How Well Do Classically Produced Correlations Match Quantum Theory?

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Abstract

A two-dimensional vector can be made from a constant signal component plus a randomly oriented noise component. This simple model can exploit detection and post-selection loopholes to produce Bell correlations within 0.01 of the theoretical cosine expected from quantum mechanics.

McEachern's Polarized Coin

A classical model for producing "quantum correlations" was recently introduced by McEachern [1]. The model involves two polarized coins, with a cloud of noise spread over the surface of each coin. A trial consists of sending one coin to Alice who uses an instrument to perform a measurement of the coin's polarity at a random angle, α, and sending the other coin to Bob who also measures at a random angle, β. Polarity varies continuously with the setting of the angle, but the measurement procedure is limited to reporting $+1$ if the polarity is positive, -1 if the polarity is negative, or 0 to indicate that the magnitude of the polarity fails to meet a preset threshold. The magnitude of both measurements must exceed this threshold in order for a trial to be counted in the analysis, which consists of reducing the observations to four totals (N_{++}, N_{+-}, N_{-+}, N_{--}) of detected polarity pairs reported by Alice and Bob for each instance of angular difference, $\theta = \alpha - \beta$. For example, N_{+-} is the number of trials, for a given angular difference between measurement settings, that Alice reports a positive polarity while Bob reports a negative polarity.

The correlation is defined as

$$C = \frac{N_{++} + N_{+-} - N_{-+} - N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}} \tag{1}$$

Values for noise amplitude and threshold can be found so that the cloud of noise influences the distribution of correlations against angular difference to approximate $C(\theta) \approx -\cos(\theta)$ expected from quantum theory, as demonstrated by a computer program [2]. Fig. 1 was produced by a similar program and shows the correlation *vs* angular difference for 10 million trials. It is clear that the model does not fit the cosine perfectly, with an error of 2% at the maximum.

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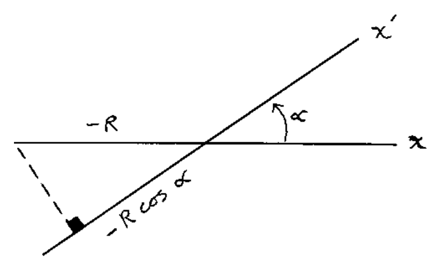
There is a consistent pattern in the difference between the model and the cosine. The pattern becomes clearer as the number of trials is increased, but substantial computation is required.

The Noisy Vector Model of McEachern's Coin

The computational load can be reduced by considering a simpler model which exhibits similar characteristics. The first step in simplifying the noisy coin model is to imagine that the cloud of noise can be replaced by a single random vector corresponding to the centroid of the cloud. This conceptual step of vectorization is then applied to the coin (the signal). Each of McEachern's polarized coins has a positive half and a negative half. This symmetry prevents vectorization of the coin because only the zero vector would be allowed. The way to get around this obstacle to vectorization is to cut the coins in half, discarding the negative halves, then sending one positive half (aligned with the instrument's zero-setting) to one observer, and sending the positive half of the other coin (pointing in the opposite direction) to the other observer. The image of each half-coin can be replaced by its centroid in the form of a vector.

The vectorized coin model then consists of two parts: a constant signal vector aligned with the x-axis, added to a randomly oriented noise vector. The coordinates of the signal vector sent to Alice are $(-R, 0)$, while the signal sent to Bob is $(R, 0)$. The signal amplitude is taken to be $R = 1$. The standard deviation of the noise and the threshold were found empirically to be rather close to $\sigma_r = 1/3$ and $\epsilon = 1/4$ respectively by a least squares fit to the theoretical cosine.

Alice's measurement process consists of setting an angle, α, on the instrument, evaluating the projection onto the x-axis in the rotated coordinate system, then determining whether the magnitude of the projection exceeds the threshold, ϵ, before reporting the polarity as positive or negative. For example, the projection of Alice's noiseless signal would be calculated using the following geometry.



Three versions of the noisy vector model are considered – the only difference is the nature of the randomness of the noise part of the vector. In all versions, the angle, γ, of the noise vector for Alice's instrument is randomly selected for each trial, all angles being equally probable. The projection onto the x-axis is given by

$$P_{\text{Alice}} = -R\cos(\alpha) + r\cos(\gamma) \tag{2}$$

where the magnitude, r, of the noise vector is either a normal variable with standard deviation, $\sigma_r = 1/3$, or is a constant, $r = 1/3$.

Similarly, the angle, δ, of the noise vector for Bob's instrument is randomly selected for each trial, and the projection onto the x-axis is given by

$$P_{\mathrm{Bob}} = R\cos(\beta) + r\cos(\delta) \tag{3}$$

where the magnitude, r, of the noise vector is either a normal variable (independent of Alice's noise vector) with standard deviation, $\sigma_r = 1/3$, or is a constant, $r = 1/3$.

Polarity in McEachern's coin model is determined by a polarized mask rotated to the measurement angle, and varies linearly with angle. In the vector model, polarity is determined by projection and varies with the cosine of the angle, although linear polarity can be made to work too. Despite these differences, both the coin model and noisy vector model 1 produce the same sort of correlations at 10 million trials. Even the error is distributed similarly, as can be seen by comparing Figures 1 and 2. See [3] for a computer program which uses the noisy vector model to produce Bell correlations.

In model 1, Alice and Bob are each presented with an independent random variable[1] as the magnitude of the noise vector, with only the standard deviation common to both of their random variables. Fig. 3 shows the correlation and error after one billion trials, which is sufficient to reveal a pattern in the error.

Model 2 has either Alice or Bob with a random variable and the other with a noise vector of constant magnitude. Fig. 4 shows that the error between the model and cosine for this hybrid model is noticably reduced. The error can be approximately reproduced by a simple linear combination of $\cos(\theta)$ and $\cos(5\theta)$ where $\theta = \alpha - \beta$. These components can be inferred from visually inspecting the error curves from any of the models after a sufficient number of trials.

Model 3 has both Alice and Bob with noise vectors of constant magnitude, This model is simplest, fastest and best fits the cosine. It can be seen from Figs. 5 and 6 that the difference between the model and the cosine is the smallest, about 1% at the maximum. When Alice's instrument is set to zero, perfectly in line with the negative signal component but in the opposite direction, there is no possibility that a positive polarity will be reported, or that the trial will be nullified because the magnitude of the measured polarity falls beneath the threshold. This is not the case for the other two models, where the randomness of the magnitude of the vector could reverse the detected polarity even when alignment is perfect.

As mentioned previously, there are two variants of the method for determining polarity. The cosine projection was selected over linear simply because it results in a signal to noise ratio which is closer to an integer. The threshold was similarly selected because $\epsilon = 1/4$ is a simple fraction, and quite close to optimal (least

[1] A pseudo-random approximation to a normal variable with zero mean and standard deviation equal to σ_r is used.

squared error) in fitting the cosine. These values result in a simple recipe for producing correlations: 1 part signal, 1/3 part noise, 1/4 threshold. Model 3 is very slightly favoured by the chosen values of noise and threshold, but not so much as to change the appearance of the figures, or any numerical estimates presented, if the optimal values for noise amplitude and threshold were used instead for each model. Those values were found from an iterative grid search with 10 million trials at each point on the grid.

Bandwidth and Bits

McEachern bases his technique on Shannon's Capacity Theorem, and the very interesting idea that quantum correlations come from a sampling process that can provide only one bit of information,

$$I = TB \log_2(1 + S/N) \tag{4}$$

where TB is the time-bandwidth product, $S = 1$ would be signal amplitude, and $N = 1/3$ would be noise amplitude for all of the noisy vector models presented. In that case, $\log_2(1 + S/N) = 2$, so for there to be $I = 1$ bit of information, the time-bandwidth product would need to be $TB = 1/2$.

This is a curious result which otherwise implies two bits are available if $TB = 1$. In any case, it may be that Shannon's theorem cannot be applied without accounting for the effect of the threshold, ϵ, on the information obtainable from the sampling process.

Discussion

There is a great deal of literature concerning local hidden variables, and dealing with detection and post-selection loopholes. These loopholes allow a classical process to exhibit what appears to be quantum correlations, and are present in the coin and noisy vector models. The noisy vector model is a rudimentary version of the locally deterministic, detector-based model of quantum measurement presented by La Cour [4, 5], who has applied it to examples such as the CHSH inequality and Mermin-Peres magic square.

It is clear that a classical process can *almost* duplicate the sinusoidal correlations expected from quantum theory. It is not clear how the small differences, on the order of 0.01, might affect the processing power of classical systems with many artificial qubits. On the other hand, it may be appropriate to compare the classical models to observed data rather than quantum theory. After all, there is the possibility that the processes underlying quantum mechanics can be understood as fundamentally classical, which might be revealed through analysis of experimental data.

References

[1] McEachern R.H., A Classical System for Producing "Quantum Correlations". (2016)
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[2] McEachern R.H., Matlab program for producing quantum correlations. (2016)
http://fqxi.org/forum-attachments/Quantum_Correlations.txt

[3] Walker C., Matlab program for producing quantum correlations using the noisy vector model. (2017)
https://sites.google.com/site/quantcorr

[4] La Cour B.R., A Locally Deterministic, Detector-Based Model of Quantum Measurement. Found. Phys. 44, 10591084 (2014)
http://link.springer.com/article/10.1007%2Fs10701-014-9829-6

[5] Zyga L., Quantum computer emulated by a classical system. (2015)
https://phys.org/news/2015-05-quantum-emulated-classical.html

Coin model with 10 million trials

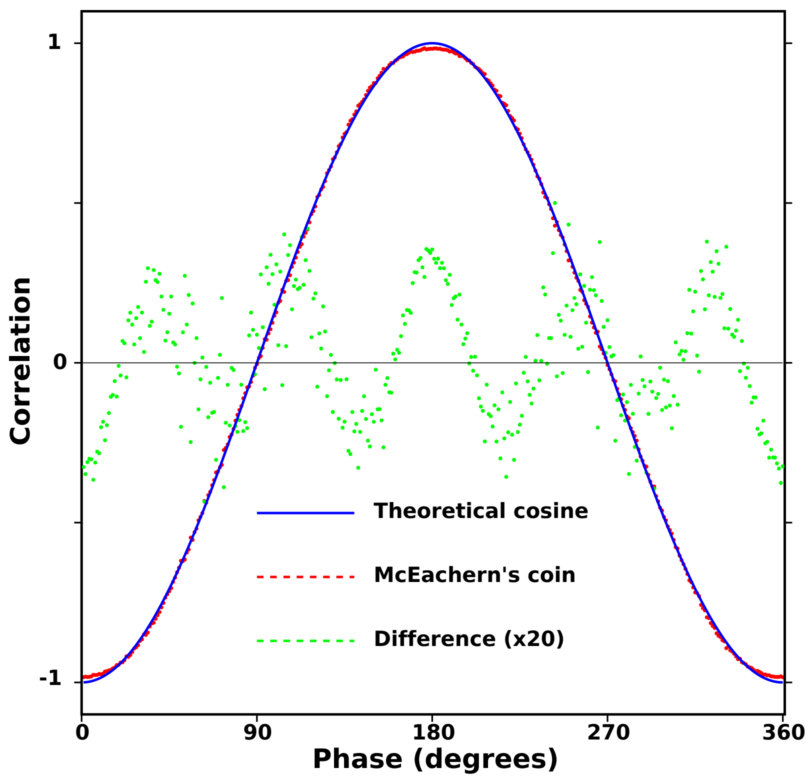

Figure 1: Correlation against angular difference between detector settings for McEachern's coin model after 10 million trials, and program parameters NoiseAmp=6.4, Threshold=1100. The phase (angular difference) is $\theta = \alpha - \beta$ where α is the setting for the angle of rotation on Alice's instrument, and β is the setting on Bob's instrument. The correlation fits to within about ± 0.02 of the theoretical cosine, and there is a pattern to the difference.

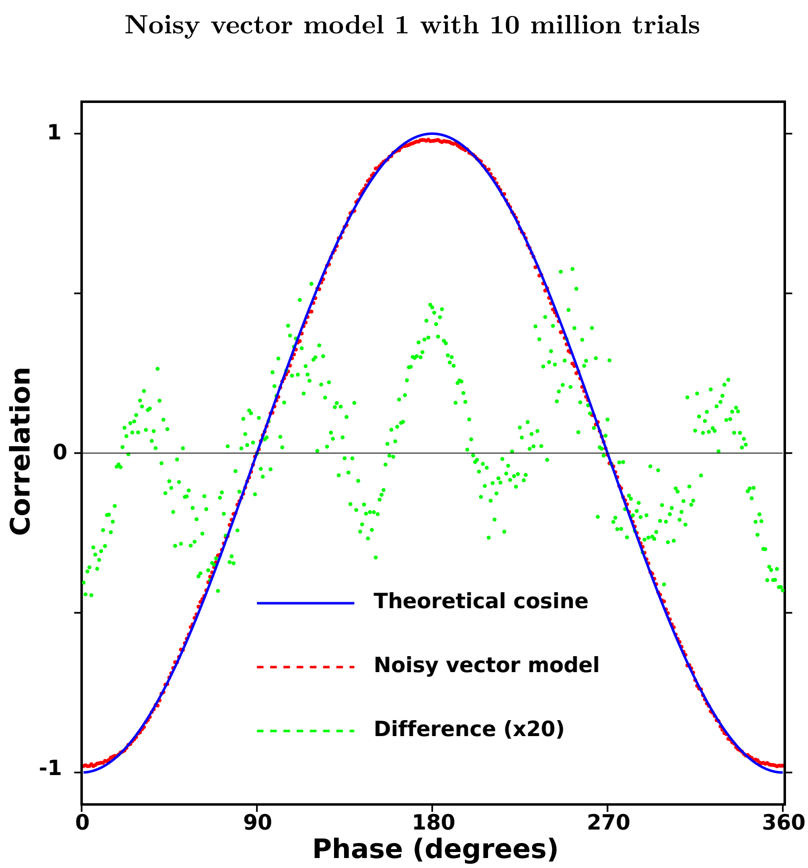

Figure 2: Noisy vector model 1 after 10 million trials and parameters $\sigma_r = 1/3$, $\epsilon = 1/4$. The vector model with 10 million pairs of measurements gives a result similar to the coin model with the same number of trials. Notably, the error in the fit to the cosine appears to be distributed similarly to the coin model.

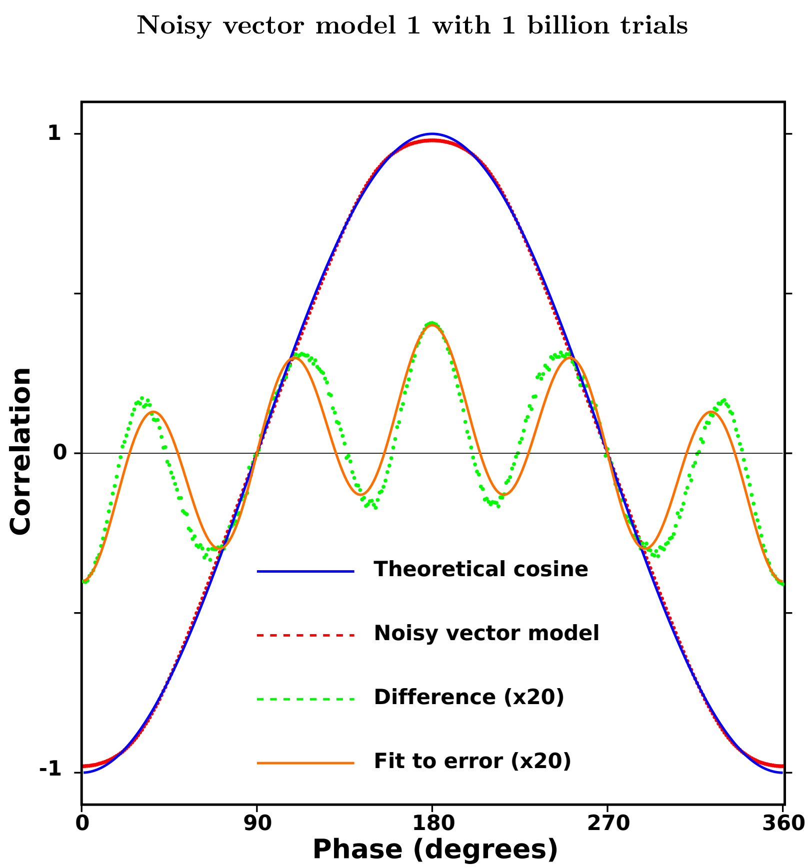

Figure 3: Noisy vector model 1 after 1 billion trials, and parameters $\sigma_r = 1/3$, $\epsilon = 1/4$. This model has a random variable as the magnitude of the noise vector for both Alice and Bob. Increasing the number of trials has not improved the fit of the correlation to the cosine which remains at about ± 0.02. The higher number of trials has reduced the scatter, revealing a clear pattern in the difference between the cosine and the measured correlation. This pattern has $\cos(\theta)$ and $\cos(5\theta)$ as its major components, and a least-squares fit of these two components to the error is shown.

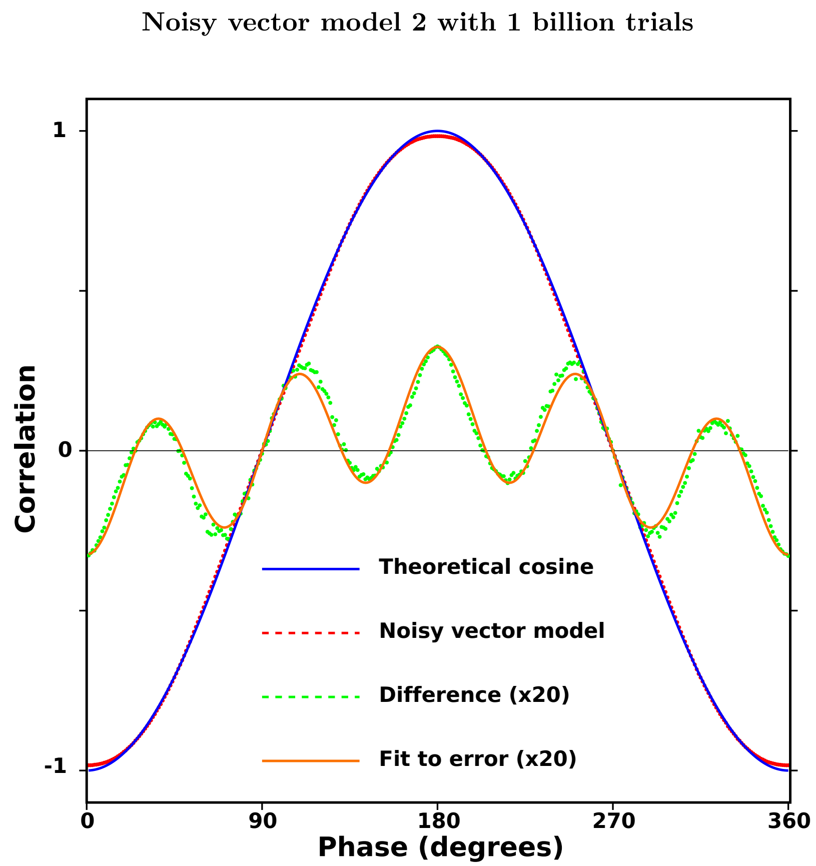

Figure 4: Noisy vector model 2 after 1 billion trials, and parameters $r = 1/3$ (or $\sigma_r = 1/3$), $\epsilon = 1/4$. In model 2, if either Alice or Bob has a noise vector of constant magnitude, then the other has a random variable for the magnitude. The fit of the correlation to the cosine lies within approximately ± 0.015. The components $\cos(\theta)$ and $\cos(5\theta)$ fit the pattern in the difference between the cosine and the measured correlation rather well, in comparison to the other models.

Noisy vector model 3 with 1 billion trials

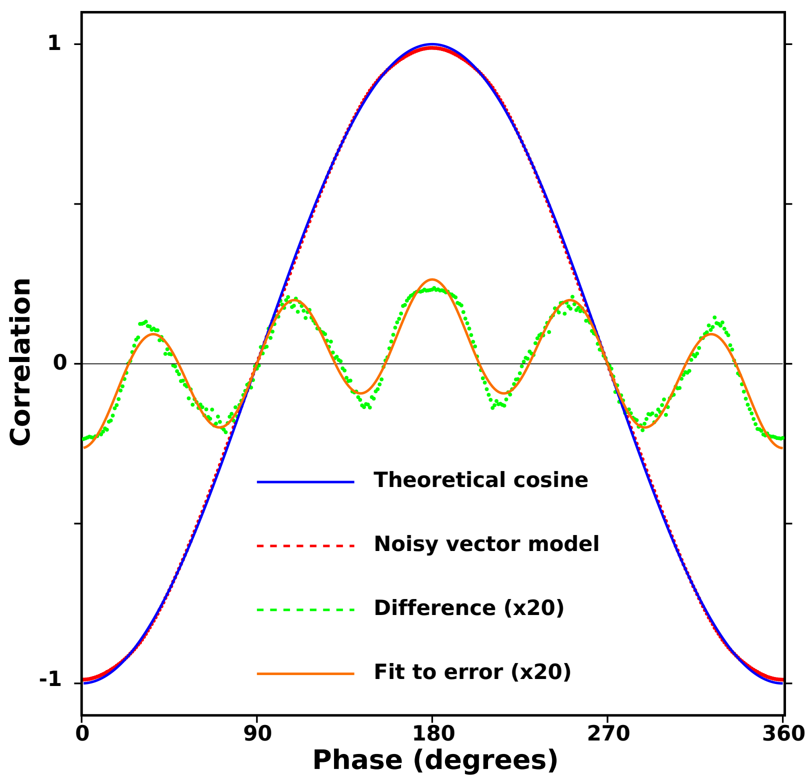

Figure 5: Noisy vector model 3 after 1 billion trials, and parameters $r = 1/3$, $\epsilon = 1/4$. Model 3, where both Alice and Bob are presented with noise vectors of constant magnitude, provides the closest fit of the correlation to the cosine, at about ± 0.01.

Noisy vector model 3 with 1 billion trials

Output from Matlab version of program

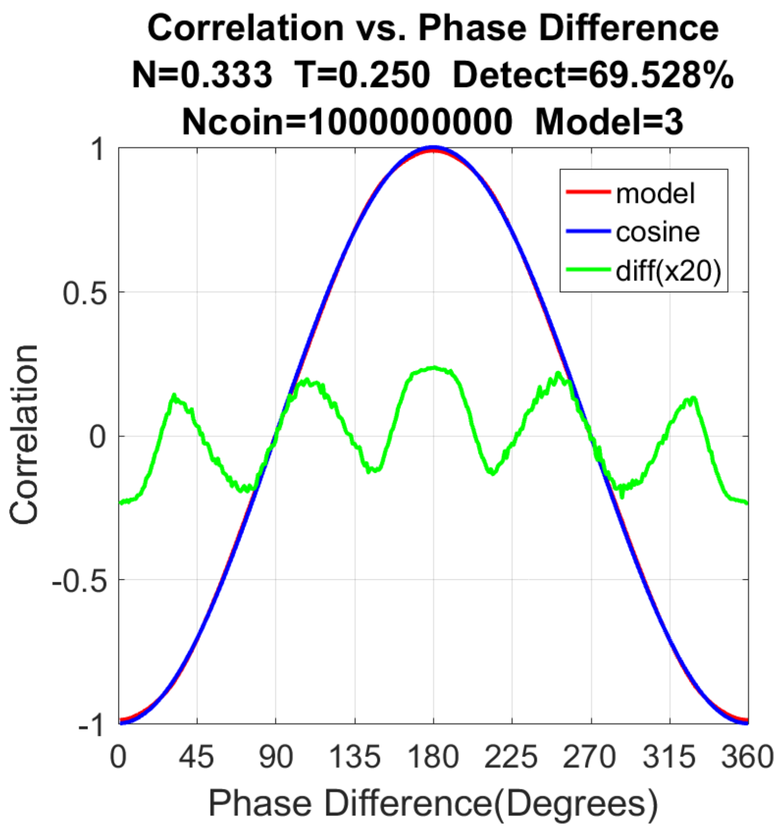

Figure 6: Noisy vector model 3 after 1 billion trials, and parameters $r = 1/3$, $\epsilon = 1/4$. This sample output from the companion Matlab program [3] shows a result nearly identical to Fig 5, which was produced by a C program with a different set of random numbers.