \in \mathrm{VP}\mathbb{P} \ .
\end{aligned}
$$

Therefore that $862 \in \mathrm{VP}\mathbb{G}_{16,128}$ is a condensed way of writing a number of "*very probable prime*" with 457 of decimal digits.

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