The Black Hole – Can the 'Irresistible Force' Overcome the 'Immovable Object?'

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Following some earlier work by Antoci and Abrams, Crothers has spent at least the past decade arguing the mathematical impossibility of the black hole. Following a brief review of the mathematical argument, a physical one is presented, based on analysis of the 'irresistible force' of increasing gravity allegedly collapsing a neutron star with an even greater 'immovable object' of increasing density into a black hole. This physical argument supports Crothers', et al., contention that a black hole is both a mathematical as well as physical impossibility.

1. Introduction

Based on work by Antoci (arXiv:physics/9912033v1, December 16, 1999), Abrams (arXiv:gr-qc/0102055v1, February 13, 2001), and Crothers (Riemannian Geometry and Applications – Riga 2008, April 25, 2008; other related works at *www.sjcrothers.plasmaresources.com/*), it has been shown that the mathematical basis upon which the concept of the black hole was founded resulted from an error by mathematician David Hilbert in interpreting the original solution by Karl Schwarzschild, who died during World War I and was never able to refute the error. From his translation of Schwarzschild's original work, "On the Gravitational Field of a Sphere of Incompressible Fluid According to Einstein's Theory" (*Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse* 1916, pp. 424-434), Antoci showed that Schwarzschild's actual equation for a line element outside the sphere is

$$ds^2 = (1 - \alpha/R)dt^2 - \frac{dR^2}{(1 - \alpha/R)} - R^2(d\theta^2 + [\sin\theta]^2\,\theta d\varphi^2)$$

where $R^3 = r^3 + \alpha^3,\ \alpha = constant > 0,\ 0 \leq r < \infty$. Therefore, in Schwarzschild's version, R cannot become zero for non-zero α, and the equation becomes indeterminate not for R = 0, which cannot occur, but for r = 0, an allowed value. Small 'r' is not an actual radius, such as that of the sphere; large 'R' is the radius of Gaussian curvature and cannot be < α, i.e., it cannot 'vanish' as would be implied for a singularity. In other words, a singularity, such as that assumed to comprise a black hole, is a mathematical impossibility.

Antoci (arXiv:physics/0310104v1, October 21, 2003) notes that, in 1917 when Hilbert revisited Schwarzschild's version of the static, spherically symmetric problem, he made two fundamental changes by reinterpreting 'r' as 'R', now allowing 'r' to be the radius of Gaussian curvature, and setting α = 2m, i.e., twice the mass of the sphere (Hilbert, D., 1917, *Nachr. Ges. Wiss. Göttingner, Math. Phys. Kl.,* p. 53). With these unfortunate changes, the 'Schwarzschild' equation became

$$ds^2 = (1 - 2m/r)dt^2 - \frac{dr}{(1 - 2m/r)} - r^2(d\theta^2 + [\sin\theta]^2\,\theta d\varphi^2)$$

retaining $0 \leq r < \infty$. In Hilbert's version, the equation becomes indeterminate for r = 2m, and 'r,' allegedly corresponding to the spherical radius, can decrease to zero, creating a singularity. Upon this (mis)interpretation has the mathematical basis for the black hole (and 'Big Bang,' also alleged to be a singularity?) been built. Note that, in the preceding equations, the constants 'G' (gravitational constant) and 'c' (speed of light) are arbitrarily set to unity for simplicity. With these simplifications, the so-called 'Schwarzschild radius' becomes $r_s = 2Gm/rc^2 = 2m/r = 1$ (*https://en.wikipedia.org/wiki/Schwarzschild-radius*). Hence, the significance of this 'radius' for the alleged 'event horizon' of a black hole when the Hilbert version is rewritten as

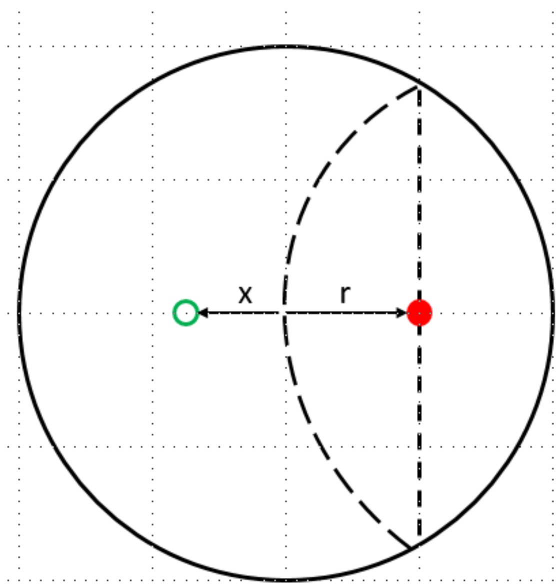
$$ds^2 = (1 - r_s)dt^2 - dr/(1 - r_s) - r^2(d\theta^2 + [\sin\theta]^2\,\theta d\varphi^2),$$

which becomes indeterminate at the 'Schwarzschild radius.'

2. The Black Hole – Is it Even Physically Plausible?

The preceding is a mathematical description of the (erroneous) basis for the existence of a black hole. What about a physical interpretation? Is it conceivable that a star could gravitationally collapse indefinitely, i.e., beyond even the density of neutronium (approximately 4 x 10^{17} kg/m^3), into a singularity? Let us begin with a neutron star, at least three times the mass of the sun (*https://en.wikipedia.org/wiki/Neutron_star*), with a scaled radius of 1. At its surface, the gravity, scaled to a gravitational constant and stellar mass of unity, is 1. As the star collapses upon itself, its radius decreases, corresponding to an increase in density (but not mass) and an increase in surface gravity with the inverse square dependence of gravity on radius. Decreasing the star's radius by 25% each time (i.e., the radius reduces to 0.75^n with each decrease 'n'), it takes only eight such decreases to reduce the radius to 0.1, an additional eight to drop to 0.01, and another eight to 0.001. Surface gravity, proportional to the inverse square of the radius, increases one-thousand fold after n = 24, while the star's density, proportional to the inverse cube of the radius, increases one-million-fold. In fact, with each decrease, the density-to-surface gravity ratio increases by a factor of $\frac{1}{0.75^3}/\frac{1}{0.75^2} = 4/3$ until, after n = 24 decreases, it is 1000. Given this ratio, does it make sense that the 'irresistible force' of increasing gravity should continue to overcome the 'immovable object' of even greater increasing density until this density is one-million times that of neutronium? Or would the collapse arrest at some point where the density cannot be increased and either an unimaginable 'implosion' occurs, releasing up to the energy equivalent of the star's mass, or the densest possible neutron star exists (but not a black hole)?

Meanwhile, as the star's radius decreases, what is the net gravity at various locations within the star itself? To estimate this, consider the following cross-sectional view of the star (Figure 1).



<u>FIGURE 1</u>. Cross-sectional View of Star

At a position 'r,' the net gravitational force will arise only from that portion of the star beyond the symmetric double 'spherical cap' (see Figure 2 [*https://en.wikipedia.org/wiki/Spherical_cap*]) volume (in two-dimensions, a double 'spherical segment' area), with a center of mass at 'x.'

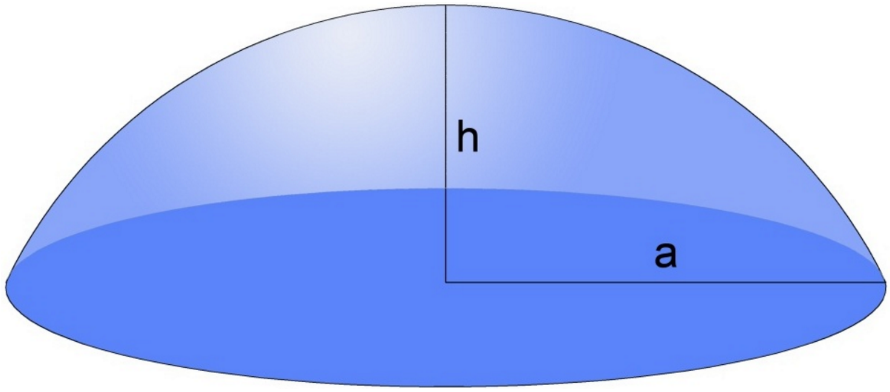


FIGURE 2. **Spherical Cap** (*http://www.intechopen.com/source/html/48232/media/image3.jpeg*)

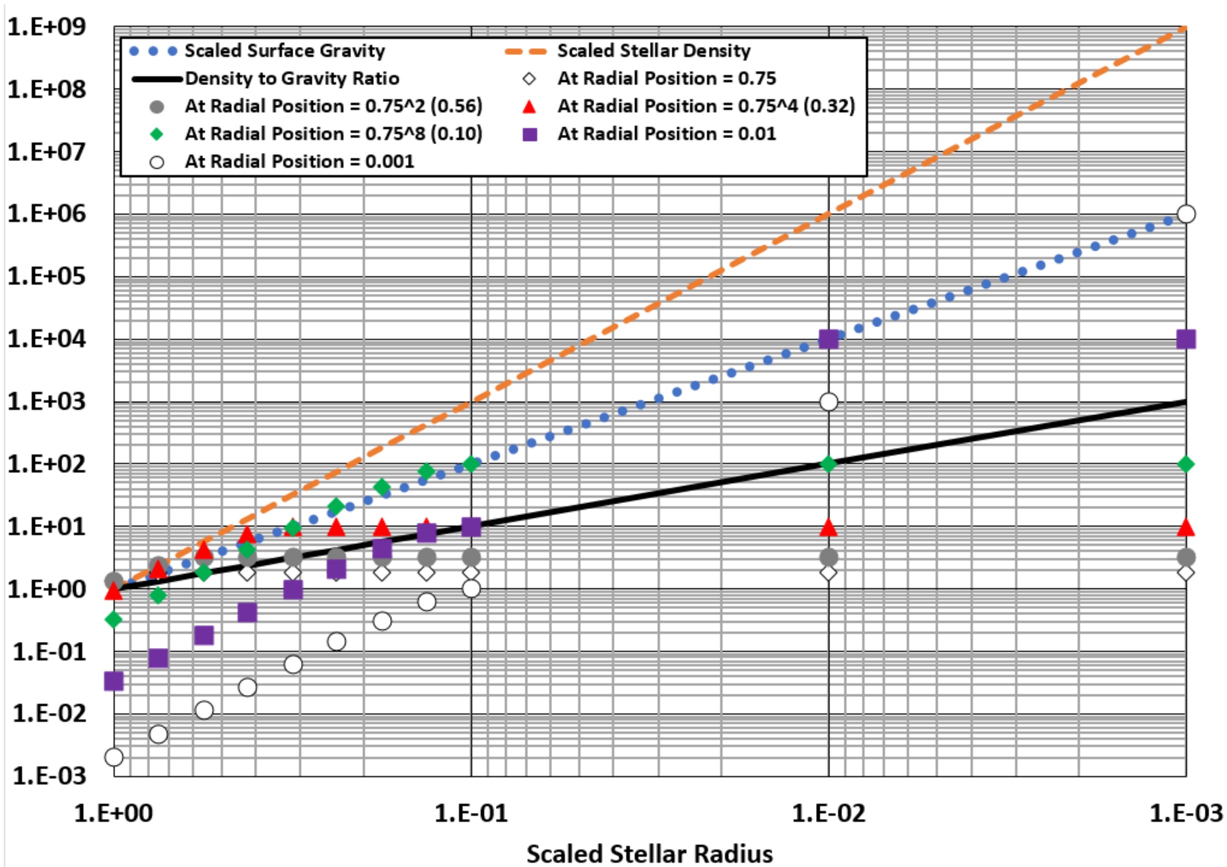
The volume of the double spherical cap, formed by angle 2β corresponding to the chord shown passing through 'r,' is $\frac{2\pi[1]^3}{3}(\cos\beta + 2)(\cos\beta - 1)^2$, for a stellar radius = 1. Therefore, the volume of the portion exerting a net gravitational force is $4\pi[1]^3/3$ minus the volume of the double spherical cap. Since the star's center of mass must always be at the center, the position 'x' for the center of mass of this remaining volume can be found by solving x ● (remaining volume) = r ● (double spherical cap volume), i.e., x = r ● (volume ratio). Thus, for any given value of 'r' $(0 < r \leq 1)$, the corresponding value for 'x' can be calculated and the net gravitational force at 'r' calculated as $1/(r + x)^2$, relative to the scaled surface gravity of unity when r = 1.

Figure 3 displays the following: (1) the scaled surface gravity, scaled stellar density and density-to-surface gravity ratio as the stellar radius decreases from 1 to 0.001 (dotted, dashed and solid lines, respectively) and (2) the scaled gravity at various radial positions (shown as individual points), inside, on the surface, and outside the star as it collapses (e.g., radial position $0.75^4 = 0.32$ is inside the star for smaller stellar radii [shown down to 0.001], on the surface at that stellar radius, and outside the star for larger stellar radii [up to 1]). Clearly evident is the one-billion-fold increase in stellar density, one-million fold increase in surface gravity, and one-thousand fold increase in their ratio when the stellar radius decreases one-thousand fold. Also evident at each radial position is the increase in gravity while that position lies within the star, with it reaching its maximum, constant value once the stellar radius reaches that position. Since this never exceeds the maximum gravity at the stellar surface, the density-to-gravity ratio must always be greater than or equal to that at the stellar radius. This further suggests the dominance of the 'immovable object' aspect of the increasing density over the 'irresistible force' aspect of the increasing gravity, lending a physical basis to the mathematical one for the impossibility of a black hole.

3. Conclusion

Just what orders of magnitude are we talking about? For a neutron star of, say, 10 solar masses (10 x 1.99x10^{30} kg = 1.99x10^{31} kg) and a density of 4 x 10^{17} kg/m^3, the radius would be $\sqrt[3]{\frac{(3)(1.99x10^{31}kg)}{4\pi(4x10^{17}\frac{kg}{m^3})}} = 22{,}800\ m\ \approx 23\ km.$ Its surface gravity (for a 1-kg test mass) would be $\frac{(6.67x10^{-11}\frac{m^3}{kg-s^2})(1.99x10^{31}kg)(1\ kg)}{(22{,}800\ m)^2} = 2.55x10^{12}nt$, over one-hundred-billion time greater than earth's. The density-to-surface gravity ratio would be over $100{,}000\ \frac{kg}{nt-m^3}$, over 100 times that of earth. Collapsing the star's radius one-thousand fold to 22.8 m (less than a football field) increases the gravity to 2.55x10^{18} nt and the density to $4x10^{26}\,\frac{kg}{m^3}$, for a ratio now of over one-hundred million, i.e., now over 100,000 times that of earth. Such a density, if even conceivable, should be more than sufficient as an 'immovable object' to arrest any 'irresistible force' of gravitational collapse, preventing the formation of the miscalculated singularity essential for the existence of a black hole. Therefore, both mathematically and physically, the black hole appears to be impossible, and anything allegedly confirming the existence of black holes (e.g., "Gravitational Waves Discovered from Colliding Black Holes," Scientific American, February 11, 2016 [*https://www.scientificamerican.com/article/gravitational-*

waves-discovered-from-colliding-black-holes1/]) is likely the result of some yet-to-be-discovered version of a neutron star.[1]



<u>FIGURE 3</u>. Increases in Scaled Values with Decreasing Stellar Radius

[1] In his book <u>Our Undiscovered Universe</u> (2007), Terence Witt assumes "that a black hole [as] heavy [as the mass of the Milky Way's core, about three million times that of our sun,] can compress its innermost matter to hyperdensity $1.2x10^{19}\frac{kg}{m^3}$ … [M]atter is not infinitely compressible … Black holes are not *singularities*, gravitational or otherwise, regardless of their size. They are merely compact objects with deep gravitational potentials."