Progresses in Brownian motion of thermal spin defects on non-orientable manifolds with broken inversion symmetry

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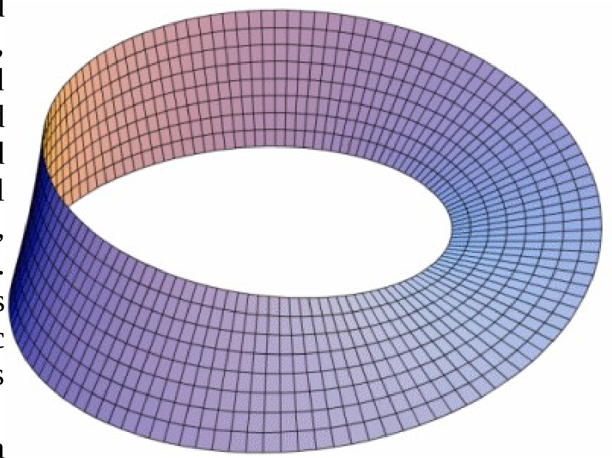
Abstract

Due to insufficient research on the (condensed) matter, we took to ourselves the task of computing the ground state for the Ising model on non-orientable manifolds, this is important because of the recent results regarding broken Lorentz invariance on Condensed matter systems, namely some crystals as seen by Jorge Ranja. By using the Metropolis algorithm, we proved that the ground state for the "simple" case of the Möbius band, contains a spin defect which thermally behaves as a Brownian particle. This is a simple consequence of the breaking of Lorentz Invariance. It can also be seen as the degenerate limit of the Heisenberg model, on a fourth quantized, non-commutative Klein bottle (see Mir Faizal's work). Then, by inserting the resolution of identity, we show that a magnetic field can induce a coherent Brownian wave, which is expected from the Kolmogorov Arnold Moser Theorem. We interpret the results on the light of the theological theory of topological invariants.

Introduction

The literature is saturated with results from simulations of the Ising lattice model, on several dimensions, but there has been no progress regarding the behaviour of this model on non-orientable manifolds. Just to recall, the ising model is defined with discrete variables Si= +-1 , and the Hamiltonian H= -J Sum(Si . Sj) + h Sum(Si) where the first sum is over first neighbours if the neighbours don't complain about loud music. This is practical to implement on a computer and admits easily imposable adequate boundary conditions.

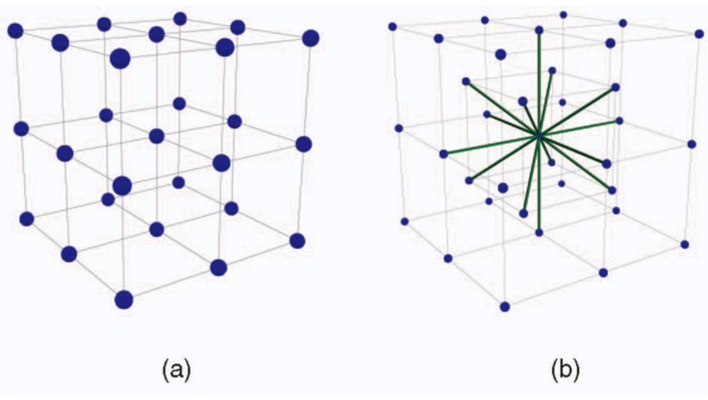
On the other hand, the work by Jorge Ranja has sugested that certain crystals break Lorentz invariance, and as we now, by the Gauss-Bonet theorem in a non-orientable surface, non-orientability is a sufficient condition to break Lorentz Invariance. Surprisingly this has not been explored explicitly by taking simple lattice models on the mobius band which is by far the simplest non-orientable manifold, as is known from elementary geometrical arguments.



Of course, the Mobius Band can be embedded in R^{11}, because of the Whittney embedding theorem, note that the Hausdorff property on the induced topology, is of course Para-compact. Second Countability comes naturaly from the existence and uniqueness of solutions of Partial Differential Equations with analytic Cauchy Boundary conditions, as in the Cauchy-Kovalevskaya Theorem. Considering a point in the Mobius Band(N) P € N is always a regular value of some complex analytic function, usually Riemann's Zeta function. This embedding in 11 dimensions is useful because we can see the mobius behaviour on a Type II B Superstring theory after compactification.

Metropolis algorithm

A key feature of our article is the Metropolis algorithm. This algorithm is a particular example of a more general class of algorithms collectively known as Monte-Carlo logarithms. This is how it works: Our system is a statistical one, so it can be in any spin configuration belonging to a configuration space S. The Metropolis logarithm is a way to run through the whole configuration space. If the system is currently in a state S, then it has a probability P(S, S') to be in state S' in the next iteration. To do this, we grab a spin and try to flip it. Due to inertia, the spin of mass S may be difficult to flip, but with a strong enough tweezer, this can be done. The spin can be modeled as a pendulum, i.e., a massless rod with a weight of mass S. If the weight is down, we say the spin is down. If the weight is up, we say the spin is up. The energy to flip the spin is the energy necessary (and sufficient) to overcome the constant gratvational pull of the Earth on the weight. If the energy of the system after this flip is lower than the previous state, we accept the new state. Otherwise, we accept the state with probability P = numpy.exp(E - E')/KT. This algorithm was a breakthrough in computational physics because it allows us to obtain in finite time what would otherwise only be possible in infinite time.

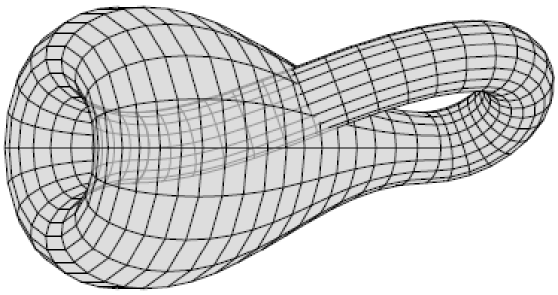


Ising Model

The Ising Model (sometimes erroneously pronounced Aising model, it is actually Eezing, because the scientist was german) is a model originally described by Ising (read Eezing), born in Germany 1900. In each point in a lattice, we put a spin (which can be modeled by a pendulum in a gravitational field) that can be either down or up. There is an interaction energy between adjacent spins because when a spin which was up goes down, it produces wind, which destabilizes the adjacent spins. To adequately describe this interaction, we would need to study the motion of a collection of pendula in a fluid. Therefore, as a first approximation, we simply say that the interaction energy is a fenomenological parameter J between spins. This is a rough approximation, that has important repercussions in the 1d ferromagnetic phase transition. In higher dimensions, this effect is less noticeable because mean-field theories become more and more accurate as the dimension goes up. Ising's Hamiltonian can therefore be described by the following equation:

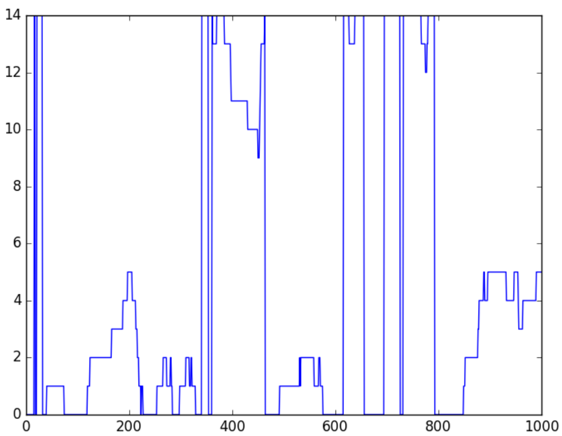
$$H(\sigma) = -\sum_{\langle i\ j\rangle} J_{ij}\sigma_i\sigma_j - \mu\sum_j h_j\sigma_j$$

We have introduced an external magnetic field. This magnetic field cannot be too high, because otherwise the field would interact strongly with the spins, possibly deforming the Möbius manifold in which they are embedded. If the field is strong enough, it is possible that the Möbius strip becomes a Klein Bottle, but there is still not enough experimental data to confirm this. This would be a prime example of a topological phase transition.



Conclusions and Discussion

As we showed, in the case of zero magnetic field, the spin defect behaves Brownianlly as is seen in the graph. This is expected but Lorentz symmetry breaking behaviour is relative, who are we to say that it is broken or not? It depends on the point of view, of course. For example, one time, I accidentally dropped a valuable vase in my grandmother's house, and she said it was broken, but I didn't think it was, so the breaking was certainly relative, as one expects from Einstein's theory. However, from the Schrodinger point of view, the lorentz symmetry can be in a superposition of being broken and not being broken, so we should be careful around cats in boxes.



In the case of a magnetic field, the result is more difficult to interpret, but using the theory of deities as topological invariants it can be seen that the non-brownian part simply comes from the non-trivial Euler characteristic of the Mobius band under a magnetic field.

In the future we expect to obtain results for a finitely generated family of non-orientable manifold including the projective plane Rpn and generalizations of the klein bottle. In what respects the fourth quantized version of this problem, we expect the results to be a beta-deformed non-Abelian gauge theory, but Mir Faizal's work is not yet completely understood by anyone other tha him, so let's hope for the best.

We enjoyed doing this work.

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