A Classical Analysis of the Szilard Engine

G. R. Prok
December. 2016

Abstract:

Maxwell's demon challenges our interpretation of thermodynamics and our understanding of the Second Law of thermodynamics. The Szilard engine is a gedanken instantiation of Maxwell's Demon that is amenable to standard thermodynamic analysis. The paradox of Maxwell's demon as presented by the Szilard engine is considered to have been solved by Landauer's principle. A classical analysis of the Szilard engine, presented here, shows that Landauer's principle is not needed to resolve the paradox of the demon. Classical thermodynamics is all that is needed.

Keywords: Maxwell's demon, Szilard engine, thermodynamics, entropy

Introduction:

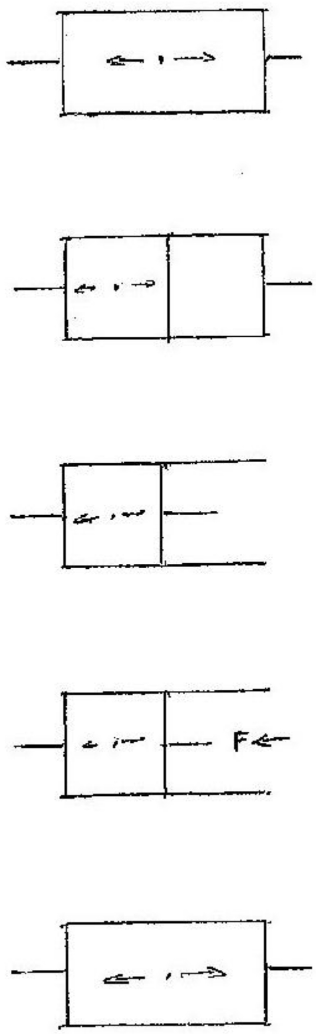
Clerk Maxwell in 1867 presented the paradox of Maxwell's demon. This is an intelligent agent that works at the microscopic level to separate gas particles with high kinetic energy from those with low kinetic energy, thereby creating two reservoirs with a temperature difference starting with a uniform gas at thermal equilibrium. This violation of the Second Law of thermodynamics presented a paradox.

Leo Szilard, in 1929, presented the Szilard engine[1], which reduced the Maxwell demon paradox to a minimal system. The Szilard engine also uses the demon's intelligent agency to violate the Second Law. This engine is very amenable to standard thermodynamic analysis and is the subject of significant studies into the nature of entropy, information and computing.

The Szilard Engine:

The Szilard engine is constructed of molecule scale components and a working material of a single gas molecule. The components and the working fluid only interact via kinetic and radiant transfer of thermal energy. All motion of moving parts is frictionless. These idealizations are not imagined to be attainable, however, the ability to convert heat directly into mechanical energy should not be dependant upon the attainability of this idealized design.

The Szilard engine is composed of an enclosure bounded on opposite ends by identical moveable boundaries, or pistons, which can freely slide into the stationary boundaries of the enclosure, which can be considered as a cylinder (Fig 1). The mechanism is isothermal and is in thermal equilibrium with a constant temperature bath, or surroundings. No force is initially



Initial state

Barrier inserted; Particle detected

Working piston inserted

Force applied; barrier removed

Isothermal expansion; cycle complete

Figure 1 The Szilard Engine

applied to the pistons. The enclosure has a means to introduce a removable barrier that divides the enclosure into two equal parts, thereby trapping the single molecule in one half of the enclosure. At this point the entropy of the system has reduced by k(ln2). The demon determines on which side of the partition the molecule is trapped and then sets in motion the reversible sliding of the piston on the opposite side until it touches the removable barrier. An inward force is applied to this piston and the barrier is then removed. The inward force is then

slowly reduced to zero so that mechanical energy is extracted as the piston returns to its initial position. At the start of this power stroke, the working molecule impacts the piston twice as frequently as at the end of the stroke. Energy can then be extracted at each collision by allowing the piston to work against the inward force. Energy is added back to the molecule from the boundary walls of the engine, driven by an incremental temperature difference of the particle and the surroundings. The engine is most efficient when the expansion occurs reversibly and thermal equilibrium is maintained. A kinetic theoretic analysis shows that the maximum work extracted is W = -kT(ln2) and the entropy increases by k(ln2), offsetting the entropy reduction earlier in the cycle. The Szilard engine thus presents us with a perpetuum mobile of the second kind, converting thermal energy directly into useful work in apparent violation of the Second Law of thermodynamics.

The Paradox:

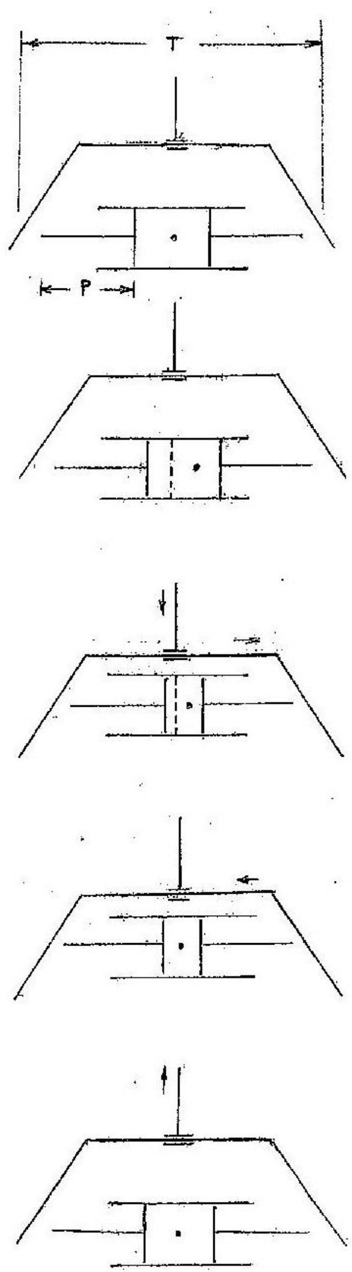
The demon has been argued to tie measurement and information processing to thermodynamic entropy in seemingly profound ways. These arguments traditionally assume that the demon is a material entity and not an ethereal ghost. The most famous of these analyses treat the demon as a physical object employing principles of physics to measure and to process information and then to somehow perform microscopic tasks. The reduction of these processes to microscopic practice has been largely ignored. A thorough analysis must include the reduction to practice.

The currently accepted navigation out of this paradox is attributed to Charles Bennett who, in 1982, proposed that the demon produces entropy as it goes through its logical cycle.[2] Specifically, he proposed that Landauer's principle causes the demon to generate kT(ln2) heat during an erasure step, which brings the demon back to its initial state. Critics have argued that Landauer's principle is based on flawed use of statistical mechanical ensembles. A well-constructed criticism is given by John Norton (2004).[3]

However, a Szilard engine can be embellished in a way that requires no thinking or computing demon at all. Such an engine is depicted in Figure 2. It has trapezoidal cam which strokes upward and downward by its vertex. The cam is restricted to one degree of freedom of movement, side-to-side by a slide mechanism on its topside. The cam is also restricted by stops (the ends of the top boundary of the particle enclosure) so that as it slides side-to side, it can never force a piston past the center of the cylinder; the geometry is such that this is true regardless of the upward-downward position of the cam. During its downward movement, whichever piston presents no opposing force from a contained particle is the piston that is moved toward the removable barrier.

The cam is forced upward as the working particle expands its occupied volume during the power stroke. The cycle is complete with one reciprocation. This simple cam acts as a mechanical "demon", forcing the piston on the side lacking the particle to move towards the inserted barrier. Regardless of the location of the particle, work is always extracted in the vertical direction by the cam.

This Szilard engine embellished with a mechanical "demon" can be readily analyzed through microscopic thermodynamic analysis.



Initial state

Barrier inserted

Cam lowered

Barrier removed

Cam raised

Figure 2 Embellished Szilard Engine

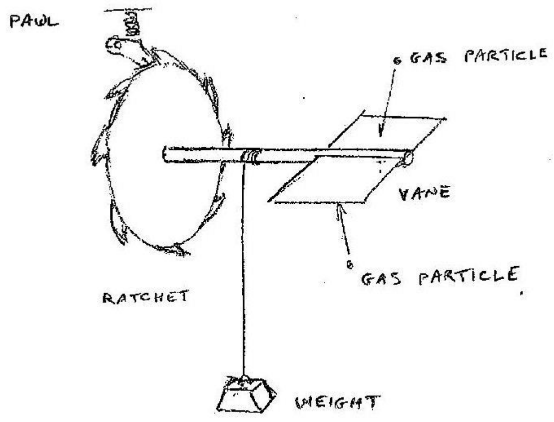


Figure 3   Ratchet and Pawl Engine

A microscopic analysis of the Szilard engine cycle:

The Szilard engine paradox can be solved by a careful classical analysis of this entire engine. The analysis is similar to Richard Feynman's analysis of another proposed engine that apparently violates the Second Law.[4] This engine is the ratchet and pawl mechanism of Figure 3. Feynman's elegant analysis teaches that the random thermal motion of the pawl exactly disables the mechanism from converting heat into mechanical energy (The Feynman Lectures on Physics Vol I chapter 46, 1964). A similar analysis, albeit not as elegant, is here applied to the Szilard engine.

Consider the engine at its initial state with the removable barrier removed and the pistons at their equilibrium states. The stationary barrier, or cylinder, is here considered fixed to the surroundings and not subject to thermal motion. Every component of the mechanism must be analyzed as having time-average thermal vibration energy of ½kT per degree of freedom. Being free to move in the direction of the cylinder axis subjects the pistons and the trapezoidal cam to random thermal fluctuations, as was Feynman's pawl. These mechanical components can be therefore treated as particles with mass which are allowed to move in this one dimension (Fig 4a).

The working particle in this analysis is free to move in all three spatial dimensions. Collisions of the particle with the stationary boundaries are important for the transfer of heat between the particle and the surroundings and for maintenance of isothermal operation. Since the boundaries in two of these dimensions are fixed, this analysis need only focus on the third dimension. Thus this analysis will only consider the motion of the working particle in the axial dimension. The removable barrier is difficult to realistically describe at the microscopic scale. If some hypothesized mechanism alternately inserts and removes the barrier, it also will be subject to thermal vibration. This analysis hypothesizes that an effective insertion and removal mechanism exists and can operate reversibly.

The initial state is therefore one of three co-linear particles constrained by the trapezoidal cam. The cam transfers kinetic energy from one end of the cylinder to the other and the system of four components behaves like four particles inside a torus, with only motion in the circumferential direction considered. The particles are the trapezoidal cam, the left piston, the right piston, and the working particle.

The four particles are constrained to an effective length of:

$$L_0 = T - 2P$$

where: T is the distance separating the contact points of the trapezoidal cam with the two pistons, and P is the length of each piston rod.

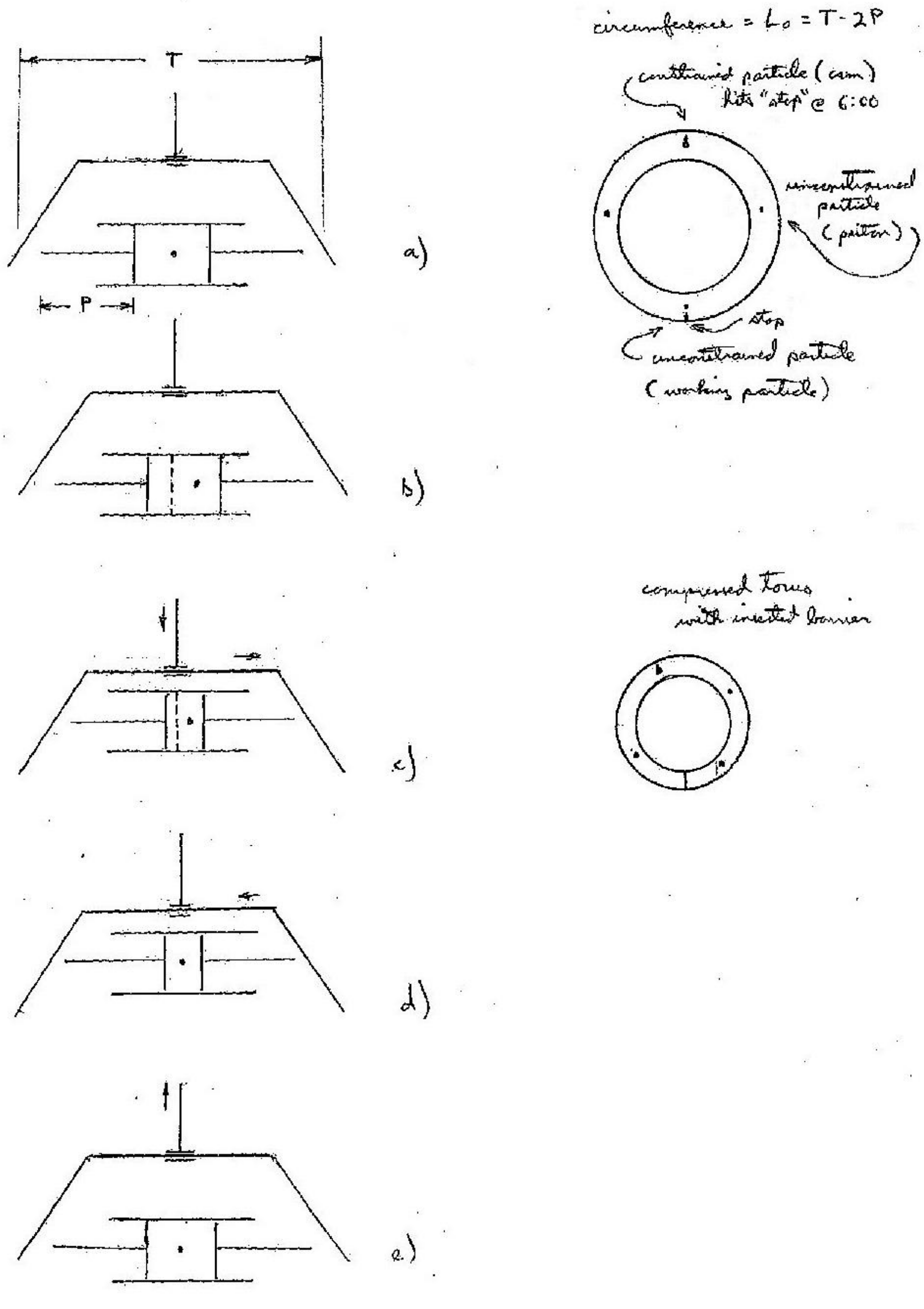


Figure 4 Microscopic Analysis of the Embellished Szilard Engine

If there were no constraint on the cam by the ends of the top enclosure boundary, then on average each of the free particles would occupy approximately equal fractions of the effective length and each will exert the same time-average force on its neighboring particle. Hence the pistons could not be considered to be fully extended at the beginning of the cycle. Invoking the ergodic hypothesis and assuming a Boltzman distribution of energies, the time-average separation between each moving component, or particle, would be nominally $^{1}/_{4}L_0$.

The unconstrained particles-in-a-torus analogy, though pedantically useful, does not reflect the action of the Szilard engine. In the engine, the particles are constrained to stay within certain regions. The cam is only free to move a certain distance, C, left or right and this constrains the other particles in a similar manner. The nominal time-average separation between the constrained particle and it neighbors is $< {}^{1}/_{4}L_{0,}$ and all other separations are $> {}^{1}/_{4}L_{0.}$ The number of available microstates is reduced by this constraint. The engine is constructed so that C is equal to L_0, and the configuration has the same microscopic states as that of four particles free to move about a torus of length L_0 to a torus with one particle, representing the cam, constrained to length $C = L_0$.

Upon addition of the removable barrier, the particle will be trapped on one side of the barrier. The piston on the same side as the particle will move to a time-average position further from the barrier. The piston on the other side will move closer to the barrier and the cam will move in the same direction. The cam is no longer centered in its constraint, on average and will more frequently bump one of its two mechanical stops and less frequently bump the piston being forced toward the removable boundary. This causes the working particle to have a time-average position closer to the barrier than the opposite piston (Fig 4b). While the work represented by these changes may be difficult to quantify, the associated entropy change is relatively straightforward to quantify.

The entropy decreases with insertion of the barrier as the number of microstates is reduced. An integration over all microstates, or allowed combination of positions for all four particles, shows that the entropy change is equal to –kln(4). As will be seen, the actual value is insignificant. It is this entropy decrease that could potentially be exploited by the engine.

The engine must now cause the piston opposite the particle to slide toward the barrier. Because of the thermal motion of the particle and engine components, work is required at this step. Sliding this piston to an average position closer to the barrier will cause the particle to impact the barrier more frequently and requires work to be performed on the piston. The work required to move this piston toward the barrier is accompanied by work performed on the rest of the system. The trapezoidal cam is supplied the required work as it is incremented downward. The piston can be forced arbitrarily close to the barrier as the downward force is increased (Fig 4c). However, forcing the piston into continuous contact with the piston requires infinite work. If the piston is forced to a closer distance from the barrier, then each component or particle will be forced to closer together as well, reducing the effective length from L_0 to a new length, L_1. The geometry of the cam and stops is chosen to allow the configuration to maintain geometric similarity as the cam moves downward. The torus representing the system merely becomes smaller. Although other geometric configurations can be chosen, this configuration turns out to generate the lowest entropy per cycle. The isothermal work required for this step to be easily calculated as $4kT\ln(L_0/L_1)$ and the entropy change is $-4k\ln(L_0/L_1)$.

The expansion step is where work is extracted from the engine. The barrier is removed and the pistons are allowed to move to their initial states as the applied force is reduced to zero. Upon removal of the barrier, the working particle irreversibly moves to an average position in the center of the cylinder (Fig 4d). This is the reverse of the insertion of the barrier and the entropy change can be determined either by again integrating microstates before and after the barrier is removed or by invoking the symmetry between this step and that of the earlier step where the barrier is inserted. The entropy increase of k ln(4) is the same as the entropy decrease seen when the barrier was inserted. The total work extracted upon expansion can also be easily calculated. The isothermal work is a magnitude of $4kT\ln(L_0/L_1)$ and the entropy change is $4k\ln(L_0/L_1)$. The engine therefore merely returns the work used to force the piston toward the barrier with no overall change in entropy. No useful work is generated and the Second Law of Thermodynamics remains secure.

Conclusion

The Szilard engine, subject of much analysis and controversy, does not offer any thermodynamic paradox. A real thermodynamic paradox would present a useful way to violate the Second Law on either the macroscopic or microscopic scale. It is the energy per degree of freedom, or temperature, of every moving part that prevents such a violation. The suggestion that a microscopic engine with nearly non-vibrating moving parts violates the Second Law by direct conversion of heat into useful work requires suspension of the laws of physics. The cold, nearly motionless parts will be heated by the working fluid and then become vibrating parts. The analysis of a simple mechanical "demon" demonstrates that the thermal motion of the engine's and the "demons" parts prevents the engine from working.

Although a deep connection between information theory and thermodynamics may exist, it is not necessary to invoke a connection such as Landauer's principle to explain why this engine and mechanical "demon" fails to violate the Second Law of thermodynamics.

References:

[1] Szilard, L., “On the Decrease of Entropy in a Thermodynamic System by the Intervention of Intelligent Beings”, *Zeitschrift fur Physik* 53:840–856, 1929

[2] Bennet, C. H., “The thermodynamics of computation---a review”, *International Journal of Theoretical Physics*, Section 5, 21(12):905-940 · December 1982

[3] Norton, J. D., "Eaters of the Lotus: Landauer's Principle and the Return of Maxwell's Demon." *Studies in History and Philosophy of Modern Physics*, 36:375-411, 2005

[4] Feynman R, Leighton R, and Sands M. *"The Feynman Lectures on Physics, Volume I"* Chapter 46, 1964, Library of Congress Catalog Card No. 63-20717