Links of Gravity with Electromagnetic and Weak Couplings at Low Energies

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ABSTRACT – Electrons interacting with the QED vacuum and distorting the space-time tissue are considered as a means to express the gravitational constant in terms of electromagnetic parameters. The link between gravity and weak interactions is also worked out in this paper. Finally we extend the first treatment to hadrons, in order to evaluate the strong interaction coupling at low energies (at the energy scale of the proton mass).

1 – Introduction

Leptons are elementary particles which may interact through the electromagnetic, the weak and the gravitational forces, but are not sensible to the strong interactions [1]. The electron is the most stable lepton and seems to be a good candidate to look for relations between gravity and electromagnetic couplings and also between weak and gravity forces.

Five characteristics lengths can be assigned to the electron, namely: - the classical radius, the Schwarzschild radius, the weak radius, the Compton length and the Bohr radius. Although in the evaluation of the Bohr radius the proton enters in the game, its internal structure governed by the strong force is not taken in account.

In section 2 we deduce a relation which ties the gravitational constant to the electromagnetic coupling and the electron mass. In section 3 is deduced a new relation which links the gravitational constant to the electroweak coupling and the electron mass. Perhaps surprisingly, relation developed in section 2 is adapted to the strong interaction case in section 4. There, this adapted relation is used in order to evaluate the strong coupling at the energy of the proton mass. Section 5 is reserved to concluding remarks.

This paper is largely inspired in a previous one published by Jesús Sánchez [2], and entitled: "Calculation of the gravitational constant G using electromagnetic parameters". The results obtained in section 2, reproduces Sánchez result [2], but we have used an alternative path as a means to get it.

2 – The gravitational constant G, the electromagnetic coupling α and the electron mass.

Modern description of the electron proposes that it is immersed in the quantum electrodynamics (QED) vacuum, where it can interact with virtual photons and electron-positron pairs [3]. We can write an general relation representing this possibility, namely

$$\sigma \, n \, \ell = 1. \tag{1}$$

In (1), σ represents the electron scattering cross-section, n is the number of virtual particles (N) per unit of volume (V), and ℓ is the electron mean free path. As was pointed out by T. D. Lee [4]: "in QED there are three important lengths, differing from each other by powers of α" . Next we define these lengths. We write (ħ = c = 1)

$$R_e = \alpha / m_e, \quad \text{and} \quad \lambda_e = 2\pi R_e = 2\,\pi\alpha / m_e. \tag{2}$$

$$\lambda_C = 1 / m_e, \quad \text{and} \quad R_B = 1 / (\alpha\, m_e). \tag{3}$$

In (2) and (3), R_e is the classical radius of electron, being λ_e a wave length related to it, λ_C is the Compton wavelength of electron, and R_B is its Bohr radius.

Now we propose that the scattering cross-section is given by "the quantum size of the electron" squared and write

$$\sigma = \lambda_C^2 = 1/m_e^2. \tag{4}$$

Meanwhile, the characteristic volume where we will count the number of elementary excitations, must consider the electromagnetic strength α and we define

$$V = \lambda_e^2\, \lambda_C = 4\pi\, \alpha^2/m_e^3. \tag{5}$$

The evaluation of the electron mean free path ℓ, will take in account that the electron gravity causes a distortion in the tissue of the space-time and therefore we are taking it as half of the Schwarzschild radius of electron. We write

$$\ell = G\, m_e. \tag{6}$$

Next step is counting the number N of elementary excitations occurring inside the volume V. We make use of the Boltzmann relation and write

$$S - S_0 = \ln \Omega, \qquad (k_B = 1). \tag{7}$$

In (7) S and S_0 are respectively the entropy and the reference entropy that we link to a string, which size is related to the Compton length of electron. Besides this, we take

$$\Omega \equiv V. \tag{8}$$

In order to evaluate the entropies S and S_0, let us think about a mass of a electron-positron pair which oscillates as

$$M = (2m_e) \cos(\omega t). \tag{9}$$

We have

$$< M^2 > = (4m_e^2) <\cos^2 (\omega t)> = 2m_e^2, \qquad \text{and} \tag{10}$$

$$\mu = <M^2>^{1/2} = \surd 2\ m_e. \tag{10A}$$

For a chain of size $1/\mu = 1/(\surd 2\ m_e)$, we define the entropy S [5,6] as

$$S = [1/(\surd 2\ m_e)] \div (\alpha/m_e) = 1/(\surd 2\ \alpha). \tag{11}$$

In (11), $R_e = \alpha/m_e$, is the size of the unit cell used to cover the chain. We verify that the classical radius of electron is the unit of length used to do the partitioning of the chain.

Meanwhile the hydrogen atom can be adopted to define the residual or reference entropy S_0, and we take $1/4$ of the perimeter of the first Bohr orbit as a means to measure the chain. We have

$$S_0 = [1/(\surd 2\ m_e)] \div [\ \pi/(2\alpha\ m_e)] = \alpha\pi/(2\surd 2). \tag{12}$$

Inserting the results (11) and (12) into (7), solving for Ω and considering (8), we get

$$N \equiv \Omega = \exp[\, 1/(\sqrt{2}\,\alpha) - \pi\,\alpha/(2\sqrt{2})]. \qquad (13)$$

We remember that in (1), we have

$$n = N/V. \qquad (14)$$

Putting the results of (4), (14), (13), (5) and (6) in equation (1), and solving for G we find

$$G = [(4\pi^2\,\alpha^2\,\hbar c)/m_e^2]\,\exp[\,\pi\alpha\,/(2\sqrt{2}) - 1/(\sqrt{2}\alpha)]. \qquad (15)$$

In (15), we have also restored the constants ħ and c, which were made equal to the unity during the calculations. This result is identical to that first obtained in reference [2], but here through an alternative path to that followed by Sánches [2]. Numerical evaluation of (15) reproduces the measured value of G with a great accuracy (please see reference [2]).

3 – Connection between the gravitational constant and the weak coupling

In this section we are going to show that the gravitational constant G can also be expressed in terms of the mass of the W-boson of the weak interactions, besides the electromagnetic coupling and the electron mass. First let us define the weak radius of the electron R_w. We write

$$m_e = \alpha_w/R_w, \qquad \text{with} \qquad \alpha_w = \alpha\,(m_e\,/M_w)^2. \qquad (16)$$

Relation (16) implies that

$$R_w = \alpha\, m_e \,/ M_w^2. \qquad (17)$$

Now let us to consider that the electron behaves as a spherical universe of radius R_e. Besides this, we propose that the Holographic Principle (HP) [7,8,9] can be applied to this universe if we use R_w (the weak radius) as the size of the unit cells which cover its surface. On the other hand we assume that the entropy of this universe can also be computed, by considering the perimeter of one of its maximum circles, partitioned in unit cells of size L, where L is a modified Planck length. Let us put these ideas in terms of the relation

$$N_w = \pi\, R_e^2 / R_w^2 = \pi\, R_e \,/ L = N_G. \qquad (18)$$

The modified Planck length is given by

$$L = L_{Pl} / (4\pi)^{1/2} = [G / (4\pi)]^{1/2}. \qquad (19)$$

The use of (2) and (17) in (18) gives after some little algebra

$$L^2 = \alpha^2\, m_e^6 / M_w^8. \qquad (20)$$

Finally considering (19) we get

$$G = 4\pi\, \alpha^2\, m_e^6 / M_w^8. \qquad (21)$$

We observe that the value of G is easily evaluated by inserting in it the known measured values of α, m_e and M_w.

However comparing relations (15) and (21), which are two different ways of expressing G, we find

$$(m_e / M_w)^8 = \pi \exp[\pi\alpha / (2\sqrt{2}) - 1/(\sqrt{2}\alpha)]. \qquad (22)$$

Solving (22) for M_w, and making use of the measured value of the electron mass, we find

$$M_w = 80.32 \text{ GeV}. \qquad (23)$$

The above value must be compared with

$$M_w \big|_{\text{measured}} = 80{,}385 \text{ MeV} \pm 15 \text{ MeV}, \qquad (24)$$

This last one quoted from a reporter of the Particle Data Group [10].

4 – The strong interaction case

Working in an analogous way we have done for the electro magnet coupling case, we can write

$$G_s = [(4\pi^2 \alpha_s^2 \hbar c)/m_p^2] \exp[\pi\alpha_s / (2\sqrt{2}) - 1/(\sqrt{2}\alpha_s)]. \qquad (25)$$

In (25), m_p is the proton mass, α_s is the strong coupling and G_s is the equivalent of the Newton constant for the strong interaction case. Now we impose that

$$G_s m_p^2 = \pi\hbar c. \qquad (26)$$

Indeed (26) defines G_s. The use of (26) into (25) yields

$$\pi(4\alpha_s^2)\exp[\pi\alpha_s/(2\sqrt{2}) - 1/(\sqrt{2}\alpha_s)] = 1. \tag{27}$$

Solving numerically Equation (27) , we find

$$\alpha_s = 0.465. \tag{28}$$

This value can be compared with $\alpha_s = 4/9$, as evaluated in reference [11].

5 – Concluding remarks

It is interesting to verify that the Schwarzschild radius of electron, which has a scale of length very much smaller than the Planck length, plays a fundamental role in the present derivation linking the gravitational constant to the electromagnetic parameters. Indeed this fact has been considered before by Sánchez [2], in the work which inspired the present paper.

In section 3, we have used the weak radius of electron in a variation of the HP. It is worth to stress that this radius is closely related to the Fermi constant of the weak interactions as can be seen in reference [12].

Finally, the adaptation of the calculations to the strong interaction case comes as a bonus got by this work.

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