On the Squaring of the Circle

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Abstract: "An Exploration of Circle-Square Properties & Identities"

1> Tabulate some CS data

Case	CircleRadius R	CircleArea $CA=\pi R^2=SA$	SquareEdge $SE=\sqrt{(SA)}$	SquareDiagonal $SD=\sqrt{(2*SE^2)}$	SquareArea $SA=SE^2=CA$
1	1	π	$\sqrt{\pi}$	$\sqrt{(2\pi)}$	π
2	$\sqrt{(\pi)}$	π^2	π	$\sqrt{(2\pi^2)}$	π^2
3	π	π^3	$\sqrt{(\pi^3)}$	$\sqrt{(2\pi^3)}$	π^3
4	$\sqrt{(\pi^3)}$	π^4	π^2	$\sqrt{(2\pi^4)}$	π^4
5	π^2	π^5	$\sqrt{(\pi^5)}$	$\sqrt{(2\pi^5)}$	π^5
6	$\sqrt{(\pi^5)}$	π^6	π^3	$\sqrt{(2\pi^6)}$	π^6
7	π^3	π^7	$\sqrt{(\pi^7)}$	$\sqrt{(2\pi^7)}$	π^7
8	$\sqrt{(\pi^7)}$	π^8	π^4	$\sqrt{(2\pi^8)}$	π^8
9	π^4	π^9	$\sqrt{(\pi^9)}$	$\sqrt{(2\pi^9)}$	π^9
10	$\sqrt{(\pi^9)}$	π^{10}	π^5	$\sqrt{(2\pi^{10})}$	π^{10}

Fig 1: Circle-Square Properties

2> Consider the data, namely case 2

3> Maximal Circle_Overlap of Square = $(\sqrt{\pi}) - (\pi/2)$

4> /* Occurs at the *cardinal* points or SquareEdge midpoints */

5> Maximal Square_Overlap of Circle = $(\sqrt{(2\pi^2)}) / 2 - \sqrt{\pi}$

6> /* Occurs at the *intercardinal* points or Square vertices */

7> It seems ***MagicNumber1*** at the ***Cardinals*** is:

8> ½ SquareEdge / Circle_Radius

9> When numerically expressed:

10> Case 1 : $((\sqrt{\pi}) / 2) \ / 1 = 0.886226925...$

11> Case 2 : $(\pi/2) / \sqrt{(\pi)} = 0.886226925...$

12> Case 3: $((\sqrt{\pi^3}) / 2) / \pi = 0.886226925...$

13> Case 4 : $((\pi^2) / 2) / \sqrt{(\pi^3)} = 0.886226925...$

14> Double-check using SquareEdge=2:

15> $SE = 2 \quad SA = 4 = CA = \pi R^2 \quad R^2 = 4/\pi \quad R = \sqrt{(4 / \pi)}$

16> $1 / \sqrt{(4 / \pi)} =$ <u>0.88622692545275801364908374167057</u>

17> Compute the reciprocal namely R / (½ SE):

18> $\sqrt{(4/\pi)} / 1 =$ <u>1.1283791670955125738961589031215</u>

19> The general case for SquareEdge N:

20> ***MagicNumber1*** $= (N / 2) / \sqrt{(N^2 / \pi)}$ or <u>$N / \sqrt{(4N^2 / \pi)}$</u>

21> Formulate the identity from the ***Cardinal*** data:

	SE/2		R
	$(\sqrt{\pi}) / 2$	divided by	1
equals	$(\pi) / 2$	divided by	$\sqrt{\pi}$
equals	$(\sqrt{\pi^3}) / 2$	divided by	π
equals	$(\pi^2) / 2$	divided by	$\sqrt{(\pi^3)}$
equals	$(\sqrt{\pi^5}) / 2$	divided by	π^2
equals	$(\pi^3) / 2$	divided by	$\sqrt{(\pi^5)}$
equals	$(\sqrt{\pi^7}) / 2$	divided by	π^3
equals	$(\pi^4) / 2$	divided by	$\sqrt{(\pi^7)}$
equals	$(\sqrt{\pi^9}) / 2$	divided by	π^4
equals	$(\pi^5) / 2$	divided by	$\sqrt{(\pi^9)}$
equals	1	divided by	$\sqrt{(4/\pi)}$
equals	2	divided by	$\sqrt{(16/\pi)}$
equals	3	divided by	$\sqrt{(36/\pi)}$
equals	4	divided by	$\sqrt{(64/\pi)}$
equals	5	divided by	$\sqrt{(100/\pi)}$
equals	6	divided by	$\sqrt{(144/\pi)}$
equals	7	divided by	$\sqrt{(196/\pi)}$
equals	8	divided by	$\sqrt{(256/\pi)}$
equals	9	divided by	$\sqrt{(324/\pi)}$
equals	10	divided by	$\sqrt{(400/\pi)}$

22> The above table entries are equal

23> ***MagicNumber1*** = 0.88622692545275801364908374167057

24> ***Reciprocal*** = 1.1283791670955125738961589031215

25> And ***MagicNumber2*** at the ***Intercardinals*** (Square vertices) is:

26> Circle_Radius / ½ SquareDiagonal

27> When numerically expressed:

28> Case 1 : $1 / (\sqrt{(2\pi)} / 2)$ = 0.797884561...

29> Case 2 : $\sqrt{\pi} / (\sqrt{(2\pi^2)} / 2)$ = 0.797884561...

30> Case 3: $\pi / (\sqrt{(2\pi^3)} / 2)$ = 0.797884561...

31> Case 4 : $\sqrt{(\pi^3)} / (\sqrt{(2\pi^4)} / 2)$ = 0.797884561...

32> Double-check using SquareDiagonal = 2 :

33> $SE = \sqrt{2}$ $SA = 2 = CA = \pi R^2$ $R^2 = 2/\pi$ $R = \sqrt{(2/\pi)}$

34> $\sqrt{(2/\pi)} / 1$ = <u>0.79788456080286535587989211986876</u>

35> Compute the reciprocal namely (½ SD) / R :

36> $1 / \sqrt{(2/\pi)}$ = <u>1.2533141373155002512078826424055</u>

37> The general case for SquareDiagonal N :

38> ***MagicNumber2*** = $(\sqrt{(X / \pi)}) / N$ where $X = 2 * (N^2)$

39> Formulate the identity from the ***Intercardinal*** data:

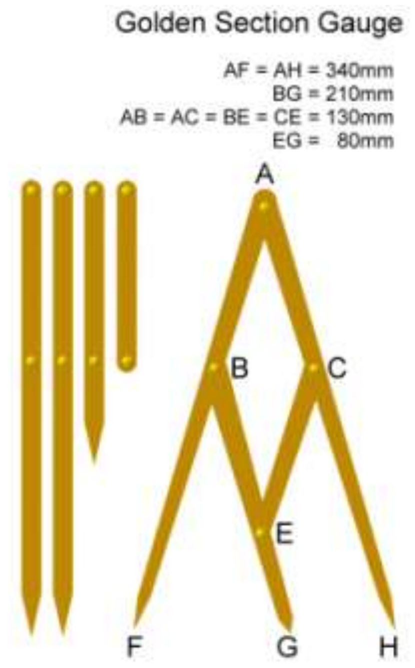
	R		SD/2
	1	divided by	$\sqrt{(2\pi)}$ / 2
equals	$\sqrt{(\pi)}$	divided by	$\sqrt{(2\pi^2)}$ / 2
equals	π	divided by	$\sqrt{(2\pi^3)}$ / 2
equals	$\sqrt{(\pi^3)}$	divided by	$\sqrt{(2\pi^4)}$ / 2
equals	π^2	divided by	$\sqrt{(2\pi^5)}$ / 2
equals	$\sqrt{(\pi^5)}$	divided by	$\sqrt{(2\pi^6)}$ / 2
equals	π^3	divided by	$\sqrt{(2\pi^7)}$ / 2
equals	$\sqrt{(\pi^7)}$	divided by	$\sqrt{(2\pi^8)}$ / 2
equals	π^4	divided by	$\sqrt{(2\pi^9)}$ / 2
equals	$\sqrt{(\pi^9)}$	divided by	$\sqrt{(2\pi^{10})}$ / 2
equals	$\sqrt{(2/\pi)}$	divided by	1
equals	$\sqrt{(8/\pi)}$	divided by	2
equals	$\sqrt{(18/\pi)}$	divided by	3
equals	$\sqrt{(32/\pi)}$	divided by	4
equals	$\sqrt{(50/\pi)}$	divided by	5
equals	$\sqrt{(72/\pi)}$	divided by	6
equals	$\sqrt{(98/\pi)}$	divided by	7
equals	$\sqrt{(128/\pi)}$	divided by	8
equals	$\sqrt{(162/\pi)}$	divided by	9
equals	$\sqrt{(200/\pi)}$	divided by	10

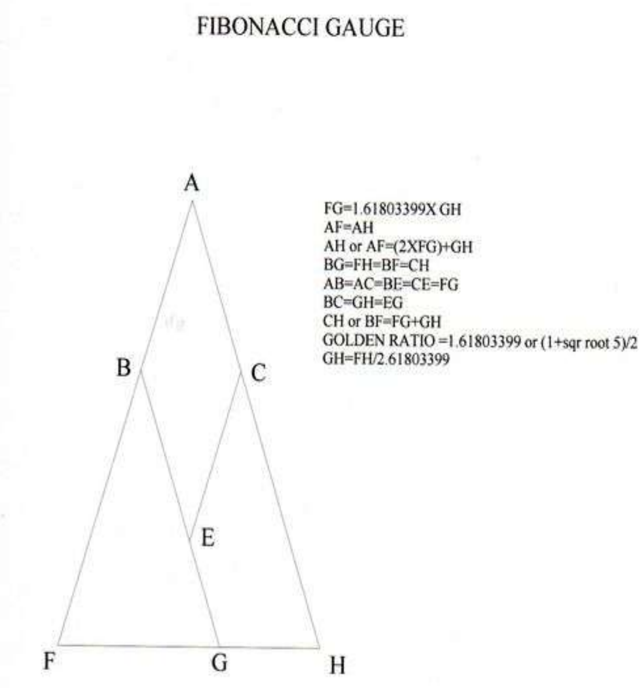
40> The above table entries are equal

41> ***MagicNumber2*** : 0.7978845608028653558798921198687 6

42> ***Reciprocal*** : 1.2533141373155002512078826424055

43> Consider the Fibonacci Gauge:





44> It appears that the following hold true:

45> $$1 / \cos 18^\circ \approx \sqrt{\tan^2 18^\circ + 1}$$

46> $$\tan 18^\circ \approx \sqrt{(1/\cos^2 18^\circ) - 1}$$

47> $$\text{phi } \varnothing \approx 1 / (2 * \sin 18^\circ) \approx 1.61803398...$$

48> $$(2\varnothing + 1) \text{ or } (2 + \sqrt{5}) \approx 4 + (1 / (2 + \sqrt{5}))$$

49> /* The approximation in <48> seems true for the first 37 decimal places */

50> **Question:** Is it possible to construct a CS gauge to facilitate circle-squaring?

51> **Answer:** Probably not, as the Golden Ratio is reciprocal in that

1/phi + 1 = phi (Ø)

MagicNumber1 can be computed from phi in radians as 0.5*SQRT(5*ACOS(phi/2))

However the numbers simply 'do not compute' in the same way