

A Stroll around $E=mc^2$ and Planck's Constant

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Since generations it has been taught that the relationship between energy and mass for E-M waves is $E = m c^2$. However, in this paper we will discover that this equation unveils the intrinsic potential energy of the carrier of waves, formerly called 'aether'. We will find the mass of an electron and Planck's constant in terms of the aether's density.

Keywords: Planck's constant, electron, aether

1. The meaning of $E = mc^2$

The classic waves theory is an excellent guidance to help us understanding some concepts in relativity theory and in quantum mechanics. The propagation velocity of a wave along a stretched string in classic theory is given by the following equation [1]:

$$v = \sqrt{\frac{T_l}{\mu_l}} \quad \text{wherein } T_l \text{ stands for the tension}$$

in the string per unit of length of the string, and μ_l for the density per unit of length.

What is meant by the tension? It is the intrinsic potential energy that is ruling the compression and extension of the string, when a wave has been initiated in it.

A wave on an idealized water channel of a certain width b can be represented in the same way, where the surface tension T_s and the density per unit of surface μ_s result in the

$$\text{propagation velocity: } v = \sqrt{\frac{T_s}{\mu_s}}$$

Also for three dimensions, one could imagine an analogous reasoning. The three dimensional tension is in that case the intrinsic

potential energy \mathcal{E} . The equation becomes then:

$$v = \sqrt{\frac{\mathcal{E}}{\rho}} \quad \text{wherein } \rho \text{ is the three-}$$

dimensional density of matter.

Remember that this equation is only describing the intrinsic properties of the medium, whereby the propagation velocity is found. It doesn't describe the wave properties that are initiated through that medium.

Remark that the latter equation has been written in the dimensions of energy density and mass density. Compare them with the equation if the numerator and denominator are multiplied with the unit of volume. Then, we get the equation:

$$v = \sqrt{\frac{E}{m}} \quad \text{wherein } E \text{ is the intrinsic tension,}$$

or its potential energy of contraction and expansion in that unit of volume, and m is the mass of the medium in that unit of volume.

Rearranging, and by changing the notation v by c , we get the well-known: $E = mc^2$. But this equation has nothing to do with transmitted waves themselves. It only determines the propagation velocity in a

medium with an intrinsic (or specific) tension and a mass density.

Below, we will find the transmitted energy of an initiated wave, which of course will differ from the former equations.

2. The energy transmitted by a wave in a string

Consider a small element of a string with a mass Δm . The infinitesimal potential energy of that element depends from its distance to the neutral line, its angular frequency and its infinitesimal mass (Fig.1).

$$\Delta U = \frac{1}{2}(\Delta m)\omega^2 y^2 \quad (1)$$

Herein $\omega = 2\pi\nu$ and ν is the frequency.

That same small element got an infinitesimal kinetic energy depending from its infinitesimal mass and its velocity.

$$\Delta E_k = \frac{1}{2}(\Delta m)v_y^2 \quad (2)$$

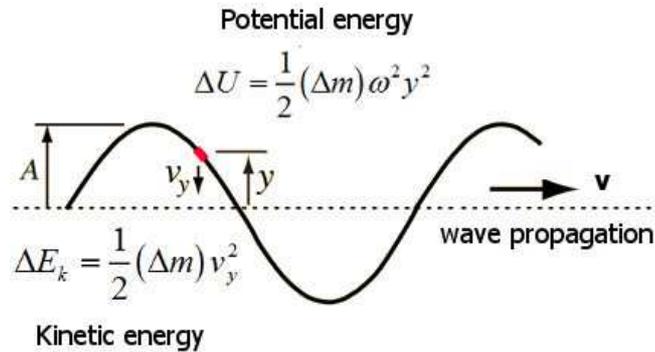


Fig. 1.

It is easy to understand that for a whole wavelength, half the small elements of the string are rather in a high potential stage and half are in a rather high kinetic stage. Moreover, when integrating the potential and the kinetic energy over the total wavelength, the total energy becomes :

$$E_{\lambda \text{ tot}} = \frac{1}{2}(\mu \lambda)\omega^2 (2\pi R A) = \pi \mu R A \lambda \omega^2 \quad (3)$$

wherein λ is the wavelength, A is the wave's amplitude for a maximal y , so that $A/2$ is the average distance y over the wavelength λ ,

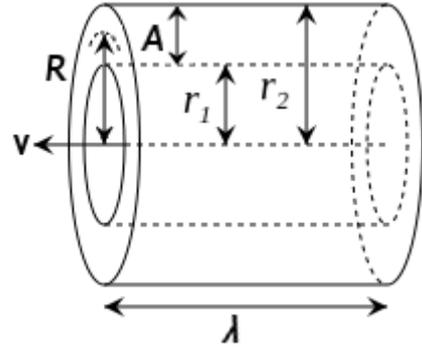


Fig.2.

R is the average radius of the wave, and μ the mass per unit of length.

When extrapolating these equations to a three-dimensional wave carrier, its mass $m = \mu \lambda$ can be written as $m = \rho V$ wherein ρ is the three-dimensional density of the wave carrier.

If we can consider that the maximal amplitude is obtained for half the wavelength while the other half is at its minimal, the dark energy mass (aether mass) of a wavelength can be written in terms of the three-dimensional mass density of the aether:

$$m = \mu \lambda = \rho V = 2\pi R \lambda \rho A/2 = \pi R A \lambda \rho \quad (4)$$

Herein, it is considered that the relevant mass of the wave is limited to the part that is vibrating. The average volume considered is a cylinder with a radius of half the maximal amplitude.

Replacing the mass per unit of length μ from eq. (4) in $E_{\lambda \text{ tot}} = \pi \mu R A \lambda \omega^2$, we get :

$$E_{\lambda \text{ tot}} = \pi^2 \rho R^2 A^2 \lambda \omega^2 \quad (5)$$

Now, we only need to replace the angular frequency ω by the frequency expression $2\pi\nu$, and the wavelength λ by c/ν and we get:

$$E_{\lambda \text{ tot}} = (4\pi^4 \rho R^2 A^2 c) \nu \quad (6)$$

wherein we recognize Planck's constant h between the brackets of the equation.

$$\text{Hence, } h = 4\pi^4 \rho R^2 A^2 c \quad (7)$$

and the mass of the wave is :

$$m_{\lambda} = 4\pi^4 \rho R^2 A^2 \nu / c = 4\pi^4 \rho R^2 A^2 / \lambda \quad (8)$$

wherein the right hand of the equations contains properties of the 'aether', now called 'dark energy', and also contains the amplitude A of the wave.

Since Planck's constant has been observed with phenomena that are related to energy-levels from electrons, it is probable that this constant is related to the electrons' properties. This would be consistent with the presence of the amplitude A .

One has to conclude that the famous equation $E = mc^2$ for an electron should be replaced by :

$$E_{\lambda \text{ tot}} = \left(\frac{4\pi^4 \rho R^2 A^2 \nu}{c} \right) c^2 \quad (9)$$

When we look at $h = 4\pi^4 \rho R^2 A^2 c$, it is interesting to check the dimensions. To analyze this, let us simplify a bit the expression.

We can now eliminate R by considering that the volume inside the radius r_1 (see fig.2) is not

participating to the vibration of the wave. The radius r_1 only determines the size of the vibrating cylinder shell. The minimal value of r_1 is zero. In that case, we get:

$$E_{\lambda \text{ tot, min}} = \left(\frac{\pi^4 \rho A^4 \nu}{c} \right) c^2 \quad (10)$$

$$\text{and } h_{\text{min}} = \pi^4 \rho A^4 c \quad (11)$$

Taking into account the known constants, one can find (ρA^4) , whereof A is the vibration's amplitude value for an electron, while the aether density ρ is expected to be a constant on the Earth's surface, where all the measurements were made.

3. Conclusions

We conclude that the mass of an electron is defined by the aether density, and by both the electron's vibration amplitude and its wavelength.

The question is whether other particles would result in the same amplitude and the same Planck's constant.

4. References

1. Vibrations and Waves, B. Crowell, 2003, ISBN 0-9704670-3-6
2. Classical Mechanics, H. Goldstein, C. Poole, J. Safko, Ed. Addison Wesley