V.V. Dudko* and A. A. Yushkanov**

* *Bauman Moscow State Technical University*

** *Moscow State Regional University*

Temperature effects in second Stokes' problem

Considered the second Stokes's problem about the behavior of rarefied gas filling half-space, when limiting the half-space the plane performs harmonic oscillations in its plane. Continuum mechanics equations with the slip are used. It is shown that in quadratic in the velocity of wall approximation in gas have taken place the temperature effects due to influence of viscous dissipation. In this case there is a temperature difference between the surface of the body and the gas away from the surface.

Key words: the second Stokes's problem, nonlinearity, temperature drop.

The history of the problem of the behavior of gas and liquid over the wall, oscillating in his plane begins with the work of John. G. Stokes equations [1]. This problem commonly called the second Stokes's problem.

The second Stokes's problem in the last years is the subject of many studies [2]–[9]. This is due to the development of modern technology, in particular, nanotechnology.

For a different case the second Stokes's problem equations was studied in [2], in [3] were investigated different phenomena of friction accompanying the process. In the work [4] the analysis of a number of applications of this task have been done.

In [5] have been considered an example of the practical application of vibrational systems similar to the problem in the field of nanotechnology.

In [6] the second Stokes's problem considered in the hydrodynamic approximation in the slip regime.

In the experiments [7] had been studied the gas flow generated by the mechanical resonator at different oscillation frequencies. In the case of low frequencies the problem is solved based on the Navier—Stokes equation. In the case of arbitrary oscillation velocities of the surface numerical methods based on kinetic equations with the collision integral in the BGK form (Bhatnagar, Gross, Krook) had been used.

In gases the second Stokes's problem considered in the works [7]–[11] with using kinetic equations.

Work [8] devoted to application of numerical methods to the solution of the second Stokes's problem in the kinetic approach. In the work [9] to solve the problem was used moment solution method of the kinetic equation.

In the works [10] and [11] an analytical solution of the second Stokes's problem for rarefied gas have been obtained. In these works a model Boltzmann kinetic equation with collision integral in the BGK form has been used. In [10] the problem was solved with diffuse boundary conditions. It is shown that the results of the works [8] and [9] are very close to the results obtained from the analytical solution [10]. Generalization of the obtained results to the case of boundary conditions Cercignani done in the work [10].

In the present work it is shown that the consideration of nonlinear terms in the second Stokes's problem leads to the temperature effects. Earlier this problem without taking into account isothermal slip and temperature jump was considered in [12].

Let the rarefied gas occupies the half space $x > 0$ on a solid flat surface lying in the plane $x = 0$. The surface is (y, z) performs harmonic oscillations along the axis y by law, $u_s(t) = u_0 \cos(\omega t)$. We want to find the temperature difference (temperature drop) between the wall temperature and fluid temperature far from the wall that occurs due to the oscillatory movement of the wall.

We will consider the case of small Knudsen numbers. For the considered problem this corresponds to the case of low frequencies. As shown in [6] this is equivalent to the condition

$$\omega \ll \frac{\nu}{\lambda^2}.$$

Here ν - kinematic viscosity coefficient, λ – the average free path length of molecules in the gas.

When this condition is fulfilled for solving the problem it is possible to use the hydrodynamics equations.

In addition, we assume that the velocity of the wall is a much less then thermal velocity of the gas molecules. When this occurs, there is a small parameter $\varepsilon = u_0/v_T \ll 1$, where $v_T = \sqrt{2kT/m}$ — thermal velocity of the gas molecules.

In the presence of a small parameter ε the problem can be solved by successive approximations.

In the linear approximation for ε, the problem becomes isothermal and Isobaric [13]. The gas velocity has only one component coinciding in direction with the direction of oscillation of the wall. In our case this is the axis y. That is, the gas velocity can be presented as $u = u_y(t, x)$.

The velocity field can be found from the equation [13]

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}. \tag{1}$$

The boundary condition is formulated from the condition that the wall does, as already indicated, harmonic oscillations in its plane. With the account of isothermal slip effect boundary condition on the wall has the following form [14]:

$$u(t, 0) = u_0 \cos \omega t + c_m \lambda \Big(\frac{\partial u}{\partial x}\Big)_{x=0} \tag{2}$$

Wall velocity can be represented in the form

$$u_s(t) = u_0 \cos(\omega t) = \operatorname{Re}\left(u_0 e^{-i\omega t}\right)$$

.

Here u_0 – the amplitude of the wall velocity oscillations.

The solution of equation (1) it is convenient to search in the complex form

$$u = \mathrm{Re}\,(ae^{-i\omega t + k_1 x}). \tag{3}$$

For values of k_1 from equation (1) we obtain [9, 13]:

$$k_1 = \sqrt{-\frac{i\omega}{\nu}} = \sqrt{\frac{\omega}{2\nu}}(i-1) = k(i-1), \quad k = \sqrt{\frac{\omega}{2\nu}}.$$

For values of a with account of the boundary conditions (2) we have

$$a = \frac{u_0}{1 + c_m \lambda k_1} = a_0 u_0, \quad a_0 = \frac{1}{1 + c_m \lambda k_1}.$$

Consequently, the solution of equation (5) has the form

$$u = u_0\,\mathrm{Re}\,(a_0 \exp(-i\omega t + k_1 x)) = u_0\,\mathrm{Re}\,(a_0 \exp[i(kx - \omega t) - kx]). \tag{4}$$

In the linear approximation for ε the temperature of the gas is constant and the thermal conductivity equation is satisfied automatically. In the quadratic approximation for ε the thermal conductivity equation becomes non-homogeneous [13]

$$c_p \rho \frac{\partial T}{\partial t} = \varkappa \triangle T + \eta \Big(\frac{\partial u}{\partial x}\Big)^2. \tag{5}$$

Here $\varkappa$ — coefficient of thermal conductivity of the gas, η — the dynamic viscosity coefficient, ρ — gas density, c_p — specific heat at constant pressure, u — obtained in the linear approximation solution (4).

The last term in (5) corresponds viscous dissipation.

The derivative of the function u (the solution (4)) is equal to

$$\frac{\partial u}{\partial x} = \frac{u_0 k}{2}\Big[a_0(i-1)e^{i(kx-\omega t)-kx} - a_0^*(i+1)e^{i(-kx+\omega t)-kx}\Big].$$

Here the asterisk denotes complex conjugation.

The square of the derivative has the form:

$$\left(\frac{\partial u}{\partial x}\right)^2 = \frac{u_0^2 k^2}{4}\Big[4|a_0|^2 e^{-2kx} - 2ia_0^2 e^{2i(kx-\omega t)-2kx} + 2ia_0^{*2} e^{2i(-kx+\omega t)-2kx}\Big] =$$

$$= u_0^2 k^2 \Big[|a_0|^2 e^{-2kx} - i \frac{a_0^2 e^{2i(kx-\omega t)-2kx} - a_0^{*2} e^{2i(-kx+\omega t)-2kx}}{2} \Big]. \qquad (6)$$

In this case the first summand in the square brackets is constant over time.

Taking into account (6) inhomogeneous thermal conductivity equation (5) will have the following form

$$c_p \rho \frac{\partial T}{\partial t} = \varkappa \triangle T + \eta u_0^2 k^2 \Big[|a_0|^2 e^{-2kx} -$$
$$- i \frac{a_0^2 e^{2i(kx-\omega t)-2kx} - a_0^{*2} e^{2i(-kx+\omega t)-2kx}}{2} \Big]. \qquad (7)$$

The boundary condition for the temperature on the surface taking into account the temperature jump has the form [14]

$$T(0) = T_s + K_t \lambda \Big(\frac{\partial T}{\partial x} \Big)_{x=0}. \qquad (8)$$

Here K_t — temperature jump coefficient, T_s — surface temperature.

The structure of a particular solution of the inhomogeneous equation (7) has the appearance defined by the form (6) of this heterogeneity

$$T = T_\infty + T_0 e^{-2kx} + T_1 e^{2i(kx-\omega t)-2kx} + T_1^* e^{2i(-kx+\omega t)-2kx}. \qquad (9)$$

In the present work we are interested in time-independent difference between surface temperature and temperature of gas away from the surface. Time-independent distribution the gas temperature is described by the first two summands of the solution (9).

Substituting the expression (9) into equation (7) and boundary condition (8), we obtain

$$4k^2 \varkappa T_0 = -k^2 |a_0|^2 \eta u_0^2,$$

$$T_\infty + T_0 = T_s - 2K_t k \lambda T_0.$$

Hence, we find:

$$T_0 = -\frac{|a_0|^2 \eta u_0^2}{4\varkappa}.$$

$$T_\infty - T_s = \frac{\eta |a_0|^2 u_0^2}{4\varkappa}(1 - 2K_t k\lambda).$$

It follows that the temperature difference between the temperature surface and temperature away from the wall (i.e., temperature drop δT) is equal to

$$\delta T = T_\infty - T_s = \frac{\eta |a_0|^2 u_0^2}{4\varkappa}(1 - 2K_t k\lambda) =$$
$$= \frac{\eta u_0^2 (1 - 2K_t k\lambda)}{4\varkappa[(1 + c_m \lambda k)^2 + c_m^2 \lambda^2 k^2]}. \qquad (10)$$

It was considered the case of the gaseous medium. The obtained results remain valid for the liquid medium. It should be noted that in liquid there are no the isothermal slip and the temperature jump, that is, the coefficients of the temperature jump and isothermal slip is equal to zero. Formally, a transition to case of liquid is as the limit $\lambda \to 0$. Then from (14) we obtain

$$\delta T_{liquid} = \frac{\eta u_0^2}{4\varkappa}.$$

Conclusion

The paper discusses the Stokes's second problem about the behavior of gas over oscillating impenetrable surface. It is shown that in the quadratic approximation in the amplitude of the velocity oscillation the problem loses its isothermal character. Near the gas surface a heterogeneous temperature profile emerges. Thus between the surface and the volume of gas there is a constant difference in temperature (temperature jump). Problem is considered in hydrodynamic approximation in the regime with slip.

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