Hybrid vector similarity measure of single valued refined neutrosophic sets to multi-attribute decision making problems

Surapati Pramanik[1*], Partha Pratim Dey[2], Bibhas C. Giri[3]

[1]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.- Narayanpur, District –North 24 Parganas, Pin code-743126,West Bengal, India

[2,,3] Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India

[*]Corresponding Author: E-mail: sura_pati@yahoo.co.in

Abstract: This paper proposes hybrid vector similarity measures under single valued refined neutrosophic sets and proves some of its basic properties. The proposed similarity measure is then applied for solving multiple attribute decision making problems. Lastly, a numerical example of medical diagnosis is given on the basis of the proposed hybrid similarity measures and the results are compared with the results of other existing methods to validate the applicability, simplicity and effectiveness of the proposed method.

Keywords: Single valued neutrosophic sets; Single valued refined neutrosophic sets; Hybrid vector similarity measures; Multi-attribute decision making.

1. Introduction

Smarandache [1] originated the theory of neutrosophic sets (NSs) which is characterized by a truth membership T_A (x), an indeterminacy membership I_A (x) and a falsity membership F_A (x) to cope with indeterminate, incomplete and inconsistent information. However, single valued neutrosophoc sets (SVNSs) defined by Wang *et al*. [2] is useful tool for practical decision making purposes. MADM under SVNSs attracted many researchers and many methods have been proposed for MADM problems such as TOPSIS [3], grey relational analysis [4, 5, 6, 7], outranking approach [8], maximizing deviation method [9], hybrid vector similarity measure [10], etc.

Hanafy *et al*. [11] proposed a method to determine the correlation coefficient of NSs by using centroid method. Ye [12] defined correlation of SVNSs, correlation coefficient of SVNSs, and weighted correlation coefficient of SVNSs. Then, a multi-criteria decision making method (MCDM) was proposed based on weighted correlation coefficient or the weighted cosine similarity measure. Ye [13] developed another form of correlation coefficient between SVNSs and presented a MADM method. Broumi and Smarandache [14] proposed a new method called extended Hausdroff distance for SVNSs and a new series of similarity measures were developed to find the similarity of SVNSs. Majumdar and Samanta [15] introduced the concept of distance of two SVNSs and discussed its properties. They also presented some similarity measures between SVNSs based on distance, a matching function, membership grades and defined the notion of entropy measure for SVNSs. Ye [16] proposed cross entropy of SVNSs and solved a MCDM based on the cross entropy of SVNSs. Ye and Zhang [17] formulated three similarity measures between SVNSs by utilizing maximum and minimum operators and investigated their characteristics. The developed weighted similarity measures were then employed for solving MADM problems under single valued neutrosophic setting. Ye [18] suggested three similarity measures between simplified NSs as an extension of the Jaccard, Dice and cosine similarity measures in vector space for solving MCDM problems. Pramanik *et al*. [10] investigated a new hybrid vector similarity measure under both single valued neutrosophic and interval neutrosophic assessments by extending the notion of variation coefficient similarity method [19] with neutrosophic information and proved some of its fundamental properties.

Smarandache [20] generalized the conventional neutrosophic logic and defined the most *n*- symbol or numerical valued refined neutrosophic logic. Each neutrosophic element *T*, *I*, *F* can be refined into $T_1, T_2, \ldots, T_m$, and $I_1, I_2, \ldots, I_p$, and $F_1, F_2, \ldots, F_q$, respectively, where *m*, *p*, *q* (≥ 1) are integers and $m + p + q = n$. Broumi and Smarandache [21] proposed cosine similarity measure for refined neutrosophic sets due to cosine similarity measure based Bhattacharya's distance [22] and an improved cosine similarity measure for SVNSs proposed by Ye [23]. Ye and Ye [24] introduced the idea of single valued neutrosophic multi sets (SVNMSs) (refined sets) by combining SVNSs along with the theory of multisets [25] and presented several operational relations of SVNMSs. Ye and Ye [24] proposed Dice similarity measure and weighted Dice similarity measure for SVNMSs and investigated their properties. Chatterjee *et al*.

[26] slightly modified the definition of SVNMSs [24] and incorporated few new set-theoretic operators of SVNMSs and their properties. Broumi and Deli [27] defined correlation measure of neutrosophic refined sets and applied the proposed model to medical diagnosis and pattern recognition problems. Ye *et al*. [28] further defined generalized distance and its two similarity measures between SVNMSs and applied the concept to medical diagnosis problem. Mondal and Pramanik [29] proposed neutrosophic refined similarity measure based on cotangent function and presented an application to suitable educational stream selection problem. Deli *et al*. [30] studied several operators of neutrosophic refined sets such as union, intersection, convex, strongly convex in order to deal with indeterminate and inconsistent information. In their paper, Deli *et al*. [30] also examined several results of neutrosophic refined sets using the proposed operators and defined distance measure of neutrosophic refined sets with properties. Karaaslan [31] developed three methods based on similarity measure for single valued refined neutrosophic sets (SVRNSs) and interval neutrosophic refined sets by extending Jaccard, Dice and Cosine similarity measures of SVNSs- and interval neutrosophic sets proposed by Ye [18]. Broumi and Smarandache [32] developed a new similarity measure between refined netrosophic sets based on extended Housdorff distance of SVNSs and proved some of their basic properties. Mondal and Pramanik [33] discussed refined tangent similarity measure for SVNSs and they applied the proposed similarity measure to medical diagnosis problems. Juan-juan and Jian-qiang [34] defined several multi-valued neutrosophic aggregation operators and established a MCDM method based on the proposed operators. Ye and Fu [35] presented new similarity measure of SVNSs through tangent function and a multi- period medical diagnosis method using the proposed similarity measure and the weighted aggregation of multi-period information for solving multi-period medical diagnosis problems under single valued neutrosophic environment. Ye and Smarandache [36] presented a MCDM method with single valued refined neutrosophic information by extending the concept of similarity method with single valued neutrosophic information of Majumdar and Samanta [15].

In this paper, we propose another form of cosine similarity measures under SVRNSs by extending the concept given in [37, 38] and prove some of its basic properties. We propose hybrid vector similarity measure with single valued refined neutrosophic information by extending hybrid vector similarity measure of SVNSs [10] and prove some of its properties. The proposed similarity measure is a hybridization of Dice and cosine similarity measures under single valued refined neutrosophic information. Moreover, we establish weighted hybrid vector similarity measure under single valued refined neutrosophic environment and prove its basic properties. Now the article is structured in the following way. Section 2 presents some mathematical preliminaries which are required for the construction of the paper. In Section 3 defines hybrid similarity and weighted hybrid similarity measures of SVRNSs and proves some of their properties. Section 4 is devoted to develop two algorithms for solving MADM problems involving single valued refined neutrosophic information. An illustrative example of medical diagnosis is solved to demonstrate the applicability of the proposed procedure in Section 5. Conclusions and future scope of research are presented in Section 6.

2. Mathematical preliminaries

In this Section, we recall some basic definitions concerning neutrosophic sets, single valued neutrosophic sets, single valued refined neutrosophic sets.

2.1 Neutrosophic set [1]

Let U be a universal space of objects with a generic element of U denoted by z. Then, a neutrosophic set P on U is defined as given below.

$$P = \{z, \langle T_P(\text{z}), I_P(z), F_P(z) \rangle \mid z \in U\}$$

where, $T_P(z)$, $I_P(z)$, $F_P(z)$: $U \rightarrow\ ]^-0,\ 1^+[$ stand for the degree of membership, the degree of indeterminacy, and the degree of falsity-membership respectively of a point $z \in U$ to the set P satisfying the condition $^-0 \leq T_P(z) + I_P(z) + F_P(z) \leq 3^+$.

2.2 Single valued neutrosophic sets [2]

Consider U be a space of points with a generic element of U denoted by z, then a SVNS Q is defined as follows:

$Q = \{z, \langle T_Q(z), I_Q(z), F_Q(z) \rangle \mid z \in U\}$

where, $T_Q(x)$, $I_Q(x)$, $F_Q(x)$: $U \rightarrow [0, 1]$ denote the degree of membership, the degree of indeterminacy, and the degree of falsity-membership respectively of a point $z \in U$ to the set Q satisfying the condition and $0 \leq T_Q(x) + I_Q(x) + F_Q(x) \leq 3$ for each point $z \in U$.

2.3 Single valued neutrosophic refined sets [24]

A SVNRS R in the universe $U = \{z_1, z_2, \ldots, z_n\}$ is defined as follows:

$R = \{ \langle z, (T_{1R}(z), T_{2R}(z), ..., T_{sR}(z)), (I_{1R}(z), I_{2R}(z), ..., I_{sR}(z)), (F_{1R}(z), F_{2R}(z), ..., F_{sR}(z)) \rangle \mid z \in U\}$

where $T_{1R}(z), T_{2R}(z), ..., T_{sR}(z): U \rightarrow [0, 1]$, $I_{1R}(z), I_{2R}(z), ..., I_{sR}(z)$: $U \rightarrow [0, 1]$, $F_{1R}(z), F_{2R}(z), ..., F_{sR}(z)$: $U \rightarrow [0, 1]$ such that $0 \leq T_{iR}(z) + I_{iR}(z) + F_{iR}(z) \leq 3$ for $i = 1, 2, \ldots, s$. where, s is said to be the dimension of R.

Definition 2.1 [24]: Let R_1 and R_2 be two SVRNSs in U, where

$R_1 = \{ \langle z, (T_{1R_1}(z), T_{2R_1}(z), ..., T_{sR_1}(z)), (I_{1R_1}(z), I_{2R_1}(z), ..., I_{sR_1}(z)), (F_{1R_1}(z), F_{2R_1}(z), ..., F_{sR_1}(z)) \rangle \mid z \in U\}$,

$R_2 = \{ \langle z, (T_{1R_2}(z), T_{2R_2}(z), ..., T_{sR_2}(z)), (I_{1R_2}(z), I_{2R_2}(z), ..., I_{sR_2}(z)), (F_{1R_2}(z), F_{2R_2}(z), ..., F_{sR_2}(z)) \rangle \mid z \in U\}$, then the relations between R_1 and R_2 are presented as follows:

(1). Containment:

$R_1 \subseteq R_2$, if and only if $T_{iR_1}(z) \leq T_{iR_2}(z)$, $I_{iR_1}(z) \geq I_{iR_2}(z)$, $F_{iR_1}(z) \geq F_{iR_2}(z)$ for i = 1, 2, …, s.

(2). Equality:

$R_1 = R_2$, if and only if $T_{iR_1}(z) = T_{iR_2}(z)$, $I_{iR_1}(z) = I_{iR_2}(z)$, $F_{iR_1}(z) = F_{iR_2}(z)$ for i = 1, 2, …, s.

(3). Union:

$R_1 \cup R_2 = \{ \langle z, (T_{iR_1}(z) \vee T_{iR_2}(z)), (I_{iR_1}(z) \wedge I_{iR_2}(z)), (F_{iR_1}(z) \wedge F_{iR_2}(z)) \rangle \mid z \in U\}$ for i = 1, 2, …, s.

(4). Intersection:

$R_1 \cap R_2$ = { $\left\langle z, (T_{iR_1}(z) \wedge T_{iR_2}(z)), (I_{iR_1}(z) \vee I_{iR_2}(z)), (F_{iR_1}(z) \vee F_{iR_2}(z)) \right\rangle \mid z \in U$} for i = 1, 2, …, *s*.

3. Hybrid vector similarity measures of SVRNSs

Definition 3.1 [37]: Let P = {$z, \left\langle T_P(z), I_P(z), F_P(z) \right\rangle \mid z \in U$} and Q = {$z, \left\langle T_Q(z), I_Q(z), F_Q(z) \right\rangle \mid z \in U$} be two SVNSs (non-refined) in the universe of discourse U. Then, the Dice similarity measure of SVNSs is defined as follows.

$$Dice\,(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{2\,(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \quad (1)$$

and if $w_i \in [0, 1]$ be the weight of z_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$, then the weighted Dice similarity measure of SVNSs can be defined as follows.

$$Dice_w\,(P, Q) = \sum_{i=1}^{n} w_i \frac{2\,(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \quad (2)$$

Definition 3.2 [38]: Let P = {$z, \left\langle T_P(z), I_P(z), F_P(z) \right\rangle \mid z \in U$} and Q = {$z, \left\langle T_Q(z), I_Q(z), F_Q(z) \right\rangle \mid z \in U$} be two SVNSs (non-refined) in the universe of discourse U = {z_1, z_2, …, z_n}. Then, the cosine similarity measure of SVNSs is defined as given below.

$$Cos\,(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))}^2 . \sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))}^2\right]} \quad (3)$$

and if $w_i \in [0, 1]$ be the weight of z_i for i = 1, 2, …, n satisfying $\sum_{i=1}^{n} w_i = 1$, then the weighted cosine similarity measure of SVNSs can be defined as follows.

$$Cos_w\,(P, Q) = \sum_{i=1}^{n} w_i \frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))}^2 . \sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))}^2\right]} \quad (4)$$

Definition 3.3 [10]: Hybrid vector similarity measure of SVNSs

Consider Q_1 = $\{z, \langle T_{Q_1}(z), I_{Q_1}(z), F_{Q_1}(z) \rangle \mid z \in U\}$ and Q_2 = $\{z, \langle T_{Q_2}(z), I_{Q_2}(z), F_{Q_2}(z) \rangle \mid z \in U\}$ be two SVNSs in U. Then, the hybrid vector similarity measure of Q_1 and Q_2 is defined as follows:

$$Hyb\,(Q_1, Q_2) = \frac{1}{n}\left[\begin{array}{l} \alpha \sum_{i=1}^{n} \frac{2(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \\ + (1-\alpha) \sum_{i=1}^{n} \frac{(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \end{array}\right] \quad (5)$$

where $\alpha \in [0, 1]$.

Definition 3.4 [10]: Weighted hybrid vector similarity measure of SVNSs

The weighted hybrid vector similarity measure of Q_1 = $\{z, \langle \mathrm{T}_{Q_1}(z), \mathrm{I}_{Q_1}(z), \mathrm{F}_{Q_1}(z) \rangle \mid z \in U\}$ and Q_2 = $\{z, \langle \mathrm{T}_{Q_2}(z), \mathrm{I}_{Q_2}(z), \mathrm{F}_{Q_2}(z) \rangle \mid z \in U\}$ can be defined as follows:

$$WHyb\,(Q_1, Q_2) = \left[\begin{array}{l} \alpha \sum_{i=1}^{n} w_i \frac{2(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \\ + (1-\alpha) \sum_{i=1}^{n} w_i \frac{(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \end{array}\right] \quad (6)$$

where $w_i \in [0, 1]$ be the weight of z_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$, and $\alpha \in [0, 1]$.

Definition 3.5 [24]: Dice similarity measure between two SVNRSs Q_1, Q_2 is defined as follows.

$Dice_{SVRNS}\,(Q_1, Q_2)$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \frac{1}{n}\left[\sum_{i=1}^{n} \frac{2(T^j_{Q_1}(z_i).T^j_{Q_2}(z_i) + I^j_{Q_1}(z_i).I^j_{Q_2}(z_i) + F^j_{Q_1}(z_i).F^j_{Q_2}(z_i))}{\left[((T^j_{Q_1}(z_i))^2 + (I^j_{Q_1}(z_i))^2 + (F^j_{Q_1}(z_i))^2) + ((T^j_{Q_2}(z_i))^2 + (I^j_{Q_2}(z_i))^2 + (F^j_{Q_2}(z_i))^2)\right]}\right]\right\rangle \quad (7)$$

Definition 3.6 [24]: Weighted Dice similarity measure between two SVNRSs Q_1, Q_2 is presented as follows.

$WDice_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i) + I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i) + F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[((T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2) + ((T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2)\right]}\right]\right\rangle \quad (8)$$

Definition 3.7: Cosine similarity measure between two SVNRSs Q_1, Q_2 can be defined in the following way:

$Cos_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n} \frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i) + I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i) + F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{((T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2}).\sqrt{(T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2})\right]}\right]\right\rangle. \quad (9)$$

Proposition 3.1 The defined cosine similarity measure Cos_{SVNRS} (Q_1, Q_2) between SVRNSs $Q_1$and Q_2 satisfies the following properties:

P_1. $0 \leq Cos_{SVRNS}(Q_1, Q_2) \leq 1$

P_2. $Cos_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$

P_3. $Cos_{SVRNS}(Q_1, Q_2) = Cos_{SVRNS}(Q_2, Q_1)$.

Proof.

P_1: According to Cauchy-Schwarz inequality:

$(\mu_1.\nu_1 + \mu_2.\nu_2 + ... + \mu_n.\nu_n)^2 \leq (\mu_1^2 + \mu_2^2 + ... + \mu_n^2).(\nu_1^2 + \nu_2^2 + ... + \nu_n^2)$, where $(\mu_1, \mu_2, ..., \mu_n) \in \Re^n$

and $(\nu_1, \nu_2, ..., \nu_n) \in \Re^n$, we have

$$(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i)) \leq$$

$$\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}$$

Therefore, $\frac{1}{n}\sum_{i=1}^{n}\left[\frac{(T_P(z_i).T_Q(z_i)+I_P(z_i).I_Q(z_i)+F_P(z_i).F_Q(z_i))}{\sqrt{(T_P(z_i))^2+(I_P(z_i))^2+(F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2+(I_Q(z_i))^2+(F_Q(z_i))^2}}\right]\leq 1,$

So, Cos_{SVRNS} (Q_1, Q_2) =

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}}\right]\right]\right\rangle\leq 1,$$

Obviously, $Cos_{SVRNS}(Q_1, Q_2)\geq 0$, thus $0\leq Cos_{SVRNS}(Q_1, Q_2)\leq 1$

P_2: If $Q_1 = Q_2$, then, $T_{Q_1}^j(z_i)=T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i)=I_{Q_2}^j(z_i)$ and $F_{Q_1}^j(z_i)=F_{Q_2}^j(z_i)$ for i = 1, 2, …, n; j = 1, 2, …, p.

Therefore, $Cos_{SVRNS}(Q_1, Q_1)$ =

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_1}^j(z_i))}{\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}}\right]\right]\right\rangle=1.$$

P_3: Cos_{SVRNS} (Q_1, Q_2) =

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}}\right]\right]\right\rangle=$$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\left[\frac{(T_{Q_2}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_2}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_2}^j(z_i).F_{Q_1}^j(z_i))}{\sqrt{((T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}}\right]\right]\right\rangle=$$

Cos_{SVRNS} (Q_2, Q_1).

Definition 3.8: Weighted cosine similarity measure between SVNRSs Q_1, Q_2 can be defined as follows:

$WCos_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n}w_i\left[\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}}\right]\right]\right\rangle. \quad (10)$$

Proposition 3.2 The defined weighted cosine similarity measure $WCos_{SVNRS}$ (Q_1, Q_2) between SVRNSs $Q_1$and Q_2 satisfies the following properties:

P1. $0 \leq WCos_{SVRNS}(Q_1, Q_2) \leq 1$

P2. $WCos_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$

P3. $WCos_{SVRNS}(Q_1, Q_2) = \mathrm{Cos}_{\mathrm{SVRNS}}(Q_2, Q_1)$

Proof.

P1: From Cauchy-Schwarz inequality, we have

$$(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i)) \leq$$

$$\sqrt{(T_{\mathrm{P}}(z_i))^2 + (I_{\mathrm{P}}(z_i))^2 + (F_{\mathrm{P}}(z_i))}^2.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))}^2$$

So, $\sum_{i=1}^{n} w_i \left[\frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))}^2.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))}^2}\right] \leq 1$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

$WCos_{SVRNS}$ (Q_1, Q_2)

$$= \frac{1}{p}\sum_{j=1}^{p} \left\langle \left[\sum_{i=1}^{n} w_i \left[\frac{(T^j_{Q_1}(z_i).T^j_{Q_2}(z_i) + I^j_{Q_1}(z_i).I^j_{Q_2}(z_i) + F^j_{Q_1}(z_i).F^j_{Q_2}(z_i))}{\sqrt{((T^j_{Q_1}(z_i))^2 + (\mathrm{I}^j_{Q_1}(z_i))^2 + (F^j_{Q_1}(z_i))^2)}.\sqrt{(T^j_{Q_2}(z_i))^2 + (I^j_{Q_2}(z_i))^2 + (F^j_{Q_2}(z_i))^2)}}\right]\right]\right\rangle \leq 1,$$

where $w_i \in [0, 1]$ be the weight of z_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$. Obviously, $WCos_{SVRNS}$ (Q_1, Q_2) ≥ 0, and therefore $0 \leq WCos_{SVRNS}(Q_1, Q_2) \leq 1$

P2: If $Q_1 = Q_2$, then, $T^j_{Q_1}(z_i) = T^j_{Q_2}(z_i)$, $I^j_{Q_1}(z_i) = I^j_{Q_2}(z_i)$ and $F^j_{Q_1}(z_i) = F^j_{Q_2}(z_i)$ for i = 1, 2, …, n; j = 1, 2, …, p.

$WCos_{SVRNS}(Q_1, Q_1) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \left[\frac{(T_{Q_1}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\sqrt{((T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}).\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2})}\right]\right]\right\rangle$$

=1.

P3: $WCos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \left[\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\sqrt{((T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}).\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2})}\right]\right]\right\rangle$$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \left[\frac{(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\sqrt{((T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}).\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2})}\right]\right]\right\rangle$$

$= WCos_{SVRNS}(Q_2, Q_1)$.

Next, we have defined hybrid vector similarity methods between SVRNSs by extending the concept of Pramanik *et al*. [10] as given below.

Definition 3.9: Hybrid vector similarity measure between SVNRSs Q_1, Q_2 can be defined as follows:

$Hyb_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\left[\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{((T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2)+((T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2)}\right]\\+(1-\alpha)\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\sqrt{((T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}).\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2})}\right]\end{array}\right]\right\rangle \quad (11)$$

where $\alpha \in [0, 1]$.

Proposition 3.3 The defined single valued refined hybrid vector similarity measure Hyb_{SVNRS} (Q_1, Q_2) between two SVRNSs Q_1 and Q_2 satisfies the following properties:

P1. $0 \le Hyb_{SVRNS}(Q_1, Q_2) \le 1$

P_2. $Hyb_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$.

P_3. $Hyb_{SVRNS}(Q_1, Q_2) = \mathrm{Hyb}_{\mathrm{SVRNS}}(Q_2, Q_1)$.

Proof.

P_1. From Dice and cosine measures of SVRNSs defined in Eq. (7) and Eq. (9), we have $0 \le Dice_{SVRNS}(Q_1, Q_2) \le 1$, $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$.

Therefore, we have, $Dice_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{n}\sum_{i=1}^{n}\left[\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right] \le 1, \text{for } j = 1, 2, \ldots, p,$$

$Cos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{n}\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\sqrt{((T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2)}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2)}}\right] \le 1, \text{ for } j = 1, 2, \ldots, p,$$

Here, $(\alpha)\, Dice_{SVRNS}(Q_1, Q_2) + (1-\alpha)\, Cos_{SVRNS}(Q_1, Q_2)$

$$=$$

$$(\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\left[\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]\right)+$$

$$(1-\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\left[\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}}\right]\right)$$

$\le \alpha + (1-\alpha)$

$= 1$, for j = 1, 2, …, p.

Therefore, $Hyb_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle \frac{1}{n}\left[\begin{array}{l}\alpha \sum_{i=1}^{n} \frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]} \\ +(1-\alpha)\sum_{i=1}^{n} \frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\end{array}\right]\right\rangle$$

≤ 1.

Also, $Dice_{SVRNS}$ (Q_1, Q_2), Cos_{SVRNS} (Q_1, Q_2) ≥ 0, for j = 1, 2, …, p .

Obviously, Hyb_{SVRNS} (Q_1, Q_2) ≥ 0.

This proves that $0 \leq$ Hyb_{SVNRS} (Q_1, Q_2) ≤ 1.

P2. For any two SVNRSs $Q_1$and Q_2, if $Q_1 = Q_2$, this implies $T_{Q_1}^j(z_i) = T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i) = I_{Q_2}^j(z_i)$,

$F_{Q_1}^j(z_i) = F_{Q_2}^j(z_i)$, for i = 1, 2, …, n and j = 1, 2, …, p.

$Dice_{SVRNS}$ (Q_1, Q_2) =

$$\frac{1}{n}\sum_{i=1}^{n} \frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]} = 1, \text{ for } j = 1, 2, \ldots, p,$$

and

$$Cos_{SVRNS}(Q_1, Q_2) = \sum_{i=1}^{n} \frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}$$

=1, for j = 1, 2, …, p.

Hence, Hyb_{SVRNS} (Q_1, Q_2) =1.

P3. Hyb_{SVRNS} (Q_1, Q_2)

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle \frac{1}{n}\left[\begin{array}{l}\alpha \sum_{i=1}^{n} \frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]} \\ +(1-\alpha)\sum_{i=1}^{n} \frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\end{array}\right]\right\rangle$$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \frac{1}{n} \left[\begin{array}{l} \alpha \sum_{i=1}^{n} \frac{2(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i) + I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i) + F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[(T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2 + (T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2\right]} \\ +(1-\alpha) \sum_{i=1}^{n} \frac{(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i) + I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i) + F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[\sqrt{(T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2}.\sqrt{(T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2}\right]} \end{array} \right] \right\rangle$$

$= Hyb_{SVRNS}(Q_2, Q_1)$.

Definition 10: Weighted hybrid vector similarity measure between SVRNSs can be defined as follows.

$WHyb_w(Q_1, Q_2)$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \left[\begin{array}{l} \alpha \sum_{i=1}^{n} w_i \frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i) + I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i) + F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2 + (T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2\right]} \\ +(1-\alpha) \sum_{i=1}^{n} w_i \frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i) + I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i) + F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2 + (I_{Q_1}^{j}(z_i))^2 + (F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2 + (I_{Q_2}^{j}(z_i))^2 + (F_{Q_2}^{j}(z_i))^2}\right]} \end{array} \right] \right\rangle \quad (12)$$

Here, $w_i \in [0, 1]$ represents the weight of z_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$,

where $\alpha \in [0, 1]$, and $WHyb_w(Q_1, Q_2)$ should satisfy the following properties.

Proposition 3.4

P_1. $0 \le WHyb_w(Q_1, Q_2) \le 1$.

P_2. $WHyb_w(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$.

P_3. $WHyb_w(Q_1, Q_2) = WHyb_w(Q_2, Q_1)$.

Proof.

P_1. Using Dice and cosine measures of SVRNSs, we have $0 \le Dice_{SVRNS}(Q_1, Q_2) \le 1$, $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$.

$(\alpha)\ Dice_{SVRNS}\ (Q_1, Q_2) + (1-\alpha)\ Cos_{SVRNS}\ (Q_1, Q_2)$

$$=(\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\right)+$$

$$(1-\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\right)$$

$\leq \alpha + (1-\alpha) = 1$, for j = 1, 2, …, p.

Therefore, $WHyb_w\ (Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\begin{array}{l}\alpha\sum_{i=1}^{n}w_i\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\\+(1-\alpha)\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\end{array}\right]\right\rangle$$

$\leq 1.$

$Dice_{SVRNS}\ (Q_1, Q_2)$, $Cos_{SVRNS}\ (Q_1, Q_2) \geq 0$, for j = 1, 2, …, p .

Obviously, $WHyb_w\ (Q_1, Q_2) \geq 0$, therefore $0 \leq WHyb_w\ (Q_1, Q_2) \leq 1$.

P2. If $Q_1 = Q_2$, then $T_{Q_1}^j(z_i) = T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i) = I_{Q_2}^j(z_i)$, $F_{Q_1}^j(z_i) = F_{Q_2}^j(z_i)$, for i = 1, 2, …, n and j = 1, 2, …, p.

$Dice_{SVRNS}\ (Q_1, Q_2)$

$$=\frac{1}{n}\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}$$

=1, for j = 1, 2, …, p, and

$$Cos_{SVRNS}\ (Q_1, Q_2)=\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}$$

=1, for j = 1, 2, …, p.

Hence, $WHyb_w(Q_1, Q_2)$ =1.

P3. $WHyb_w(Q_1, Q_2)$

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\begin{array}{l}\alpha\sum_{i=1}^{n} w_i \frac{2\,(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2\right]} \\ +(1-\alpha)\sum_{i=1}^{n} w_i \frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle$$

$$= \frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\begin{array}{l}\alpha\sum_{i=1}^{n} w_i \frac{2\,(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2+(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2\right]} \\ +(1-\alpha)\sum_{i=1}^{n} w_i \frac{(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}.\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle$$

= $WHyb_w(Q_2, Q_1)$.

4. MADM with single valued refined neutrosophic information based on hybrid similarity measure

Consider $P = \{P_1, P_2, \ldots, P_m\}$ ($m \geq 2$) be a discrete set of m candidates, $C = \{C_1, C_2, \ldots, C_n\}$, ($n \geq 2$) be the set of attributes of each candidates, and $A = \{A_1, A_2, \ldots, A_k\}$, ($k \geq 2$) be the set of alternatives of each candidate. The decision maker or expert presents the ranking of alternatives with regard to each candidate. The ranking represents the performances of P_i, i = 1, 2, …, m against the attributes C_j, j = 1, 2, …, n and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of the attributes C_j, j = 1, 2, …, n with $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. The relation between candidates and attributes, and the relation between attributes and alternatives can be presented as follows (see Table 1 and Table 2 respectively).

Table 1. The relation between candidates and pre-defined attributes

$$\begin{pmatrix} & C_1 & C_2 & \dots & C_n \\ P_1 & \beta^t_{11} & \beta^t_{12} & \dots & \beta^t_{1n} \\ P_2 & \beta^t_{21} & \beta^t_{22} & \dots & \beta^t_{2n} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ P_m & \beta^t_{m1} & \beta^t_{m2} & \dots & \beta^t_{mn} \end{pmatrix}$$

where $\beta^t_{11} = \left\langle T^t_{ij}, I^t_{ij}, F^t_{ij} \right\rangle$ represents single valued neutrosophic numbers (SVNNs), i = 1, 2, …, *m*; j = 1, 2, …, *n*; t = 1, 2, …, *s*.

Table 2. The relation between attributes and alternatives

$$\begin{pmatrix} & A_1 & A_2 & \dots & A_k \\ C_1 & \gamma_{11} & \gamma_{12} & \dots & \gamma_{1k} \\ C_2 & \gamma_{21} & \gamma_{22} & \dots & \gamma_{2k} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ C_n & \gamma_{n1} & \gamma_{n2} & \dots & \gamma_{nk} \end{pmatrix}$$

Here, $\gamma_{j\ell} = \left\langle T_{j\ell}, I_{j\ell}, F_{j\ell} \right\rangle$ denotes SVNNs, j = 1, 2, …, *n*; ℓ = 1, 2, …, *k*.

We now develop two algorithms for MADM problems based on hybrid similarity measure with single valued refined neutrosophic information as given below.

Algorithm 1

Step 1. Calculate the single valued refined hybrid similarity measures between Table 1 and Table 2 by using Eq. (11).

Step 2. Rank the alternatives based on the descending order of hybrid similarity measures. The biggest value reflects the best alternative.

Step 3. Stop.

Algorithm 2

Step 1. Compute the single valued refined weighted hybrid similarity measure between Table 1 and Table 2 by means of Eq. (12).

Step 2. The alternatives are ranked in descending order of the weighted hybrid similarity measures and the bigger value means the better alternative.

Step 3. Stop.

5. Application of the proposed method to medical diagnosis problem

We consider the illustrative example of medical diagnosis with single valued refined neutrosophic information studied in [33]. Medical diagnosis has to deal with a large amount of uncertainties and huge amount of information available to the medical practitioners using new and advanced technologies. The procedure of classifying dissimilar set of symptoms under a single name of diseases is not easy [21]. Also, it is possible that every object has different truth, indeterminate and false membership functions and the proposed similarity measures among the patients versus symptoms and symptoms versus diseases will provide the appropriate medical diagnosis. In practical situation, there may occur errors in diagnosis if we consider data from one time observation and therefore multi time inspection, by considering the samples of same patient at different times will provide best medical diagnosis [39].

Consider $P = \{P_1, P_2, P_3, P_4\}$ be the set of four patients, C = {viral fever, malaria, typhoid, stomach problem, chest problem} be the set of five diseases, A = {temperature, headache, stomach pain, cough, chest pain} be the set of six symptoms. Now our objective is to examine the patient at different time intervals and we will obtain different truth, indeterminate and false membership functions for every patient. Let three observations are taken in a day: 7 am, 1 pm and 6 pm (see the Table 3) [33].

Table 3. The relation between patients and symptoms

	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8, 0.1, 0.1) (0.6, 0.3, 0.3) (0.6, 0.3, 0.1)	(0.6, 0.1, 0.3) (0.5, 0.2, 0.4) (0.5, 0.1, 0.2)	(0.2, 0.8, 0.0) (0.3, 0.5, 0.2) (0.2, 0.3, 0.4)	(0.6, 0.1, 0.3) (0.4, 0.4, 0.4) (0.4, 0.3, 0.3)	(0.1, 0.6, 0.3) (0.3, 0.4, 0.5) (0.2, 0.5, 0.4)
P_2	(0.0, 0.8, 0.2) (0.2, 0.6, 0.4) (0.1, 0.6, 0.4)	(0.4, 0.4, 0.2) (0.5, 0.4, 0.1) (0.4, 0.6, 0.3)	(0.6, 0.1, 0.3) (0.4, 0.2, 0.5) (0.3, 0.2, 0.4)	(0.1, 0.7, 0.2) (0.2, 0.7, 0.5) (0.3, 0.5, 0.4)	(0.1, 0.8, 0.1) (0.3, 0.6, 0.4) (0.3, 0.6, 0.3)
P_3	(0.8, 0.1, 0.1) (0.6, 0.4, 0.1) (0.5, 0.3, 0.3)	(0.8, 0.1, 0.1) (0.6, 0.2, 0.4) (0.6, 0.1, 0.3)	(0.0, 0.6, 0.4) (0.2, 0.5, 0.5) (0.3, 0.4, 0.6)	(0.2, 0.7, 0.1) (0.2, 0.5, 0.5) (0.1, 0.6, 0.3)	(0.0, 0.5, 0.5) (0.2, 0.5, 0.3) (0.3, 0.3, 0.4)
P_4	(0.6, 0.1, 0.3) (0.4, 0.3, 0.2) (0.5, 0.2, 0.3)	(0.5, 0.4, 0.1) (0.4, 0.4, 0.4) (0.5, 0.2, 0.4)	(0.3, 0.4, 0.3) (0.2, 0.4, 0.5) (0.1, 0.5, 0.4)	(0.7, 0.2, 0.1) (0.5, 0.2, 0.4) (0.6, 0.4, 0.1)	(0.3, 0.4, 0.3) (0.4, 0.3, 0.4) (0.3, 0.5, 0.5)

The relation between symptoms and diseases in the form single valued neutrosophic assessments is given in the Table 4 below.

Table 4. The relation between symptoms and diseases

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.6, 0.3, 0.3)	(0.2, 0.5, 0.3)	(0.2, 0.6, 0.4)	(0.1, 0.6, 0.6)	(0.1, 0.6, 0.4)
Headache	(0.4, 0.5, 0.3)	(0.2, 0.6, 0.4)	(0.1, 0.5, 0.4)	(0.2, 0.4, 0.6)	(0.1, 0.6, 0.4)
Stomach pain	(0.1, 0.6, 0.3)	(0.0, 0.6, 0.4)	(0.2, 0.5, 0.5)	(0.8, 0.2, 0.2)	(0.1, 0.7, 0.1)
Cough	(0.4, 0.4, 0.4)	(0.4, 0.1, 0.5)	(0.2, 0.5, 0.5)	(0.1, 0.7, 0.4)	(0.4, 0.5, 0.4)
Chest pain	(0.1, 0.7, 0.4)	(0.1, 0.6, 0.3)	(0.1, 0.6, 0.4)	(0.1, 0.7, 0.4)	(0.8, 0.2, 0.2)

Now by using Eq. (11), Hybrid vector refined similarity measures (HVRSM) by considering α = 0.5 between Relation 1 and Relation 2 are presented as given below (see the Table 5).

Table 5. HVRSM between Relation 1 and Relation 2

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9033**	0.7953	0.7676	0.6809	0.6809
P_2	0.8135	0.7981	**0.8892**	0.8880	0.7446
P_3	**0.8846**	0.7418	0.7959	0.7074	0.6535
P_4	**0.9116**	0.8231	0.8031	0.6898	0.7526

The maximal HVRSM from Table 5 determines the proper medical diagnosis. Therefore, from Table 5, we observe that P_1, P_3, P_4 suffer from viral fever, and P_2 suffers from typhoid.

Also, using Eq. (12), weighted hybrid vector refined similarity measures (WHVRSM) with known weight information w = (0.3, 0.2, 0.15, 0.2, 0.15) and α = 0.5 between Relation 1 and Relation 2 are presented as given below (see the Table 6).

Table 6. Weighted hybrid vector refined similarity measure (WHVRSM) between Relation 1 and Relation 2

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9078**	0.7721	0.7383	0.6533	0.6607
P_2	0.7994	0.8165	**0.8989**	0.8919	0.7909
P_3	**0.8879**	0.7189	0.7664	0.6886	0.6423
P_4	**0.9189**	0.8030	0.7814	0.6788	0.7326

Here, we also see that P_1, P_3, P_4 suffer from viral fever, and P_2 suffers from typhoid.

By using Eqs. (11) and (12), HVRSMs and WHVRSMs with different values of α between Relation 1 and Relation 2 are presented in the following Tables 7-14 and which patient suffers from which disease is indicated by $\rightarrow$ mark below the Tables.

Table 7. HVRSM between Relation 1 and Relation 2 when α = 0.1

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9059**	0.7987	0.7706	0.6904	0.6849
P_2	0.8156	0.8033	0.8917	**0.8931**	0.7467
P_3	**0.8880**	0.7434	0.7976	0.7118	0.6562
P_4	**0.9157**	0.8301	0.8066	0.6979	0.7571

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Stomach problem, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 8. HVRSM between Relation 1 and Relation 2 when α = 0.25

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9049**	0.7974	0.7695	0.6868	0.6834
P_2	0.8148	0.8014	0.8908	**0.8912**	0.7459
P_3	**0.8867**	0.7428	0.7970	0.7102	0.6552
P_4	**0.9142**	0.8274	0.8053	0.6949	0.7554

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Stomach problem, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 9. HVRSM between Relation 1 and Relation 2 when α = 0.75

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9016**	0.7931	0.7658	0.6750	0.6784
P_2	0.8122	0.7948	**0.8876**	0.8848	0.7434
P_3	**0.8825**	0.7408	0.7949	0.7047	0.6517
P_4	**0.9090**	0.8187	0.8009	0.6847	0.7498

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 10. HVRSM between Relation 1 and Relation 2 when α = 0.90

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9006**	0.7918	0.7647	0.6714	0.6769
P_2	0.8114	0.7928	**0.8867**	0.8829	0.7426
P_3	**0.8813**	0.7401	0.7942	0.7030	0.6507
P_4	**0.9075**	0.8161	0.7996	0.6816	0.7482

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 11. WHVRSM between Relation 1 and Relation 2 when α = 0.1

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9136**	0.7756	0.7409	0.6616	0.6641
P_2	0.8014	0.8224	**0.9012**	0.8966	0.7890
P_3	**0.8907**	0.7208	0.7679	0.6926	0.6448
P_4	**0.9233**	0.8170	0.7852	0.6875	0.7408

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 12. WHVRSM between Relation 1 and Relation 2 when α = 0.25

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9114**	0.7743	0.7399	0.6585	0.6628
P_2	0.8006	0.8202	**0.9003**	0.8948	0.7920
P_3	**0.8397**	0.7201	0.7673	0.6911	0.6438
P_4	**0.9217**	0.8162	0.7838	0.6842	0.7378

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 13. WHVRSM between Relation 1 and Relation 2 when α = 0.75

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9041**	0.7698	0.7366	0.6482	0.6585
P_2	0.7981	0.8128	**0.8975**	0.8890	0.7897
P_3	**0.8695**	0.7178	0.7655	0.6861	0.6408
P_4	**0.9162**	0.8138	0.7790	0.6734	0.7274

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Table 14. WHVRSM between Relation 1 and Relation 2 when α = 0.90

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.9019**	0.7685	0.7356	0.6451	0.6572
P_2	0.7974	0.8106	**0.8967**	0.8873	0.7890
P_3	**0.8785**	0.7171	0.7649	0.6846	0.6400
P_4	**0.9145**	0.8130	0.7775	0.6702	0.7243

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

Note 1. By using neutrosophic refined tangent similarity measure, Mondal and Pramanik [33] obtained the results as shown in Table 15.

Table 15. The tangent refined similarity measure between Relation – 1 and Relation – 2 [32]

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	**0.8963**	0.8312	0.8237	0.8015	0.7778
P_2	0.8404	0.8386	**0.8877**	0.8768	0.8049
P_3	**0.8643**	0.8091	0.8393	0.7620	0.7540
P_4	**0.8893**	0.8465	0.8335	0.7565	0.7959

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

From the above Table 15, we found that P_1, P_3, P_4 suffer from viral fever, and P_2 suffers from typhoid.

5. Conclusion

We investigate hybrid vector similarity and weighted hybrid vector similarity measures with single valued refined neutrosophic assessments and proved some of their basic properties. Then, the proposed hybrid similarity measures have been used to solve a medical diagnosis problem. We compare the obtained results with different values of the parameter α and with the results of other existing method in order to verify the effectiveness of the proposed procedure. We hope that the proposed hybrid vector similarity measure can be applied to solve decision making problems in neutrosophic environment such as fault diagnosis, cluster analysis, data mining, investment, etc.

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