On the Riemann Zeta Function

Jonathan Tooker
Dunwoody, Georgia, USA, 30338
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We discuss the Riemann zeta function and make an argument against the Riemann hypothesis. While making the argument in the classical formalism, we discuss the argument as it relates to the theory of infinite complexity. We extend Riemann's own (planar) analytic continuation $\mathbb{R} \to \mathbb{C}^2$ into (bulk) hypercomplexity with $\mathbb{C}^2 \to {}^\star\mathbb{C}$.

Consider the analytic continuation of the Dirichlet series

$$\mathcal{D}(z) = \sum_{N=1}^{\infty} \frac{1}{N^z} \quad , \qquad \text{with} \qquad z > 1, \; z \in \mathbb{R} \quad , \tag{1}$$

onto the entire complex plane via the Riemann zeta function

$$\zeta(\mathcal{Z}) = \left(\frac{\displaystyle\int_0^\infty \frac{x^{(\mathcal{Z}-1)}}{e^x - 1}\,dx}{\displaystyle\int_0^\infty \frac{x^{(\mathcal{Z}-1)}}{e^x}\,dx} \right) , \text{ with } \mathcal{Z} \in \mathbb{C}^2, \mathcal{Z} \neq 1 \quad ,$$

Note that the domain of ζ is different than the extended complex plane $\hat{\mathbb{C}}$ which is defined as

$$\hat{\mathbb{C}} \equiv \mathbb{C}^2 \cup \hat{\infty} \quad . \tag{2}$$

$\mathcal{Z}$ only samples from finite values. The object $\hat{\infty}$ is complex infinity so $\hat{\mathbb{C}}$ contains four values that are not in $\mathbb{C}^2$. These are $\pm\infty$ and $\pm i\infty$, and when the Riemann sphere is constructed from $\mathbb{C}^2$, all of these points are mapped to one polar null point: $\theta = \pi$ in the familiar polar coordinates. If the domain of $\zeta(\mathcal{Z})$ was $\mathbb{C}^2$ then we would expect to be able to calculate $\zeta(\infty)$ but we have no such expectation. $\zeta(\infty)$ is neither well defined nor an object we will consider. The domain of $\zeta(\mathcal{Z})$ is the non-extended complex plane as in figure 1; it is the Cartesian plane with one real axis and one imaginary axis.

The Riemann hypothesis says that all of the non-trivial zeros of $\zeta(\mathcal{Z})$ are such that if $\zeta(\mathcal{Z}_n)$ is a non-trivial zero then

$$\text{Re}(\mathcal{Z}_n) = \frac{1}{2} \quad , \tag{3}$$

and in 1914 Hardy proved that there are an infinite number of of these non-trivial zeros

$$\mathcal{Z}_n = \frac{1}{2} + it \quad , \qquad \text{with} \qquad t \in \mathbb{R} \quad . \tag{4}$$

In this paper we will make an argument that there should also exist at least one nontrivial zero

$$\zeta(z_{\bar{n}}) = 0 \quad , \qquad \text{with} \qquad \text{Re}(z_{\bar{n}}) \neq \frac{1}{2} \quad . \tag{5}$$

Since the domain of definition of $\zeta(\mathcal{Z})$ is a subset of the extended complex plane, we can map the complex plane and the details shown in figure 1 onto the surface of a 2-sphere. We simply do not include the the polar point that holds $\hat{\infty}$ so that the polar angles are

$$\theta \in [0, \pi) \quad , \qquad \text{and} \qquad \phi \in [0, 2\pi) \quad . \tag{6}$$

The azimuthal angle ϕ cannot contain both points 0 and 2π because then that point would be double valued. The zenith angle θ would contain π if we were using the extended complex plane, but we are not so it is removed. Even if we did include it, there would be the other problem about it being infinitely multiply valued and $\theta = \pi$ would be the same point for any given ϕ. If we did need to include $\theta = \pi$ we would simply introduce a second coordinate chart per the standard prescription but we do not need to do that, and we will work with a single coordinate chart on the surface of the 2-sphere. These are the coordinates in equation (6). The infinite extent of the Cartesian coordinates in figure 1, stretching from the origin out infinity, are condensed onto the surface of the 2-sphere with conformal coordinates whose precise definitions are irrelevant. This is all totally standard. All that is required to move the domain of $\zeta(\mathcal{Z})$ onto the

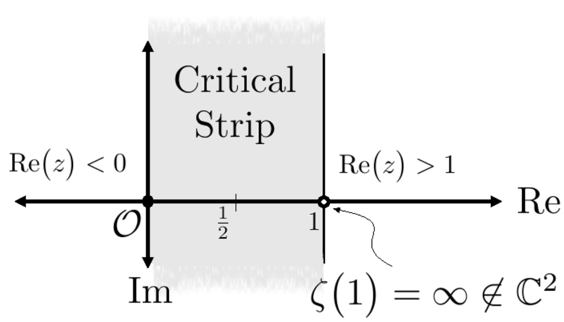

FIG. 1. This figure shows the features of the complex plane that are most relevant to the Riemann zeta function.

sphere is to make a conformal change of coordinates in the ordinary fashion.

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The object $\hat{\infty}$ is complex infinity so $\hat{\mathbb{C}}$ contains four values that are not in $\mathbb{C}^2$. These are $\pm\infty$ and $\pm i\infty$, and when the Riemann sphere is constructed from $\mathbb{C}^2$, all of these points are mapped to one polar null point: $\theta = \pi$ in the familiar polar coordinates. It is our belief that the four-to-one multiplicity here described is certainly related to the ontological resolution of the identity

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{\imath} \quad . \tag{7}$$

We define an avenue for chronos and chiros to act independently when we act on a qubit with $\hat{2}$. Then on the chirological copy of the qubit we act again to replace the ordinary concept of orthonormalism. The traditional idea of orthogonal fields usually relies on a phase difference of $\pi/2$ in the sines and cosines that make use of $\mathbb{C}^2$ through the Euler's identity

$$e^{i\theta} = \cos\big(\theta\big) + i\sin\big(\theta\big) \quad . \tag{8}$$

We have physically motivated $\hat{2}$ in quantum mechanics by only making a connection between non-relativistic quantum theory and QFT. Although the ontological basis $\{\hat{\imath}, \hat{\sqrt{2}}, \hat{\Phi}, \hat{\pi}\}$ has four channels like the Dirac vector, we have been able to fully describe non-relativistic quantum theory with only $\hat{\Phi}$ and $\hat{\pi}$ because in that limit the third and forth components, $\hat{\sqrt{2}}$ and $\hat{\imath}$, which are directly proportional to the special relativistic factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad , \tag{9}$$

all vanish in the nonrelativistic limit of the theory, always.

In the limit of mathematical analysis for physics, they never vanish due to the finite precision of the relevant testing apparati and, and even when $\beta \to 0$ is imposed as analytical constraint, the all topological channels of the spinors and the bispinor in the Dirac vector exist in the ensemble that describes the statistical mechanics. The present analysis regards determinism and we will shy away from stochasticism because it is not immediately relevant to the argument about the Riemann hypothesis.

If we did need to extend the TOIC connection to QFT, which we do not because QM is already connected to QFT, then we would have to operate with $\hat{2}$, and make four copies of the qubit by the commutativity of multiplication

$$\hat{2} \times \hat{2} = \hat{1} + \hat{1} + \hat{1} + \hat{1} \equiv 4 \quad . \tag{10}$$

We can achieve this same operation with

$$\hat{2} + \hat{2} = \hat{1} + \hat{1} + \hat{1} + \hat{1} \equiv 4 \quad . \tag{11}$$

The symmetry between the additive and multiplicative properties of $\hat{2}$ is similar, in some way to the unique symmetry of $\hat{\Phi}$

$$\hat{2} = \hat{1} + \hat{1} \equiv 2 \in \mathbb{Z} \tag{12}$$

$$\hat{\Phi}^2 = \hat{\Phi} + \hat{1} := \Phi^{n+\Delta} + \Phi^{n-1+\Delta} \tag{13}$$

Now also consider the four primary objects in $\zeta(\mathcal{Z})$. The numerator in each integrand is identical so we see the O(1,3) topology emerging. Take one qubit and operate act with $\hat{2}$ twice to make four objects. We separate chronos and chiros by making the direct and inverse sectors as in $\zeta(\mathcal{Z})$. Then we act with $\hat{2}$ on the entire system to make direct and inverse sectors within each integrand. Then we conjure the identity from nowhere, which we can do because it is an axiom of the theory that the identity exists, and then decompose it onto the ontological basis, and then put each of these four identical qubits onto the orthogonal but not orthonormal ("not normal" and "non-unitary" mean the same thing in this sense.).

Why do we say that the four qubits are identical? Everything we have said about making them direct or inverse relates to their arrangement inside $\zeta\mathcal{Z}$. The slight "-1" phase shifts, which are all on the scale of any of the ontological vectors, have an obvious association with the non symmetry of ζ's critical strip existing in the positive half plane and not being centered on the origin of coordinates in $\mathbb{C}^2$. Then we can say they are identical because after we operate with $\hat{2}$ twice, we would also have to apply some translation operations to achieve the analytical form of the double-double direct and inverse qubits in ζ. We can put the four identical qubits onto the four orthogonal basis vectors after $\hat{2}$ but before the translation operators. Perhaps the fullest utility of the ontological basis will be solve for a new completely new analytic continuation. Although we have written equation (2), it is only an integral representation of the complete analytic continuation of $\mathcal{D}(z)$. The Riemann hypothesis is an interesting problem because it is known that we can't be sure if we have the correct functional form of the true analytical continuation. Perhaps the ontological resolution of the identity, equation (7), is that form and if we can show the existence of one zero $\mathcal{Z}_{\bar{n}}$ not on the critical line, then we will have disproven Riemann's hypothesis. We will certainly be happy to accept the million dollars but if it means that this writer would have to compromise on his personal ethical standards then he will be happy to show solidarity with Perelman letting it be written even more clearly than it already is: people who do the biggest things don't do them for money.

If we are to replicate $\zeta(\mathcal{Z})$ in the ontological basis, then we should first consider its domain. This is very interesting because it says the identity is not in the domain of $\zeta(\mathcal{Z})$. This is induced as an axiom that it is there. Where $\hat{\pi}$ points in the 3 directions of space, let $\hat{\Phi}$ be a vector located at $\mathcal{Z}=1$ which points in the direction of the arrow of time. **Spacetime is emergent**. Chronos and chiros are different, but we have required that the chronos component defined on $\hat{\pi}$ contain the ordinary theory. We simply aim to explain it better on the three chirological channels in the **timecube** $\{\hat{2},\hat{\Phi},\hat{\imath}\}$. We have most recently used the idea of inserting vector objects that come predefined with their own locations to fill in the null points in certain topological configurations that make certain operations cumbersome. By doing this, in reference [1], we have shown a good place to look for the precise origin of the fine structure constant α_{MCM}.

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Here we will make a disambiguation regarding what we have called the Riemann sphere. The Riemann sphere is a 2-sphere whose two dimensions are those of both $\mathbb{C}^2$ and $\hat{\mathbb{C}}$. However, for the mapping operation from planar $\mathbb{C}^2$ to the 2-sphere $\mathbb{S}^2$ to work perfectly, we have to make sure they have exactly the same topology. The domain of $\mathcal{Z}$ in equation (2), even when we include the ring at infinity $\hat{\infty}$, there is still this point at $\mathcal{Z}=1$. It means that if we sample $\mathcal{Z}$ from $\mathbb{C}^2$ in general with $\mathcal{Z}=1$, the ζ will not be a map from $\mathbb{C}^2$ everywhere in its domain $\mathbb{C}^2$. This is a critical element of what it means to be an analytic function in complex analysis. Therefore we should first complete $\mathbb{C}^2$ with $\hat{\infty}$, which is ring at infinity with U(1) topology that introduces a new degree of freedom θ which appears in equation (XXX). To complete the domain, we insert $\hat{\Phi}$ and say that it is an operator that has an odd (chirological) coordinate transformation that does something with the value $\mathcal{Z}=1$, such as storing it instead of using it to produce a divergent value. Then we expect that the topology of the 2-sphere will have interesting, and likely novel interactions with the rectangular topology of the time cube.

When we start with a classical qubit we need to unsuppress the $\hat{\pi}$ vector and then project it onto $\hat{\Phi}$ to do the math trick. $\hat{\pi}$ is the arrow of space so it gets projected onto $\hat{\Phi}$ three times as space gets contracted onto time for translation through the hypercomplex bulk. That bulk comprises is both the interior and exterior of the 2-sphere. There is no way to invert $\mathbb{S}^2$ so that the interior and exterior may be permuted, but the sphere theorem proves that this is possible for

$$\mathbb{S}^j \quad , \qquad \text{with} \qquad j>3, j\in\mathbb{Z}^+ \quad . \tag{14}$$

Riemann began with real analysis in $\mathbb{R}$ and saw he could use $\mathbb{C}^2$ Dirichlet series in the complex where it had bee

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Figure 1 shows a region of $\mathbb{C}^2$ near its origin of coordinates $\mathcal{O}$.

Also, the reader should note that we are using the definition of the Riemann sphere that includes $\hat{\infty}$ at $\theta=\pi$ and this contrasts other work where we have used the

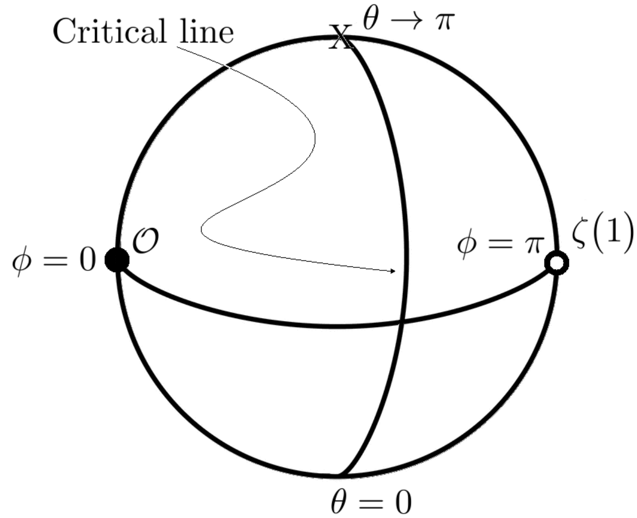

FIG. 2. We put the origin of the domain of ζ at the point $(\pi/2,0)$ in the (θ,ϕ) coordinates. The point $z=1$ goes to $(\pi/2,\pi)$. The line that connects $z=0$ and $z=1$ in the plane becomes the semicircle that goes through $\theta=0$; the horizontal line in figure 1 is not to be considered the "equator" shown here, at least for the purposes of this argument. We have brought forward the black and white dots from figure 1 and added an X to show the Reimann sphere's null point.

name "Riemann sphere" to describe the sphere without $\theta=\pi$ included. It should be clear from the context which object is being used, and we

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Now that we have decided to move the function's domain onto a sphere we need to decide how the coordinates of the plane will be oriented with respect to the polar angles. If we were going to define conformal coordiantes to condense the infinite extent of $\mathbb{C}^2$ onto the finite surface of $\mathbb{S}^2$, that would be the next step after defining the orientation. For that, consider figure 1 again. The function ζ is a map from $\mathbb{C}^2$ to $\mathbb{C}^2$ for every point in $\mathbb{C}^2$ except $z=1$. At that point ζ maps an element of $\mathbb{C}^2$ onto an element of $\hat{\mathbb{C}}^2$ as shown in figure 1. The argument presented in this paper mostly examines the relationship between this odd point $z=1$ and the null point on the Riemann sphere. Note that these cannot be the same point; the open boundaries of the complex plane at infinity will get mapped onto the open boundary of $\theta\in[0,\pi)$ after we project onto the sphere with the above mentioned conformal coordinate transformation.

So... to begin we take the coordinates of the Riemann sphere, ϕ and θ and we will say where the interesting points from figure 1 should go in these coordinates, and this is illustrated in figure 2.

Here we will reduce the argument that all the zeros lie on the line where $\mathrm{Re}(z)=1/2$ to a symmetry argument. Due to the style of the analytic continuation of ζ onto the entire complex plane, all the zeroes will lie on the critical line because it is exactly in the middle of critical strip.

Where Hardy has shown that there are an infinite number of zeros of the form of equation XXX, by symmetry, all those zeros lie on the great circle that passes through $\phi = \pi/2$ and $\phi = 3\pi/2$. We can squeeze an infinite number of them onto that circle, spaced arbitrarily far apart, because of the conformal coordinates. The upper half of the critical line in figure 1 is mapped to the semicircle with $\theta \in [0, \pi)$ at $\phi = \pi/2$, and likewise for the lower half of the critical line at $\phi = 3\pi/2$.

The null point on the Riemann sphere makes it easy to map the infinite extent of the imaginary line onto the vertical meridian. At the point $\theta = 0$, which is the point labeled $1/2$ in figure 1, there is a distance of π radians to the reach null point X in any direction so the conformal coordinates are totally symmetric. However, also consider that we must map the infinite extent of the real line onto the sphere.

Figure three clearly shows how the symmetry argument about the critical line, which is the meridian shown in figure 3, breaks down. Also consider if there was an asymmetry like the one shown in figure 3, then the critical line would simply be the one circle that does respect symmetry. This is also problematic because that could no longer be a great circle, meaning that the circle would no longer pass the null point X, and then positive imaginary infinity would connect to negative imaginary infinity somewhere around the star in figure 3. This is also not allowed because it violates the definition of of the domain of ζ as the non-extended complex plane $\mathbb{C}^2$. However, there are some well known issues associated with the idea that

$$\frac{\infty}{2} \neq \frac{\infty}{2} - 1 \quad , \tag{15}$$

so we will improve the argument.

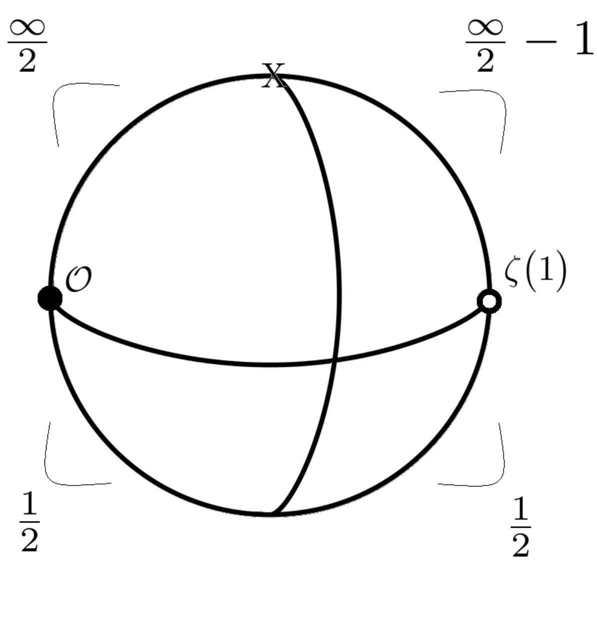

FIG. 3. XXXXXXXXXXXXXXXXXX.

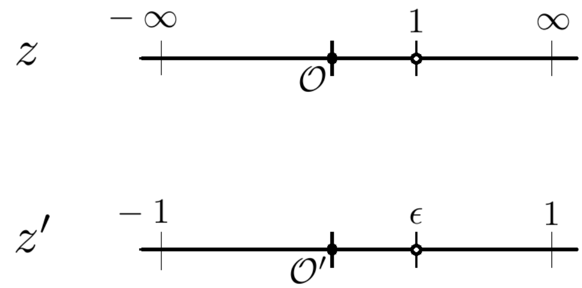

FIG. 4. Now we have redefined the coordinates so the critical line at $z = 1/2$ lies at $z' = \epsilon/2$.

From real analysis, it possible to claim that the complex plane exists solely on the basis that everything about real analysis that is true (most of it anyway), must also be true for complex numbers whose imaginary part is zero. We can also extend the reals to the hyperreals in this way by claiming that any real analysis that is true must also be true if that analysis was carried out on infinitely smaller but completely self-similar domain.

Therefore we should introduce a new hyperreal coordinate system on figure 1 that says the distance from the origin to $z = 1$ is an infinitesimal ϵ, and that $\pm\infty$ lie at ± 1. This means the new coordinates are defined on a higher tier of hyperreal infinitude than the previous coordinates. This gives us figure 4.

Now everything that was true

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In the plane we know that any off-critical line zeros will be symmetric about the critical line, but on the sphere they are symmetric about the critical line and the real line. Both of these symmetries must be respected due to the spherical symmetry.

However by going to the

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Actually as soon as we convert to figure 5 we can say that period three in the U(1) gauge symmetry implies chaos, and then there must be infinitely many points in the critical strip that are zeros such that $\mathrm{Re}(\mathcal{Z}) \neq 1/2$.

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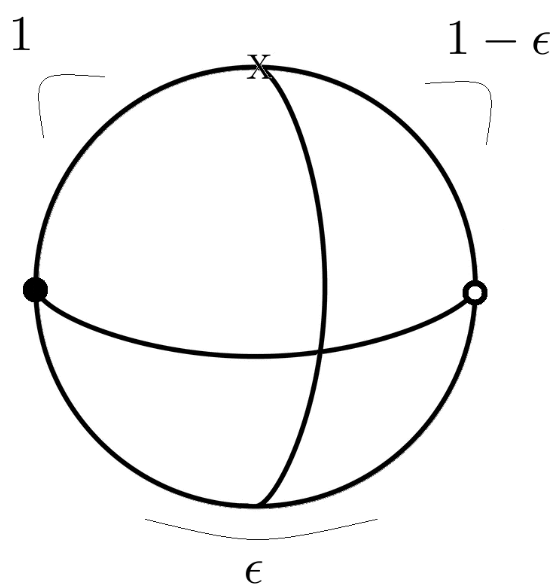

FIG. 5. Now we have redefined the coordinates so the critical line at $z = 1/2$ lies at $z' = \epsilon/2$.

[1] Jonathan W. Tooker. Infinitudinal Complexification. *viXra:1608.0234*, (2016).