Radius of single fluxon electron model identical with classical electron radius

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Abstract

Analytical determination of the magnetic flux included in the electron's dipole field - with consideration of magnetic flux quantization - reveals that it precisely comprises one magnetic flux quantum Φ_0. The analysis further delivers a redefinition of classical electron radius r_e by a factorized relation among electron radius r_e, vacuum permeability μ_0, magneton μ_B and fluxon Φ_0, exclusively determined by the electron's quantized magnetic dipole field:

$$r_e = \mu_0 \, \mu_B \, (\Phi_0)^{-1} = e^2/4\pi\epsilon_0 m_e c^2$$

The single fluxon electron model further enables analytical determination of its vector potential at r_e: $\vec{A}_{re} = \vec{\Phi}_0/2\pi r_e$ and canonical angular momentum: $e\vec{A}_{re} \, 2\,\pi r_e = \hbar/2$.
Consideration of flux-quantization supports a toroidal electron model.

1 Introduction

QFT uncertainty relations generally restrict a precise definition of whatsoever electron structure. Nonetheless it is possible to define and interpret mathematical or statistical structure elements like r_e or λ_C without violating QFT-rules. Among the deficiencies of existing electron models is the unknown magnetic flux Φ_{el} associated its magneton μ_B. [1], [2]
At the other hand, if flux quantization was a universal principle it should generally apply to all microphysical magnetic fields like that of the electron's dipole field. [3], [4] Following logical reasoning it might thus be justified to postulate flux quantization as an axiomatic basis for heuristic assignment of at least 1 fluxon (or an integer multiple) to the electron' vacuum dipole field.
In this article, r_e designates the classical (electrostatically) and r_m the QFT (magnetostatically) determined electron radius.
As there exists no unambiguous relationship among magnetic momentum and magnetic flux, a "try and error" method based on flux quantization could lead to a precise solution. In addition, such approach would substantially simplify determination and interpretation of r_m, vector-potential A_m and spin angular momentum $\hbar/2$.

Thus let us start with a general determination of magnetic flux Φ of a dipole-field traversing its own equatorial plane p_e, as a function of a delimiting circle with variable radius r. Hence r will act as lower integration limit and ∞ as hypothetical upper limit. The aim is to identify a characteristic radius r_m for which precisely one fluxon passes through p_e, outside of delimiting r_m : $(r > r_m)$. (Note that this definition of r_m only makes sense in the equatorial plane p_m.) Thus r_m should not be confused with a spherical radius.
If Parson's toroidal electron model was considered, r_m might be identical with the main toroidal radius which offers the possibility to determine the vector potential $\vec{A}_m$ at r_m as well as canonical angular momentum $\vec{L}_{cm}$. [5], [6], [7]

2 The dipole field of μ_B

Let us start with an analysis of a classical dipole induction field $\vec{B}_{(\vec{\Theta},\vec{r})}$, to approximate the electron's dipole (vacuum) field outside of a hypothetical microscopic current loop generating its magneton μ_B.

$$\vec{B}_{(\vec{\Theta},\vec{r})} \approx \mu_0 \Big(\frac{2\,\mu_B\,cos\,\Theta}{4\,\pi\,r^3}\,\hat{\mathbf{r}} + \frac{\mu_B\,sin\,\Theta}{4\,\pi\,r^3}\hat{\mathbf{\Theta}} \Big) \tag{1}$$

In (1), $\hat{\mathbf{r}}$ designates the radius unit vector, r the radial distance, $\hat{\mathbf{\Theta}}$ the polar unit angle, Θ the polar angle, μ_b Bohr's magneton and μ_0 vacuum permeability. Within the scope of our analysis we can restrict ourselves to the equatorial (x-y) plane p_e, where the $\vec{B}$-field perpendicularly crosses p_e along the z-axis. Hence in p_e the following simplifications are justified for all points in $p_e : \Theta = \pi/2 =$ const. $\rightarrow$ $cos\,\Theta = 0$, $sin\,\Theta = 1$. Thus the first term in brackets in (1) vanishes:

$$|\vec{B}(\vec{r})| = \frac{\mu_0\,\mu_B}{4\,\pi\,r^3} \tag{2}$$

Let $da = 2\,\pi\,r\,dr$ denote a circular surface differential at radius r in p_e, then the magnetic flux differential $d\Phi$ through da is:

$$d\Phi = B(r)\,da = B(r) 2\,\pi\,r\,dr \tag{3}$$

Let $\Phi(r)$ designate the flux through p_e from ∞ to a delimiting circle of radius r. Thus r acts as the lower integration limit for:

$$\Phi(r) = \int_r^{\text{inf}} d\Phi = 2\pi \int_r^{\text{inf}} B(r)\,r\,dr \tag{4}$$

Substitution with (2) in (4):

$$\Phi(r) = \frac{\mu_0\,\mu_B}{2} \int_r^{\text{inf}} \frac{dr}{r^2} = \frac{\mu_0\,\mu_B}{2\,r} \tag{5}$$

From (5) we obtain the desired function $r = f(\Phi)$:

$$r = \frac{\mu_0 \, \mu_B}{2 \, \Phi} \tag{6}$$

As the objective of this analysis is to determine the radius r_m delimiting one fluxons Φ_0 outside of $r = r_m$ let us substitute in (6) $\Phi = \Phi_0 = h/2e$ and rename r by r_m.

$$r_m = \frac{\mu_0 \, \mu_B}{2 \, \Phi_0} = \frac{\mu_0 \, \mu_B \, e}{h} = \mu_0 \, \mu_B \, (\Phi_0)^{-1} = \mu_0 \, \mu_B \, K_J \tag{7a}$$

($K_J =$ Josephson's constant)
Substitution in (7a) with $\mu_0 = 1/\epsilon_0 \, c^2$, $\mu_B = e \, h/4 \, \pi \, m_e$ and $\Phi_0 = \; h/2 \, e$ yields:

$$r_m = \frac{e^2}{4 \, \pi \, \epsilon_0 \, m_e \, c^2} = r_e \tag{7b}$$

Thus $r_m = r_e$ is a magnetically determined equivalent of the classical electron radius r_e. [1, 3]
If above single fluxon electron model was combined with a toroidal electron model, one fluxon would be confined within a circle of radius r_m, as r_m would delimit the external ($r > r_m$) from the internal ($r < r_m$) dipole field.

3 Vector-potential at A_{rm}

is determined by the condition that one fluxon $\Phi_0 = A_{rm} 2 \, \pi \, r_m$ confined within a circle of radius r_m:

$$A_{rm} = \Phi_0/2\pi r_m = \frac{h \, \epsilon_0 \, m_e \, c^2}{2 \, \pi e^3} \tag{8a}$$

4 Phase-shift

Toroidal electron models hypothesize that the charge e might propagate along a circular filament resembling a current-loop of main radius r_m with magnetic momentum μ_B. In above toroidal electron model the charge e would interact with the vector-potential A_{rm} along its circular pathway given by r_m.This implies that if a probability wave $\Psi(\vec{r})$ was assignable to e - propagating along a circle of radius r_m a phase-shift

$$\delta_{rm} = \frac{e}{\hbar} A_{rm} \, r_m \oint_0^{2\pi} d\varphi = \frac{e \, \Phi_0}{\hbar} = \pi \tag{9}$$

might occur. Full periodicity 2π would however require in (9) an upper integration limit of 4π or two fluxons ($2 \; \Phi_0 = \hbar/e$) as initially conjectured by F. London [3].

5 Canonical angular momentum $\hbar/2$

Consider a point-like charge q of mass m at location $\vec{r}$ moving in the $x-y$ plane with velocity $\dot{\vec{r}}$, through a magnetostatic field with vector-potential $\vec{A}_r$ in the x-y plane. Its Lagrangian $\mathcal{L}$ would be:

$$\mathcal{L} = \frac{m}{2}\dot{\vec{r}}^2 + q\,\vec{A}\,\dot{\vec{r}} \tag{10}$$

corresponding to canonical momentum $\vec{p}_c$:

$$\vec{p}_c = \frac{\partial\,\mathcal{L}}{\partial\,\dot{\vec{r}}} = m\,\dot{\vec{r}} + q\,\vec{A}_{(r)} \tag{11}$$

Note that even a stationary charged particle ($m\,\dot{\vec{r}} = 0$) can have a non-zero canonical momentum $q\vec{A}_r$ in presence of a magnetic field. [7] (Canonical angular momentum of an electron immersed in a macroscopic magnetic field can become by orders of magnitude larger than its own spin angular momentum.)
Hence the term $q\vec{A}_r$ in (11) can be regarded as an invisible part of canonical momentum.
The mass m in (10) and (11) usually refers to the mass of a point-like charge moving through a uniform magnetic field with vector-potential $A_{(r)}$.
Canonical angular momentum $\vec{L}_c$ of a charged mass in motion at $\vec{r}$ generally is:

$$\vec{L}_c = \vec{r} \times \vec{p}_c \tag{12}$$

After substitution with (11) in (12) and $q = e$:

$$\vec{L}_{ce} = \vec{r}_m \times (m\dot{\vec{r}} + e\,\vec{A}_{rm}) \tag{13}$$

Generally, the physical state inside r_m is unknown. The mass m in (12) presumably can't be regarded as classical inertial mass, as total inertial mass m_e is already included in the electromagnetic field essentially defined by relativistic criteria (Lorentz-Abraham). Moreover, the unknown mechanical stress condition and balance inside r_q (non-electromagnetic Poincaré stress) would require to consider the relativistic stress- energy-momentum tensor inside of r_m which could theoretically neutralize the inertia inside r_m. Hence in (13) both quantities m and $\dot{\vec{r}}$ are unknown.
However heuristic reasoning suggests that in (13) either m or $\dot{\vec{r}}$ or both are zero ($m\,\dot{\vec{r}} = 0$) thus further calculus can tentatively be restricted to the remaining term in (13): $r_m \times q\vec{A}_r$. Substitution in (13) with $r = r_m$ and $\vec{A}_{rm} = \Phi_0/2\pi r_m$:

$$\vec{L_{1c}} = e\,\vec{r_m} \times \vec{\Phi_0}/2\,\pi r_m = e\,\Phi_0/2\pi = \vec{\hbar}/2 \tag{14}$$

(14) can be interpreted as electromagnetic field angular momentum represented by the Poynting-vector field $\vec{S_{(r)}} = \vec{E_{(r)}} \times \vec{H_{(r)}}$, and corresponding momentum-density field $\vec{S}/c^2$ around e where the electron's Coulomb-field $\vec{E}$ intermeshes

with its own dipole-field $\vec{H}_{(r)}$.[7] The radius determined in [7] was slightly below the expected radius r_e, under the presumption that electromagnetic angular momentum should precisely amount $\hbar/2$. The mismatch presumably results from the conjectured spherical electron, instead of a toroidal model, which would deliver the correct electromagnetic angular momentum for a suitably adjusted small toroidal radius.
The relation $e\,\vec{A}_{rm} \times \vec{r_m} = e\,\vec{\Phi}_0/2\pi$ in (14) thus represents the spin-angular momentum incorporated in the electron's own electromagnetic field, as confirmed by substituting: $\Phi_0 = h/2e$:

$$L_{el} = \frac{1}{2\pi}\,e\,\Phi_0 = \frac{1}{2\pi}\,e\,\frac{h}{2\,e} = e\frac{\hbar}{2e} = \hbar/2 \tag{15}$$

As the identical results of (14) and (15) precisely match up with the electron's spin angular momentum the initial conjecture $m\,\vec{r} = 0$ may be proven valid.

6 Alternative dual-fluxon electron model

Obviously above calculus would also apply for a dual-fluxon hypothesis using $\Phi = 2\,\Phi_0 = h/e$). The respective ratios of results from dual- vs. single fluxon models are:
Electron-Radius $= 0,5 : 1$ - Vector-potential $= 4 : 1$ - Phase-shift $= 2 : 1$ Canonical angular momentum $= 2 : 1$.

7 Synopsis/Conclusion

Magnetic flux qantization delivers alternative criteria for determination of electron radius $r_e = r_m$, vector-potential A_{rm}, phase-shift δ_{rm} and canonical angular momentum $\mathcal{L}_e = \hbar/2$. Results are identical and commensurable with established data.
It is conjectured that spin angular momentum $\hbar/2$ is included in the electron's electromagnetic field. Figuring out the term $e\,\vec{A}_{(rm)}$ of canonical angular momentum in (13) shows that it is identical with spin angular momentum $\hbar/2$. The results support a toroidal electron model. [5], [6]

References

[1] Rohrlich, F. Classical Charged Particles, World Scientific Pub Co Inc, 2007, ISBN: 9789812700049

[2] Pearle, P. 1982 Classical electron models ; Springer US, Online ISBN 978-1-4757-0650-5;

[3] F. London, Superfluids, Macroscopic Theory of Superconductivity, Structure of Matter Vol. 1 (Wiley, New York, 1950), p. 152;

[4] L. Onsager, Phys. Rev. Lett. 7,50 (1961);

[5] Kurtnalty.com/The_Parson_Ring.pdf

[6] http://research.omicsgroup.org/index.php/Toroidal_ring_model

[7] http://isites.harvard.edu/fs/docs/icb.topic1327064.files/143bF2013Sec10.pdf

[8] Blinder, S M: Singularity-free electrodynamics for point charges and dipoles: a classical model for electron self-energy and spin, Eur. J. of Phys.24 (2003) 271-275;