Emergence of Spacetime in Quantum Shape Kinematics

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A model universe is analyzed where there is only electrical force between protons and electrons in the context of quantum shape kinematics. A similar model where there is only gravitational attraction between masses was investigated by Barbour, Koslowski and Mercati before. Our results is an expansion of the ideas there. It is found that for $N \geq 3$ particles, with at least one of them oppositely charged, absolute space emerges if the spin degrees of freedoms are neglected as it is the case for a classical observer.

INTRODUCTION

Shape dynamics is a fully relational theory of gravitation. In the case of N-body problem, it states that only the relative distances and angles between them are dynamical [1]. In this scenario universe cannot have a non-vanishing angular momentum, otherwise it would define an absolute space in which the universe is rotating [2]. Similarly non-vanishing total energy implies an external absolute time according to which the universe evolves, therefore we require total energy to vanish [2]. Another constraint comes from working in the center of mass frame, the total momentum should be zero. Some works [3] also put another constraint on the system, e.g. the vanishing of the dilational momentum: $\sum_a \mathbf{r}^a \cdot \mathbf{p}^a = 0$. This is required for scale invariance [3], however we will not impose it. Overall, we have three constraints:

- $\mathrm{H} = \sum_a E^a = 0$
- $\mathbf{P} = \sum_a \mathbf{p}^a = 0$
- $\mathbf{L} = \sum_a \mathbf{r}^a \times \mathbf{p}^a = 0$

There is a need to calculate Poisson brackets of constraints with each other in order to classify them. This distinction will be important when the theory is quantized. The list is as follows:

$$\{P_i, P_j\} = 0, \tag{1}$$

$$\{L_i, L_j\} = \varepsilon^{ijk} L_k, \tag{2}$$

$$\{L_i, P_j\} = \varepsilon^{ijk} P_k, \tag{3}$$

$$\{H, P_i\} = 0, \tag{4}$$

$$\{H, L_i\} = 0. \tag{5}$$

The results of these commmutators are either zero or another constraint. Hence they vanish weakly. Therefore all the constraints are first class in the terminology of Dirac [4]. We did the Dirac analysis in order to show that our system is consistent and have seen that it is well defined.

QUANTIZATION OF THE MODEL

The model is quantized by promoting positions and momenta to quantum operators. The Poisson bracket $\{\cdot, \cdot\}$ is mapped to $i\hbar[\cdot, \cdot]$. Between position and momenta is the following expression:

$$[\hat{r}_i^a, \hat{p}_j^b] = i\hbar \delta_b^a \delta_j^i. \tag{6}$$

Momenta are represented by operators, $\hat{p}_i^a = -i\hbar\partial/\partial r_i^a$. It is time to consider what happens to constraints in this case. As the readers can verify easily, the constraint algebra survives the quantization. In particular there is no anomaly. In the presence of dilational momentum constraint ref. [3] reports the existence of scale anomaly. It is then argued in [3] that this anomaly may give rise to a gravitational arrow of time. However, the arrow of time is outside our scope in this study.

At the quantum level the constraints become operators acting on the quantum state of the system. For example the Hamiltonian constraint becomes:

$$H\psi = \sum_a -\frac{\hbar^2}{2m}\nabla_a^2\psi + \frac{1}{2}\sum_{a\neq b}\frac{kq_a q_b}{|\mathbf{r}^a - \mathbf{r}^b|}\psi = 0, \tag{7}$$

where $k = 1/4\pi\epsilon_0$, ∇_a stands for gradient operator with respect to particle a and we neglect spin-spin interactions. We see that we have obtained a time independent Schrödinger equation. Wavefunctions do not evolve in time and are static. This is similar to what happens with the Wheeler-DeWitt equation.

The momentum and angular momentum constraints become:

$$\mathbf{P}\psi = -i\hbar \sum_a \nabla_a \psi = 0, \tag{8}$$

$$\mathbf{J}\psi = -i\hbar \sum_a \mathbf{r}^a \times \nabla_a \psi + \sum_a \mathbf{S}_a \psi = 0, \tag{9}$$

where the spin operator for each particle $\mathbf{S}_a$ is added. We interpret equations (7) (8) and (9) as operator equations that determines the allowed kinematic states of the system.

EMERGENCE OF SPACETIME

The Hamiltonian constraint, momentum constraint and angular momentum constraint adds up to total of 7 constraints in three dimensional space. For a single particle the momentum constraint implies that ψ is a constant spinor. Because it has not orbital angular momentum $\mathbf{L}\psi = 0$, however $\mathbf{S}\psi$ cannot vanish. Hence the theory disallows the existence of a single particle in the universe.

In the case of two particles, in order for $H\psi$ to vanish, the particles should be oppositely charged. Otherwise there is no solution. At this moment the solution is given by the ionized Hydrogen atom, e.g. the electron and proton are free. In order to satisfy the momentum constraint, the electron and proton should have opposite momenta and this can be satisfied by suitably choosing the wavefunction, e.g. by plane waves. As for the angular momentum constraint, the spin states should be such that it is satisfied and it can be done for specific values of momenta because the spin states are quantized. Thus, two particles in the universe can only exist if they are oppositely charged. The particles do not form a bound state due to Hamiltonian constraint.

As for $N \geq 3$, there are at least 6 spin degrees of freedom and many spatial degrees of freedom for the spatial part of the wave function. The constraints can be satisfied. However there must be at least one oppositely charged particle in order to satisfy the Hamiltonian constraint. In this case a bound state or states can exist with suitable combination of momenta and spin states.

The angular momentum constraint gives us $(\mathbf{L}+\mathbf{S})\psi = 0$. However, for a classical eye the spin part is inaccessible. Therefore a classical observer sees that the angular momentum of the universe does not vanish. It is important to notice that this fact results in the structure of absolute space in which the universe has a classical non-vanishing angular momentum. This is the emergence of spacetime in quantum shape kinematics.

CONCLUSION

In this study, quantum shape kinematics is studied for N electrons and protons in an otherwise empty universe. For $N = 1$ no solution is found. When $N = 2$ there is a solution with free electron and proton. As for $N \geq 3$, it is found that bound state or states may occur and the universe must include at least one oppositely charged particle.

Moreover, for three or more particles we have found out that universe has a non-vanishing angular momentum when the spin degrees of freedoms are disregarded as it is the case for a classical observer. This is regarded as the emergence of absolute space.

In the analysis, only the attractive feature of the Coulomb potential is used. Therefore the emergence of spacetime is expected for any theory which can exhibit attractive force. However the spin-spin interactions are neglected in this study. Future studies exploring the idea of the emergence of spacetime in line with this paper ought to search for the implications of spin-spin interactions.

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