

# Paradoxes Come from the Concept of Magnetism as a Relativistic side effect of Electrostatics

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## Abstract

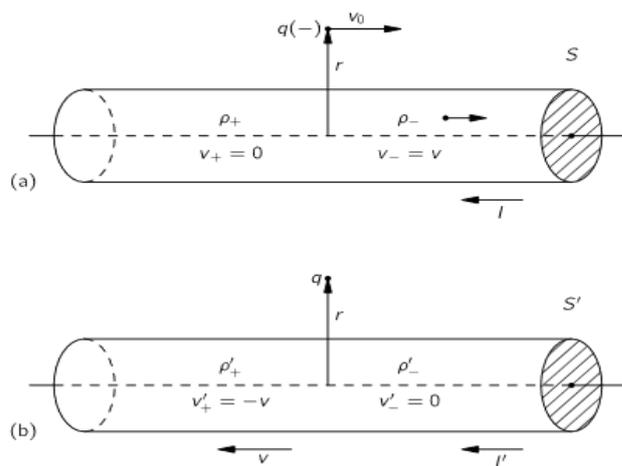
Mistakes are found in the theoretical derivation process, during which the magnetic force is explained to be the relativistic side effect of Coulomb force. As a result, some serious paradoxes will be inevitable if we accept the notion that Magnetism is a Relativistic side effect of Electrostatics.

**Key words:** magnetism and relativity, paradox of magnetism

## Introduction

The notion of Magnetism as relativistic side effect of Electro Statics can be derived from the relativistic length contraction of charges in their moving direction, which is described in many textbooks [1-4]. The theoretical derivation concludes: 1. in reality, a magnetic field is not a fundamental field; 2. the magnetic field is just an electric field with relativity applied, i.e. an electric field viewed from the moving reference frame. The brief deduction process is summarized below. For detailed derivation steps, please see the reference1-4.

Considering what happens when a negative charge moves with velocity  $v_0$  parallel to a current-carrying wire, as in Fig. 1. We will try to understand what goes on in two reference frames: one fixed with respect to the wire, as in part (a) of the fig. 1, and the other fixed with respect to the particle, as in part (b). We will call the first frame  $S$  and the second  $S'$ .



**Fig.1.** The interaction of a current-carrying wire and a particle with negative charge  $q$  as seen in two frames. In frame  $S$  (part a), the wire is at rest; in frame  $S'$  (part b), the charge is at rest.

In the  $S$ -frame, there is a magnetic force on the particle and the force is directed toward the wire. But in the  $S'$ -frame there can be no magnetic force on the particle according to Lorentz

force law, because its velocity is zero. However, the special relativity will tell you that the particle will be exerted a force toward to the wire in  $S'$  frame, too.

In a normal metal conductor, the electric current is due to the motion of the electrons—while the positive nuclear charges stay fixed in the body of the material. Let the charge density of the conduction electrons be  $\rho^-$  and their velocity in  $S$  be  $v$ . The density of the positive nuclear charges at rest in  $S$  is  $\rho^+$ , which must be equal to the negative of  $\rho^-$ , since we are considering an uncharged wire. There is thus no electric field outside the wire, and the force on the moving particle is just

$$\mathbf{F} = q\mathbf{v}_0 \times \mathbf{B}. \quad (1)$$

The force on the particle is directed toward the wire and has the magnitude

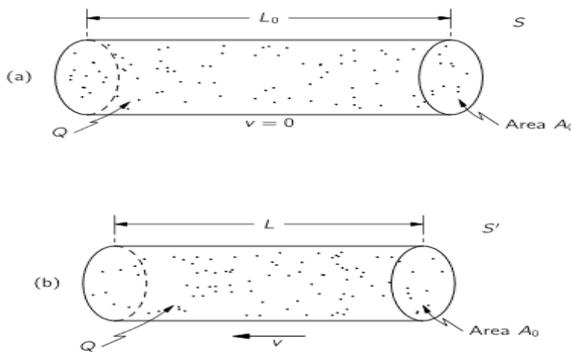
$$F = \frac{1}{4\pi\epsilon_0 c^2} \cdot \frac{2Iqv_0}{r}. \quad (2)$$

Now let's see what happens in  $S'$ , where the particle is at rest and the wire is moving past to left with the speed  $v$ . For simplicity, we set the velocity  $v_0$  of the particle is the same as the velocity  $v$  of the conduction electrons. Because the particle is now at rest, there is no magnetic force on it! If there is any force, it must come from an electric field. It must be that the moving wire has created an electric field. But it can do that only if it appears charged—it must be that a neutral wire with a current appears to be charged when set in motion.

Now, let's calculate the charge density in the wire in  $S'$  from what we know about it in  $S$ . If we take a length  $L_0$  of the wire with a charge density  $\rho_0$  of stationary charges, it will have the total charge  $Q = \rho_0 L_0 A_0$ . If the same charges are observed in a different frame to be moving with velocity  $v$ , they will all be found in a piece of the material with the shorter length

$$L = L_0 \sqrt{1 - v^2/c^2}, \quad (3)$$

but with the same area  $A_0$  (since dimensions transverse to the motion are unchanged). See Fig. 2



**Fig. 2.** If a distribution of charged particles at rest has the charge density  $\rho_0$ , the same charges will have the density  $\rho = \rho_0 / \sqrt{1 - v^2/c^2}$  when seen from a frame with the relative velocity  $v$

If we call  $\rho$  the density of charges in the moving frame, the total charge  $Q$  will be  $\rho L A_0$ . This must also be equal to  $\rho_0 L_0 A_0$ , because charge is the same in any system, so that  $\rho L = \rho_0 L_0$  or, from (3),

$$\rho = \frac{\rho_0}{\sqrt{1 - v^2/c^2}}. \quad (4)$$

The negative charges are stationary in  $S'$ . So they have their “rest density”  $\rho_0$  in this frame. In Eq. (4)  $\rho_0 = \rho_-'$ , because they have the density  $\rho_-$  when the wire is at rest, i.e., in frame  $S$ , where the speed of the negative charges is  $v$ . For the conduction electrons, we then have that

$$\rho_- = \frac{\rho_-'}{\sqrt{1 - v^2/c^2}}, \quad (5)$$

$$\rho_- = \rho_- \sqrt{1 - v^2/c^2}. \quad (6)$$

Now we can see why there are electric fields in  $S'$  —because in this frame the wire has the net charge density  $\rho'$  given by

$$\rho' = \rho_+' + \rho_-'. \quad (7)$$

Using (5) and (6), we have

$$\rho' = \frac{\rho_+}{\sqrt{1 - v^2/c^2}} + \rho_- \sqrt{1 - v^2/c^2}. \quad (8)$$

Since the stationary wire is neutral,  $\rho_- = -\rho_+$ , and we have

$$\rho' = \rho_+ \frac{v^2/c^2}{\sqrt{1 - v^2/c^2}}. \quad (9)$$

The moving wire is positively charged and will produce an electric field  $E'$  at the external stationary particle. The electric field at the distance  $r$  from the axis of the cylinder is

$$E' = \frac{\rho' A}{2\pi\epsilon_0 r} = \frac{\rho_+ A v^2/c^2}{2\pi\epsilon_0 r \sqrt{1 - v^2/c^2}}. \quad (10)$$

The force on the negatively charged particle is toward the wire. Now, we have a force in the same direction from the two points of view; the electric force in  $S'$  has the same direction as the magnetic force in  $S$ .

### Discussion

This derivation however is questionable. Mainly so because according to the relativistic analysis above, if the wire with current flowing through is neutral (This is what the derivation start from) the wire without current will become electrically charged in lab frame. This is obviously against the fact. On the contrary, if the wire without current is neutral the wire with current will become electrically charged in lab frame. This is also against the fact. We all know, for a neutral wire, it keeps electrically neutral in lab frame no matter having or not having current flowing through it.

If the above derivation begins from the point that a wire without current is neutral in lab frame (actually we have far more sufficient reason to suppose this than for a wire having current), we would obtain that a neutral wire would become electrically charged when there is a current flowing through it. For normal metal wires, because the current is due to electrons' directional migration, the wire would show negatively charged in the lab frame. This would cause these weird results which violate facts:

1. For a neutral wire, when having current inside, it would show electrically charged and would exert an electric force on a charged particle nearby even if the particle was stationary.
2. For two parallel neutral wires, when having currents inside, they would always repel each other no matter the two currents are in the same direction or in opposite direction.

### **Conclusions**

From discussions above, we can see that some serious paradoxes will be inevitable if we accept the notion that Magnetism is a Relativistic side effect of Electrostatics.

### **References:**

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