

A simple analysis of the relationship between the power of the solar constant and the temperatures of the earth's surface and the atmosphere.

Abstract

This is a very simple analysis of earth and the input of solar energy using heat transfer and geometry. I show that earth radiates as a blackbody by volume and not as a surface.

1. Heat transfer

The concept of blackbody radiation is the perfect absorber emitting a flux density in exact balance, without any variation in temperature in the volume. Emission is defined in two dimensions from a surface, as radiation exclusively. Every photon absorbed at the surface is balanced to an exact equal amount of energy emitted from the absorbing body, exactly proportional to the difference in temperature measured at the systems outer boundary.

A perfect blackbody is thought to not exist, it is an ideal model that is a frame for the variations we see in the universe.

Using the blackbody as the fundamental model of every radiating body in the universe, and make necessary modifications according to observations, I try to design a concept of earths relation to the energy absorbed from solar irradiation. The result is a toy-model based on only geometry and heat transfer that describe earth temperature accurately according to the basic concept of blackbody radiation. The earth radiates according to a perfect blackbody by it's volume of radiating mass, the earth surface combined with the atmosphere.

The modifications made are based on the difference in absorption, in a volume surrounding a spherical surface on earth, in relation to a blackbody radiating in two dimensions from a surface.

The tropopause is the boundary where earth absorbs solar energy in the volume beneath, it is the equivalent of the surface of a blackbody.

The earth surface temperature below the atmosphere must be treated as a shell within a volume surrounded by another spherical shell.

The solar constant is measured to a mean $1360.8 \pm 0.5 W/m^2$ (Kopp, Lean 2011)¹. The earth mean temperature is usually said to be 288 Kelvin, although it is hard to find a source for this value. I will use the value from wikipedias *earth page*², 288 Kelvin.

Using the simplest form of heat transfer $P_{net} = A\sigma(T_1^4 - T_2^4)$ for a blackbody radiation, where T_1^4 is the solar constant and T_2^4 is the earth surface temperature, I get:

$$P_{net} = A\sigma(393.59793425 \dots^4 - 288^4) = 5.67 * 10^{-8}(24000000000 - 6879707136) \\ = 970.72W/m^2$$

$970.72W/m^2$ is the amount of energy transferred to the earth surface at a global mean temperature of 288 Kelvin.

This is very close to the value given in the greenhouse model after albedo-correction.

The energy of solar irradiation after the energy radiated from the surface is subtracted, is equal to the radiated energy from earth as effective temperature when solar energy is distributed over the area of the sphere.

The surface temperature combined with the radiated energy of the atmosphere is perfectly balanced with incoming solar radiation.

2. The relationship between the solar constant, geometry and surface temperature of earth.

The unsatisfactory description of heat absorption in earth surface, as modeled in greenhouse theory, gave inspiration to try finding a way to balance the steady state of earth temperature in a toy model. The large gap between the flux density at the tropopause and the emitted energy of the atmosphere was the object of investigation. My approach was to start with a reasonable geometric distribution of solar energy between the tropopause and the surface. Since the atmosphere consist of mass, I decided that flux density would not be the best approach.

There is no way to know the amount of scattering, absorption and diffusion that can happen over the distance from the tropopause and the surface, and it seems to vary over time. To clarify how the solar energy transforms when absorbed by the earth, we must look at geometry.

I wanted to find a neutral starting point and used the volume of an empty spherical shell as absorbing body. The energy distributes divided by $4\pi/3$, used as $4/3$ in this toy model.

I later found out that the same fraction has been used in an old theory of electromagnetic mass, a concept that might be related to the one I arrived at. Henri Poincare defined it as a part of the electrons total energy:

$$\frac{E_{tot}}{c^2} = \frac{E_{em} + E_p}{c^2} = \frac{E_{em} + \frac{E_{em}}{3}}{c^2} = \frac{4}{3} \frac{E_{em}}{c^2} = \frac{4}{3} m_{es} = m_{em}$$

I found a slightly different solution to Earth's absorption and emission of energy. I treated the system as two spheres, the atmosphere and the solid earth, and found that a double empty shell irradiating half the sphere's surface, emitting from the whole surface, is the complete model to calculate average surface temperature in a straightforward way.

The flow of thermal energy from the sun, is accurately described as an independent flow of heat in transfer through the earth system. The earth mean temperature distribution is the same as a perfect blackbody absorbing and emitting solar energy by the volume of it's atmosphere, in addition to the solid surface.

The twodimensional ideal blackbody surface is transformed to a threedimensional body, irradiated on half the surface, and emitting from double the area. The disc at the center of earth has the intensity of solar radiation over surface πr^2 , transformed by the hemisphere absorbing at an area of $2\pi r^2$, emitted at a strength transformed by distribution through it's whole volume as a mean intensity fractioned to $4\pi r^2$.

The mean absorption of the irradiated hemisphere is the same as a perfectly black surface, but distributed throughout volume of a spherical shell representing the atmosphere.

$$\frac{1360.8W}{\frac{4}{3}} = 1020.6W/m^2$$

When adding the solid mass of the earth surface and the internal heat in combination with the atmospheric sphere, the energy transforms into a second spherical shell:

$$\frac{1020.6W}{\frac{4}{3}} = 765.45W/m^2$$

That amount of energy is balanced through emission from twice the size of the absorbing surface:

$$\frac{1}{2} \left(\frac{TSI}{\left(\frac{4}{3}\right)^2} \right) = \sigma T_{surface}^4 = 382.725W/m^2 = 286.6Kelvin$$

So, we can easily determine the basic limits of the earth system, heat transfer tells us the rate of thermal energy in transfer according to difference in temperature, and it is very close to the energy density when distributed in a volume defined as mean temperature.

This equation is balanced throughout the system, where each step correspond to the others and energy can flow at a constant rate in every part. As can be seen, no albedo or other corrections is needed, it is instead the fraction falling off from the shape of a sphere in relation to the disk receiving solar energy of $1360.8W/m^2$.

The effective temperature is the mean intensity radiated from the volume in relation to the blackbody surface. The spherical shells temperature is defined by distribution in the volume as a gradient between two concentric spheres.

The inclusion of albedo or anything else than the power of solar irradiation, seems as a backward approach. Everything inside the troposphere is a product of solar energy and cannot affect the temperature. They are caused by temperature.

The most basic version of perfect blackbody radiation is an accurate model of earth temperature, when modified to distribute the energy in a volume with two concentric shells. It also combines well with observed effective temperature.

3. Gravity

The energy needed to have gravitational acceleration of $\approx 9.8m/s$ must be included in the thermal energy coming from the sun, since it is the only energy entering and exiting the system.

I wanted to see the relationship between the work performed by gravity and earth's emitted longwave radiation. Radiation is defined as flux density at the surface, emitted as $W/m^2/s$, while gravity is defined by mass, distance and time as $kg/m/s$.

The work performed by gravity over a square meter per kg in a second:

$$g/m^2 = 9,8^2 = 96W/m^2/s$$

The amount of gravitational energy needed from the surface of earth, propagating in a radiationlike manner towards a surrounding sphere must then be:

$$4 * 96W/m^2/s = 384W/m^2/s = \sigma T_{surface}^4$$

Using the value at the equator, 9.78m/s, gives the closest value to the emitted power of the spherical distribution described in earlier chapter.

To see if the universal gravitation constant G is related to earth's g, I use the energy density of vacuum, $8\pi G$, in a terrestrial version, the energy density of earth, $8\pi g$, where $g = 9.8$

$$8\pi g = 246W/m^2 = 256Kelvin$$

$$Effective\ temperature\ (earth) \approx 255K$$

I exchanged g for π^2 , since it seems to correlate with all values of interest for earth. Reasoning behind this is that all spheres rotating around it's axis will have a point of no momentum at the absolute center of mass and energy, and that all changes in radius, density or energy will include π , no matter how small or big.

For the perfect sphere:

$$8\pi^3 = 248.05W = 257K$$

$$8\pi^4 = 2 * \sigma T_{surface}^4$$

$$7\pi^4 = \frac{1}{2} * 1363W$$

$$4\pi^4 = 389.6W/m^2 = \sigma T_{surface}^4 (287.9K)$$

$$\pi^4 + \pi^3 = 128,4\sigma T_{tropopause}^4 (218,1K)$$

$$\sigma T_{tropopause}^4 / \pi^3 = \pi + 1$$

$$\sqrt{\sigma T_{surface}^4 / 4} = \pi^2 = 9.86W/m/s$$

Conclusions:

This is nothing more than a toy model, but the relationship between earth's emitted energy at the surface, and the work done by gravity over an equally sized surface, has not been considered before as far as I know, and might be of interest for closer investigation.

The basic concept of blackbody radiation is a complete model of earth temperature, when modified to absorb and radiate in three dimensions from a volume with two concentric shells.

Earth surface is not supposed to radiate the same amount as a perfect blackbody surface, the earth surface flux density radiates according to it's temperature, and is balanced with the solar constant together with the radiated energy of the atmosphere. The emission of a perfectly black surface is radiated as a combination of surface OLR together with atmospheric OLR. The transfer

of energy according to the difference in temperature, is equal to the emitted energy of the atmosphere as effective temperature. The rate of transfer shows that there is no greenhouse effect where the atmosphere adds energy to the surface. It is entirely a product of heat transfer from the sun.

$$\frac{TSI - \sigma T_{surface}^4}{4} = T_{effective}^4$$

$$TSI = P_{net} = \sigma T_{surface}^4 + 4\sigma T_{effective}^4$$

References:

1. Kopp, G.; Lean, J. L. (2011). "[A new, lower value of total solar irradiance: Evidence and climate significance](#)" (PDF). *Geophysical Research Letters*. **38**: n/a. [Bibcode:2011GeoRL..38.1706K](#). [doi:10.1029/2010GL045777](#).

2. <https://en.wikipedia.org/wiki/Earth>