

# Does Set Theory Cause Perceptual Problems?

W. Mückenheim

University of Applied Sciences, Augsburg

wolfgang.mueckenheim@hs-augsburg.de

**Abstract** Some results of transfinite set theory are nonsensical. Nevertheless many mainstream mathematicians cannot accept but try to silence any critique. It appears as if a mass-psychosis has infected them.

## Facts

Transfinite set theory has been invented by Cantor. Cantor has emphasized over and over again that his teaching is based upon the exhaustibility of infinite sets. The complete indexing of rational numbers, for instance, causes a considerable backlog because in the first  $n$  unit intervals of the real line there are infinitely many rational numbers but only  $n$  natural indices. This situation persists during the whole real line. To believe that nevertheless all rational numbers can be indexed, requires to believe in strange things unless a strict denial of any analytical approach is assumed by the attitude that "everything happens simultaneously and yields the desired result".

The first to recognize *and talk about* this problem in depths was A. Fraenkel, a leading proponent of set theory (the F in ZFC). He writes: "Well known is the story of Tristram Shandy who undertakes to write his biography, in fact so pedantically, that the description of each day takes him a full year. Of course he will never get ready if continuing that way. But if he would live infinitely long (for instance a 'countable infinity' of years), then his biography would get 'ready', because every day in his life, how late ever, finally would get its description. No part of his biography would remain unwritten, for to each day of his life a year devoted to that day's description would correspond." [1]

Here the ratio of infinity-to-one, occurring in the enumeration problem, is already reduced to about 365.25-to-1. But I prefer an even simpler ratio: Every day Scrooge McDuck receives 10 \$ and issues 1 \$. Since a comic character lives forever his wealth grows immeasurably. However if he issues always the dollars received first and if he applies transfinite set theory, then he will go bankrupt. [2] Somehow he will lose all his money like the rational numbers without index will somehow vanish – "in the limit".

Another story concerns the little demon who resides in a home with two rooms. In the morning he leaves his bedroom for his living room, and in the evening he returns to his bedroom. This continues in eternity because demons are immortal. There is no problem, no paradox and no "final result". A similar story, often misunderstood as a paradox, has been told by Thomson

already. [3] However, if we apply set theory, then the number of the little demon's returns to the bedroom will become exhausted. [4] His bedroom will remain empty. Alas the number of returns to the living room will become exhausted too, so the living room will also remain empty. The little demon has disappeared. Note that the sequence of *days* of the little demon is *monotonically increasing*. So the often heard excuse about non-monotonicity does not apply.

## Conclusion

These results are due to the exhaustibility of infinite sets, the so-called set-limit [5], which is the foundation of transfinite set theory which in turn is considered as the foundation of modern mathematics by most mathematicians: Those many which are not experts in this field simply believe in the expertise of the experts. These comparatively few experts however have to accept exhaustibility because otherwise the important theorems of set theory collapse and the idea of "countable set" becomes untenable – and in addition a great deal of their life's work.

Obviously these results are nonsensical. But how could this credo-in-absurdum-attitude evolve? It appears as if a mass-psychosis infects students of set theory. A special section in the brain must be disabled during the study. Apparently this infect does not impair intelligence in general but it prevents any perceptibility of the ridicule of the facts explained above.

As experience shows the chances are good to save students if they are warned in time. I conclude this from hundreds of my own students who with rational scepticism refuse to accept these "results". I teach them set theory. But before the lessons can become fixed I tell them the nonsensical facts and refer to classical analysis as the only arbiter of good taste in mathematics. So they know and apply it to McDuck's wealth: It is the (improper) analytical limit of the sequence  $(9n)_{n \in \mathbb{N}}$ , namely  $9n \rightarrow \infty$ . And they know that the infinite cannot be exhausted.

## References

- [1] [A. Fraenkel: "Einleitung in die Mengenlehre" 3rd ed., Springer, Berlin \(1928\) p. 24](#)
- [2] [W. Mückenheim: Transfinity - A Source Book \(2017\) p. 252](#)
- [3] [J.F. Thomson: "Tasks and super-tasks", Analysis 15 \(1954\) 1-13](#)
- [4] [W. Mückenheim: Transfinity - A Source Book \(2017\) p. 21](#)
- [5] A sequence  $(M_n)$  of sets  $M_n$  has a limit  $\text{Lim } M_n$  if and only if

$$\text{Lim } M_n = \text{LimSup } M_n = \text{LimInf } M_n$$

where

$$\text{LimSup } M_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} M_k \quad \text{and} \quad \text{LimInf } M_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} M_k .$$