Bell's theorem refuted in our locally-causal Lorentz-invariant world

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Abstract: Adjusting EPR (to accord with Bohr's insight), and accepting Bell's principles (but not his false inferences), we:— (i) complete the QM account of EPR correlations in a classical way; (ii) deliver Bell's hope for a simple constructive model of EPRB; (iii) justify EPR's belief that additional variables would bring locality and causality to QM's completion; (iv) refute key claims that such variables are impossible — including CHSH, Mermin's three-particle *always-vs-never* variant of GHZ, and this: in the context of Bell's theorem 'it's a proven scientific fact that a violation of local realism has been demonstrated theoretically and experimentally,' *(Annals of Physics* Editors, 2016). In short: we refute Bell's theorem and endorse Einstein's locally-causal Lorentz-invariant worldview.

Keywords: Bell's errors, causality, CLR, completeness, EPRB, equivalence, GHZ, locality, realism

Notes to the Reader: (i) All paragraphs and equations are numbered to facilitate discussion, improvement, correction. (ii) Texts freely available online (see References) are taken as read. (iii) The term particle is used in accord with convention. (iv) All results here accord with QM and experiment. (v) Taking math to be the best logic, it may flow for several lines before we comment. (vi) Using Bell (1964), CHSH (1969), Mermin (1990): we refute every Bellian claim known to us. Such claims include:

> Wiseman (2005:1): Bell (1964) "strengthened Einstein's theorem (but showed the futility of his quest) by demonstrating that either reality or locality is a falsehood." Goldstein *et al.* (2011:1): "In light of Bell's theorem, [many] experiments ... establish that our world is non-local. This conclusion is very surprising, since non-locality is normally taken to be prohibited by the theory of relativity." Maudlin (2014:25): "Non-locality is here to stay ... the world we live in is non-local." Gisin (2014:4): "For a realistic theory to predict the violation of some Bell inequalities, the theory must incorporate some form of nonlocality." Norsen (2015:1): "In 1964 Bell demonstrated the need for non-locality in any theory able to reproduce the standard quantum predictions. ('Non-locality' here means a violation of a generalized prohibition on faster-than-light causal influences.)" Bricmont (2016:112): 'There are nonlocal physical effects in Nature.'

1 Introduction

> 1.0. Bell's theorem: "In a theory in which parameters are added to [QM] to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant," Bell (1964:199). In the context of Bell's theorem – according to a unanimous claim by *Annals of Physics* Editors (2016:67) – it's 'a proven scientific fact that a violation of local realism has been demonstrated theoretically and experimentally'. Dissenting, we refute both statements.

1.1. Studying EPR (1935) in the context of EPRB – the EPR-based experiment in Bohm & Aharonov (1957) – Bell (1964) claimed that EPR's program required a grossly non-local mechanism; see ¶1.0.

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1.2. Challenging such views, we find a Bellian inference that is false in quantum settings. Now from d'Espagnat (1979:158), endorsed by Bell (1980:7), we accept these principles of local realism: (i) *realism* – regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers; (ii) *locality* – no influence of any kind can propagate superluminally; (iii) *induction* – legitimate conclusions can be drawn from consistent observations. So this is not a dispute about differing principles. Rather, we show that Bell and d'Espagnat fail under (iii): ie, ignoring consistent observations re the validity of QM and Bohr's well-known insight, they draw conclusions that are quantum-incompatible in general and illegitimate under EPRB (the focus of their study).

1.3. In contrast, we identify our quantum-compatible position as *commonsense* local realism (CLR), the union of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively). Then – taking realism to include the view that physical reality exists and has definite properties – we advance EPR's program by validating their belief:

> "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible," EPR (1935:780). It is, as we show.

1.4. At the core of our approach – compatible with Bell/d'Espagnat at ¶1.2 and EPR/EPRB – is CLR's sufficient condition for an element of physical reality: 'If, without in any way disturbing a system, we can predict with adequate accuracy the result of a test (which may be a disturbance) on that system, then elements of physical reality mediate this result,' after Watson (1998:417). In 3-space, this sufficient condition delivers Bell's hope (2004:167) for 'a simple constructive model' of EPRB.

1.5. We proceed – via Analysis, Conclusions, Appendix, References – to deliver the Abstract.

2 Analysis

> 2.0. Einstein "argued that the EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way," Bell (2004:86). EPR suggest that a state, 'richer in content than the quantum state, would provide a commonsense explanation of certain perfect correlations predicted by QM, which are otherwise baffling,' after GHSZ (1990:1131). Agreeing, we analyze EPR in the context of EPRB and deliver both.

2.1. Under CLR-completeness, here's our idealization of every relevant element of the subject reality, including 3-space (since time and gravity are irrelevant here):

$$.A_i \equiv +1 \leftarrow \Delta^{\pm}_{\hat{a}} \Leftarrow q(\lambda_i) \ \lhd E \rhd \ q(\mu_i) \Rightarrow \Delta^{\pm}_{\hat{b}} \rightarrow +1 \equiv B_i. \tag{1}$$

$$.A_i \equiv +1 = \hat{a}\cdot\hat{a}^{+} \leftarrow [\hat{a}\cdot *] \Leftarrow q(\hat{a}^{+}) \leftarrow \delta^{\pm}_{\hat{a}} \Leftarrow q(\lambda_i) \ \lhd E \rhd \ q(\mu_i) \Rightarrow \delta^{\pm}_{\hat{b}} \rightarrow q(\hat{b}^{+}) \Rightarrow [\hat{b}\cdot *] \rightarrow \hat{b}\cdot\hat{b}^{+} = +1 \equiv B_i. \tag{2}$$

$$\llbracket \quad \text{Alice's locale} \quad \rrbracket \qquad \llbracket \text{Source} \rrbracket \qquad \llbracket \quad \text{Bob's locale} \quad \rrbracket \tag{3}$$

$$\lambda_i + \mu_i = 0;\ i = 1, 2, ..., n.\ A_i(\hat{a}, \lambda_i) = -B_i(\hat{a}, \mu_i);\ \text{etc.}\ P(\lambda_i = \lambda_j \,|\, E, i \neq j) = 0;\ \text{etc.} \tag{4}$$

$$.A_i \equiv +1 = \hat{a}\cdot\hat{a}^{+} \leftarrow [\hat{a}\cdot *] \Leftarrow q(\hat{a}^{+}) \leftarrow \delta^{\pm}_{\hat{a}} \Leftarrow q(\lambda_i) \ \lhd E \rhd \ q(\mu_i) \Rightarrow \delta^{\pm}_{\hat{a}} \rightarrow q(\hat{a}^{-}) \Rightarrow [\hat{a}\cdot *] \rightarrow \hat{a}\cdot\hat{a}^{-} = -1 \equiv B_i. \tag{5}$$

2.2. (1) shows experiment E and a test on (a decoherent interaction with) each member of the i-th particle-pair; thick arrows ($\Rightarrow$) denote movement toward an interaction, thin arrows ($\rightarrow$) point to the subsequent output. With pristine spin-related properties λ_i and μ_i, spin-$\frac{1}{2}$ particles $q(\lambda_i)$ and $q(\mu_i)$ emerge from a spin-conserving decay $\lhd E \rhd$ such that (4) holds. Each particle interacts with a dichotomic linear-polarizer-analyzer $\Delta^{\pm}_{\hat{x}}$ – freely and independently operated by Alice (with result A) and Bob (result B) – where $\hat{x}$ denotes a relevant orientation of its principal-axis in 3-space. Since A and B are discrete (± 1), we employ discrete variables λ_i, μ_i. This choice accords with our understanding of

EPRB, and with Bell's (1964:195) indifference to whether such variables are discrete or continuous.

2.3. Identifier i is used generically: but each particle may be tested once only in its *pristine* state, and thereafter until absorbed in an analyzer. Since the tests are locally-causal and spacelike-separated, we hold fast to this principle of local causality: the real factual situation of $q(\mu_i)$ is independent of what is done with $q(\lambda_i)$ which is spatially separated from $q(\mu_i)$ – and vice-versa – per Bell (1964: endnote 2; citing Einstein). Nevertheless, consistent with this principle, test outcomes are correlated via (4).

2.4. (2) expands on (1) to show that each polarizer-analyzer $\Delta^{\pm}_{\hat{x}}$ is built from a polarizer $\delta^{\pm}_{\hat{x}}$ and a removable analyzer $[\hat{x}\cdot *]$ which responds to the polarization-vector $(*)$ of each post-polarizer particle $q(*)$ that it receives. We assign the correct polarization $\hat{x}^{\pm}$ to $q(*)$ by observing the ± 1 output of the related analyzer; or by understanding the nature of polarizer/particle interactions. So experiment E is EPRB – per Bell (1964) – with this benign finesse: to facilitate additional analysis and experimental confirmation, we can employ additional polarizers $(\delta^{\pm}_{\hat{y}})$ to test $q(\hat{x}^{\pm})$; and $\hat{y}$ may equal $\hat{x}$.

2.5. (3), with no symmetry requirements, shows the locales for Alice and Bob arbitrarily spacelike-separated from each other and from the source. Given that (1)-(5) hold over any spacelike separation, it follows that the relevant particle properties here are stable between emission and interaction with a polarizer. Further, our theory is locally-causal and Lorentz-invariant because A_i and B_i are locally-caused by precedent local events $\delta^{\pm}_{\hat{a}}q(\lambda_i)$ and $\delta^{\pm}_{\hat{b}}q(\mu_i)$ respectively, which are spacelike-separated.

2.6. (4) shows λ_i and μ_i pairwise correlated via the conservation of total angular momentum. These CLR-based variables provide a more complete specification of particle-pairs under E. Let these pristine spin-related properties to be random multivectors in 3-space (each incorporating intrinsic spin $\boldsymbol{s}$); then for us, under our conservatism, it's probability zero that any two pristine particle-pairs are the same.

2.7. (5) shows experiment E with Alice and Bob having the same polarizer setting $\hat{a}$. Thus, as an idealized example: a physicist, observing one result, may predict the other result with certainty. Here's how Alice predicts Bob's result after observing $A_i = +1$; and vice-versa, with Bob observing $B_i = -1$:

$$A_i = +1 \because q(\lambda_i) \Rightarrow \delta^{\pm}_{\hat{a}} \rightarrow q(\hat{a}^{+}) \Rightarrow [\hat{a}\cdot *] \rightarrow [\hat{a}\cdot\hat{a}^{+}] = +1. \therefore -\text{ using (4) }- \tag{6}$$

$$q(\mu_i) = q(-\lambda_i) \Rightarrow \delta^{\pm}_{\hat{a}} \rightarrow q(\hat{a}^{-}) \Rightarrow [\hat{a}\cdot *] \rightarrow [\hat{a}\cdot\hat{a}^{-}] = -1 = B_i.\ QED. \blacksquare \text{ And vice-versa.} \tag{7}$$

2.8. This is consistent with CLR's sufficient condition for an element of physical reality (¶1.4): since, without in any way disturbing $q(\mu_i)$, Alice can predict with certainty that Bob's result will be $B_i = -1$ when he tests $q(\mu_i)$ with $\delta^{\pm}_{\hat{a}}$ (which may be a disturber), then elements of physical reality $q(\mu_i), \delta^{\pm}_{\hat{a}}$ and $q(\hat{a}^{-})$ mediate Bob's result. Thus the element of physical reality corresponding to Bob's $B_i = -1$ result will be $q(\hat{a}^{-})$; a CLR outcome — acceptable to EPR, and at one with Bell's belief (as follows):

> Here's Bell's (1980:7): "To explain this dénouement [of my (ie, Bell's) theorem] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."
>
> Here's d'Espagnat (1979:166), recast for EPRB (and our E) in our notation, *with added emphasis*: 'A physicist can infer that in every pair, one particle has the property $\hat{a}^{+}$ [a positive spin-component along axis $\hat{a}$] and the other has the property $\hat{a}^{-}$. Similarly, he can conclude that in every pair one particle has the property $\hat{b}^{+}$ and one $\hat{b}^{-}$, and one has property $\hat{c}^{+}$ and one $\hat{c}^{-}$. *These conclusions require a subtle but important extension of the meaning assigned to our notation* $\hat{a}^{+}$. Whereas previously $\hat{a}^{+}$ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis $\hat{a}$ would give the definite result $\hat{a}^{+}$, then that particle is said to have the property $\hat{a}^{+}$. *In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times.* ... This view is contrary to the conventional interpretation of QM, but it is not contradicted by any fact that has yet been introduced.'

2.9. However, under CLR as we'll show: (i) such inferences are false; (ii) a weaker more-general inference is available; (iii) there's no need to contravene known facts re QM; (iv) there's no need to negate Bohr's insight which, here supported by Bell, bolsters our case against Bell's claims:

> It's "Bohr's insight that the result of a 'measurement' does not in general reveal some pre-existing property of the 'system', but is a product of both 'system' and 'apparatus'," Bell (2004: xi-xii). So CLR's physical-realism – *some physical properties change interactively* – is consistent with Bohr's insight. Moreover: "It seems to me that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [like Bell's *impossibility* theorem, as we'll see], and most of 'quantum logic'," Bell (2004: xi-xii). We agree.

2.10. Thus, to be clear and consistent with Bohr's insight, CLR goes beyond the Bell-d'Espagnat inferences wherein the 'measured' property is *equated* to the pristine property. That is – going beyond d'Espagnat's *subtle extension* cited in ¶2.8 – we instead infer to *equivalence under a 'measurement' operator*. Equivalence – a weaker, more general relation than equality – is here compatible with QM, Bohr's view, and the need here to recognize the effect of 'the means of observation':

> "... the unavoidable interaction between the objects and the measuring instruments sets an absolute limit to the possibility of speaking of a behaviour of atomic objects which is independent of the means of observation," Bohr (1958:25).

2.11. So now – via the known effect of linear-polarizer $\delta^{\pm}_{\hat{x}}$ on polarized particles $q(\hat{x}^{+})$ – we can match ancillary interactions like $\delta^{\pm}_{\hat{x}}q(\hat{x}^{+})\to q(\hat{x}^{+})$ under CLR with interactions like $\delta^{\pm}_{\hat{x}}q(\lambda_i)\to q(\hat{x}^{+})$. Then – since $\delta^{\pm}_{\hat{x}}$ is a dichotomic operator that dyadically partitions its binary domain – we let $\sim$ here denote the equivalence relation *has the same output under the same operator*. (Here the operators are *polarizing* operators; elsewhere they might be termed *'measurement'* or *interaction* operators.) So:

$$\text{If } \delta^{\pm}_{\hat{a}}q(\lambda_i)\to q(\hat{a}^{+}) \text{ then } q(\lambda_i)\sim q(\hat{a}^{+}) \because \delta^{\pm}_{\hat{a}}q(\hat{a}^{+})\to q(\hat{a}^{+}) \text{ exclusively; and } q(-\lambda_i)=q(\mu_i)\sim q(\hat{a}^{-}). \tag{8}$$

$$\text{If } \delta^{\pm}_{\hat{b}}q(\mu_i)\to q(\hat{b}^{+}) \text{ then } q(\mu_i)\sim q(\hat{b}^{+}) \because \delta^{\pm}_{\hat{b}}q(\hat{b}^{+})\to q(\hat{b}^{+}) \text{ exclusively; and } q(-\mu_i)=q(\lambda_i)\sim q(\hat{b}^{-}). \tag{9}$$

2.12. That is, from (8) – consistent with Alice's frame of reference wherein Alice observes $A_i=+1$, per $q(\hat{a}^{+})$ – we confirm $\sim$ under $\delta^{\pm}_{\hat{a}}$ as follows: (i) polarizing-operator $\delta^{\pm}_{\hat{a}}$ delivers $q(\lambda_i)$ and $q(\hat{a}^{+})$ to the same output; (ii) it is impossible for $\delta^{\pm}_{\hat{a}}$ to deliver $q(\lambda_i)$ and $q(\hat{a}^{+})$ to two different outputs; (iii) an equivalence relation $\sim$ therefore holds between $q(\lambda_i)$ and $q(\hat{a}^{+})$ under $\delta^{\pm}_{\hat{a}}$. (9) similarly, via Bob's frame of reference wherein Bob observes $B_i=+1$, per $q(\hat{b}^{+})$; and $\sim$ holds under $\delta^{\pm}_{\hat{b}}$.

2.13. Re our equivalence relations (using $\overset{\delta^{\pm}_{\hat{x}}}{\sim}$ if required) and the related equivalence classes:

$$Q\equiv\{q(\lambda_i),q(\mu_i);q(\hat{a}^{\pm})\,|\,E,\lambda_i+\mu_i=0,i=1,2,...,n;\delta^{\pm}_{\hat{a}}\}; \tag{10}$$

$$[q(\hat{a}^{+})]\equiv\{q(\cdot)\in Q:q(\cdot)\overset{\delta^{\pm}_{\hat{a}}}{\sim}q(\hat{a}^{+})\};\ [q(\hat{a}^{-})]\equiv\{q(\cdot)\in Q:q(\cdot)\overset{\delta^{\pm}_{\hat{a}}}{\sim}q(\hat{a}^{-})\}; \tag{11}$$

$$S/_{\sim}=\{[q(\hat{a}^{+})],\ [q(\hat{a}^{-})]\};\text{ etc,} \tag{12}$$

where Q here is the set of input particles $q(\cdot)$, and output particles $q(\hat{a}^{\pm})$ under $\delta^{\pm}_{\hat{a}}$. In (11), our equivalence classes $[q(\hat{a}^{\pm})]$ show that Q is partitioned dyadically under the mapping $\delta^{\pm}_{\hat{a}}q(\cdot)\to q(\hat{a}^{\pm})$. So $\sim$ on the elements of $\delta^{\pm}_{\hat{a}}$'s domain denotes: *has the same output/image under* $\delta^{\pm}_{\hat{a}}$; etc. Thus the quotient set S in (12) – the set of all equivalence classes under $\sim$ – is a set of two diametrically-opposed extremes: a maximal antipodean discrimination; a powerful push-pull dynamic (as we'll see at ¶2.16).

2.14. We now combine (1), (2), (8) and (9) into a single test on the i-th particle-pair from two perspectives: (13), which Alice reads from left-to-right; (14), which Bob reads from right-to-left:

$$A_i \equiv +1 \cdots q(\hat{a}^+) \leftarrow \Delta^{\pm}_{\hat{a}} \Leftarrow q(\lambda_i) \;\lhd E \rhd\; q(\mu_i) = q(-\lambda_i) \sim q(\hat{a}^-) \Rightarrow \delta^{\pm}_{\hat{b}} \Rightarrow q(\hat{b}^+) \rightarrow [\hat{b}\cdot *] \rightarrow \hat{b}\cdot\hat{b}^+ = +1 \equiv B_i\,; \quad (13)$$

$$A_i \equiv +1 = \hat{a}\cdot\hat{a}^+ \leftarrow [\hat{a}\cdot *] \Leftarrow q(\hat{a}^+) \leftarrow \delta^{\pm}_{\hat{a}} \Leftarrow q(b^-) \sim q(-\mu_i) = q(\lambda_i) \;\lhd E \rhd\; q(\mu_i) \Rightarrow \Delta^{\pm}_{\hat{b}} \rightarrow q(\hat{b}^+) \cdots +1 \equiv B_i : \quad (14)$$

> Thus, in line with Bell's (1964:196) specification for his λ: (i) seeking a physical theory of the type envisioned by Einstein, our hidden variables have dynamical significance and laws of motion; (ii) our pristine λ and μ – correlated under (4) – are the initial values of such variables at some suitable instant; (iii) since different tests produce different disturbances, different relational properties may be pairwise revealed under $\sim$ without contradiction: ie, finding $q(\lambda_i \sim \hat{a}^+)$ experimentally, we learn $q(\mu_i \sim \hat{a}^-)$ relationally; etc. *QED.* ■

2.15. So, from (4) and (10)-(14), we can now provide: (i) the functional relationships missing from Bell 1964:(1); (ii) relevant EPRB probabilities and expectations; (iii) a refutation of Bell's theorem; (iv) the whole followed by explanatory comments. With $A^{\pm}$ ($B^{\pm}$) denoting Alice's (Bob's) result ± 1:

$$\Delta^{\pm}_{\hat{a}} q(\lambda) \rightarrow A(\hat{a}, \lambda) = \cos(\hat{a}, \lambda \,|\, q(\lambda) \sim q(\hat{a}^{\pm})) = \pm 1 \equiv A^{\pm};\, \langle A \,|\, E\rangle = 0 \because P(A^+ \,|\, E) - P(A^- \,|\, E) = 0. \quad (15)$$

$$\Delta^{\pm}_{\hat{b}} q(\mu) \rightarrow B(\hat{b}, \mu) = \cos(\hat{b}, \mu \,|\, q(\mu) \sim q(\hat{b}^{\pm})) = \pm 1 \equiv B^{\pm};\, \langle B \,|\, E\rangle = 0 \because P(B^+ \,|\, E) - P(B^- \,|\, E) = 0. \quad (16)$$

$$P(A^+ \,|\, E) = P(A^- \,|\, E) = P(B^+ \,|\, E) = P(B^- \,|\, E) = \tfrac{1}{2} \because \lambda \text{ and } \mu \text{ are random variables.} \quad (17)$$

$$P(A^+ \,|\, EB^+) = P(q(\lambda \sim a^+) \,|\, E, q(\mu \sim b^+)) = P(\delta^{\pm}_{\hat{a}} q(b^-) \rightarrow q(\hat{a}^+) \,|\, E) = \cos^2 s\,(\hat{a}^+, \hat{b}^-) = \sin^2 \tfrac{1}{2}(\hat{a}, \hat{b}). \quad (18)$$

$$P(B^+ \,|\, EA^+) = P(q(\mu \sim b^+) \,|\, E, q(\lambda \sim a^+)) = P(\delta^{\pm}_{\hat{b}} q(a^-) \rightarrow q(\hat{b}^+) \,|\, E) = \cos^2 s\,(\hat{a}^-, \hat{b}^+) = \sin^2 \tfrac{1}{2}(\hat{a}, \hat{b}). \quad (19)$$

$$\therefore P(A^+B^+ \,|\, E) = P(A^+ \,|\, E)P(B^+ \,|\, EA^+) = P(B^+ \,|\, E)P(A^+ \,|\, EB^+) = \tfrac{1}{2}\sin^2 \tfrac{1}{2}(\hat{a}, \hat{b}).\, QED. \blacksquare \quad (20)$$

$$\therefore \big\langle A^+B^+ \,|\, E\big\rangle = \big\langle A^-B^- \,|\, E\big\rangle = \tfrac{1}{2}\sin^2 \tfrac{1}{2}(\hat{a}, \hat{b});\, \big\langle A^+B^- \,|\, E\big\rangle = \big\langle A^-B^+ \,|\, E\big\rangle = -\tfrac{1}{2}\cos^2 \tfrac{1}{2}(\hat{a}, \hat{b}). \quad (21)$$

$$\therefore \langle AB \,|\, E\rangle \equiv \big\langle A^+B^+ \,|\, E\big\rangle + \big\langle A^+B^- \,|\, E\big\rangle + \big\langle A^-B^+ \,|\, E\big\rangle + \big\langle A^-B^- \,|\, E\big\rangle = -\hat{a}\cdot\hat{b}.\, QED. \blacksquare \quad (22)$$

2.16. That is. Given (13), the cosine function in (15) reads: with $q(\lambda)$ equivalent to $q(\hat{a}^+)$ under $\sim$, $\cos(\hat{a}, \lambda \,|\, q(\lambda) \sim q(\hat{a}^+))$ denotes the cosine of the angle $(\hat{a}, \hat{a}^+)$: ie — under the push-pull dynamic foreshadowed in ¶2.13 — the outcome is $+1 = A^+$; etc. (16) similarly, given (14). Thus, under $\sim$, CLR could embrace Bell-d'Espagnat inferences (¶2.8) to equality, but: (i) the probability that such inferences are valid is negligible; (ii) their theory does not embrace ours; (iii) being locally-realistic, we conservatively allow $P(\lambda_i = \lambda_j \,|\, E, i \neq j) = 0$, etc., under CLR, per (4).

2.17. Next, re (18); and (19) similarly. Via standard probability theory: (i) any correlation induces the probability relation at LHS (18); (ii) A^+ and B^+ are correlated via (4); (iii) such correlation is recognized by Bell (in our favor) under EPRB (as follows):

> Recasting Bell (2004:208) in line with EPRB: "There are no 'messages' in one system from the other. The inexplicable [sic] correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system [eg, from Alice's A^+] to events in the other [eg, to Bob's B^+] are possible."

2.18. Further. In (18) under $\sim$, the LHS probability relation is – from the middle term in (18) – equivalent to a classical test on spin-$\frac{1}{2}$ particles of known polarization. So RHS (18) is derived by extending our representation of Malus' *classical* $\cos^2 s\,(\hat{a}^+, \hat{b}^-)$ Law – re the relative intensity of beams of polarized photons ($s = 1$) – to spin-half particles ($s = \frac{1}{2}$). (19) similarly. Then, since our equivalence relations hold under probability functions P, P is well-defined under $\sim$ and is that same law.

> Re Aspect's (2002:5-7) discussion of Malus' Law, our trigonometric arguments represent dynamical law-based processes under (8)-(9) and ¶2.13: eg, $(q(\lambda) \sim q(\hat{a}^+)) \equiv (\delta^{\pm}_{\hat{a}} q(\lambda) \to q(\hat{a}^+))$. The $\hat{a}$ in $\delta^{\pm}_{\hat{a}}$ denotes the orientation of a non-uniform field with which $q(\lambda)$ interacts. A macroscopic analogy is provided by a wire-grid microwave polarizer. With the conducting wires represented by direction-vectors in 3-space, an impinging unpolarized beam of microwaves drives electrons within the wires, thereby generating an alternating current (Hecht 1975:104). So the wires become operators, the transmitted beam being strongly linearly polarized orthogonal to the wires. Polaroid®-sheet is a molecular analog. (This suggests an analogous local-realistic QM-based calculation of the expectation $\langle AB\,|\,E\rangle$ wherein the pairwise conservation of total angular momentum in (4) $[\lambda_i + \mu_i = 0]$ reduces to the pairwise conservation of intrinsic spin $[\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 = 0]$; see Appendix.)

2.19. Thus, from (18), $P(A^+ \mid EB^+)$ under $\sim$ is Malus' Law generalized to entangled particles: so we can let Malus' *quantum* Law be the new QM-compatible law that links the first and last terms in (18) directly. And (19) similarly. Then, seeking one unified law under CLR, we might say that Malus' Law applies classically to the relational properties of the classical beams that Malus employed: and quantum-mechanically to the relational properties of particles. (To maintain this law-based unity in physics – bypassing much debate in philosophy; and seeking a reconstruction of QM per Kochen (2015) – we would then need to define absolute (intrinsic) and relational (extrinsic) properties accordingly.) However, using (4) and (8) with the generalization $\lambda + \mu = 0$, the expanded version of (18) is:

$$P(A^+ \,|\, EB^+) \equiv P(\delta^{\pm}_{\hat{a}} q(\lambda) \to q(\hat{a}^+) \,|\, E, \delta^{\pm}_{\hat{b}} q(\mu) \to q(b^+)) = P(\delta^{\pm}_{\hat{a}} q(\lambda) \to q(\hat{a}^+) \,|\, E, q(\mu) \sim q(b^+)) \quad (23)$$

$$= P(\delta^{\pm}_{\hat{a}} q(\lambda) \to q(\hat{a}^+) \,|\, E, q(-\lambda) \sim q(b^+)) = P(\delta^{\pm}_{\hat{a}} q(b^-) \to q(\hat{a}^+) \,|\, E) = \cos^2 s\,(\hat{a}^+, \hat{b}^-) = \sin^2 \tfrac{1}{2}(\hat{a}, \hat{b}). \quad (24)$$

> Now, with (23)-(24), we're in the following good company: "Nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary. ... the so-called 'hidden variable' possibility," Bell (2004:28-29). 'The major transformation from classical to quantum physics,' in Kochen's (2015:587) approach, 'lies not in modifying the basic classical concepts such as state, observable, symmetry, dynamics, combining systems, or the notion of probability, but rather in the shift from intrinsic to extrinsic properties.' We take this no further here.

2.20. However – in passing – allowing that every extrinsic property in QM can be converted to a relational property under an equivalence relation: our unified Malus' Law (henceforth, Malus' Law without qualification) would apply to relational properties generally and we'd live in a space consistent with Einstein's 'classical' world-view. To put it another way, in Malus' 19th-century context, consider two photons: (i) $q(\lambda_j \overset{\delta^{\pm}_{\hat{x}}}{\sim} \hat{x}^+) \equiv (\delta^{\pm}_{\hat{x}} q(\lambda_j) \to q(\hat{x}^+))$ in our QM terms; (ii) $q(\lambda_k = \hat{x}^+)$, our modern notation for a polarized photon in a classical beam that Malus worked with. We then say: for each $q(.)$ here, $(.)$ is a relational property under $\sim$. For, as relational properties – with P well-defined under $\sim$ from ¶2.17 – they yield valid results:

$$P(\delta^{\pm}_{\hat{a}} q(\lambda_j \overset{\delta^{\pm}_{\hat{x}}}{\sim} \hat{x}^+) \to q(\hat{a})) \text{ and } P(\delta^{\pm}_{\hat{a}} q(\lambda_k = \hat{x}^+) \to q(\hat{a})) = \cos^2 s(\hat{a}^+, \hat{x}^+) = \cos^2(\hat{a}, \hat{x}). \quad (25)$$

> "It is not easy [maybe] to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision [as is our aim] might be possible by concentration on the *be*ables, which can be described 'in classical terms',

because they are there [like our $q(\lambda_j \sim \hat{x}^+)$ under $\delta^{\pm}_{\hat{x}}$ and $q(\lambda_k = \hat{x}^+)$]. ... 'Observables' [like A_j and A_k respectively, in our notation] must be *made*, somehow, out of beables [as they are; eg, in (25)]. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables [as is our aim]," Bell (2004:52).

2.21. Returning to our comments on the flow (15)-(22): (20) follows from (17)-(19) via Bayes' Rule, and the expectations in (21) follow from (20) via the definition of an expectation. Then, with (22) from (21) via the definition of the overall expectation, we have the expectation $\langle AB \,|\, E\rangle$. Thus – with Bell claiming in the line below his 1964:(3) that (22) is impossible – Bell's theorem is refuted. With ▲ denoting absurdity, the source of Bell's 'impossibility theorem' – ie, Bell's false inference – follows:

2.22. Bell (1964:197), under 'Contradiction: The main result will now be proved', takes us via Bell 1964:(14), direction-vector $\hat{c}$, and three unnumbered equations – say, (14a)-(14c) – to his 1964:(15):

$$|\langle AB \,|\, E\rangle - \langle AC \,|\, E\rangle| \leq 1 + \langle BC \,|\, E\rangle\,;\ \text{ie, using our (22):}\ |(\hat{a}\cdot\hat{c}) - (\hat{a}\cdot\hat{b})| \leq 1 - (\hat{b}\cdot\hat{c});\ \blacktriangle \tag{26}$$

$$\text{ie, Bell 1964:(15) is absurd under QM, CLR and mathematics:}\ \because |(\hat{a}\cdot\hat{c}) - (\hat{a}\cdot\hat{b})| \leq \tfrac{3}{2} - (\hat{b}\cdot\hat{c}). \tag{27}$$

2.23. To pinpoint the source of this absurdity (and avoid any defective intermediaries), we now link LHS Bell 1964:(14a) directly to LHS Bell 1964:(15). Using illustrative angles, Bell's 1964:(15) allows:

$$0 \leq \langle AB \,|\, E\rangle - \langle AC \,|\, E\rangle \leq 1 + \langle BC \,|\, E\rangle\,; \tag{28}$$

$$\text{ie, using (22), } 0 \leq (\hat{a}\cdot\hat{c}) - (\hat{a}\cdot\hat{b}) \leq 1 - (\hat{b}\cdot\hat{c}); \tag{29}$$

$$\text{so, if } (\hat{a},\hat{b}) = \tfrac{\pi}{4} \text{ and } (\hat{a},\hat{c}) = (\hat{c},\hat{b}) = \tfrac{\pi}{8}, \text{ then } 0 \leq 0.217 \leq 0.076 \text{ (conservatively)};\ \blacktriangle \tag{30}$$

$$\text{ie, Bell 1964:(14a)} \neq \text{Bell 1964:(14b)} = \text{Bell 1964:(14c)} = \text{Bell 1964:(15).}\ QED. \blacksquare \tag{31}$$

2.24. Thus, under EPRB, Bell's theorem (and related inequalities) stem from the $\neq$ in (31); ie, via Bell's move from his (14a) to (14b). Studying Bell's note at 1964:(14b), Bell moves from (14a) to (14b) via the generalization $[A(\hat{b},\lambda)]^2 = 1$. As we'll show, this generalization is invalid under EPRB; with the following consequences: (i) Bellian absurdities – like (26) and (30) – flow from this restriction under EPRB; (ii) Bell's theorem is limited to systems for which this equality holds; (iii) EPR/EPRB-based settings are not such systems; (iv) this assumption has nothing to do with local causality. Let's see:

2.25. Using our (1)-(5) and a particle-by-particle analysis of E, let $3n$ random particle-pairs be equally distributed over three randomized polarizer-pairings $(\hat{a},\hat{b}), (\hat{b},\hat{c}), (\hat{c},\hat{a})$. (We recall that if $i \neq j$, the product of uncorrelated scalars like $A(\hat{b},\lambda_i)A(\hat{b},\lambda_j) = \pm 1$.) Allowing each particle-pair to be unique – and here, uniquely indexed – let n be such that (for convenience in presentation and to an adequate accuracy hereafter):

$$\text{Bell 1964:(14a)} = \langle AB \,|\, E\rangle - \langle AC \,|\, E\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\hat{a},\lambda_i)A(\hat{b},\lambda_i) - A(\hat{a},\lambda_{n+i})A(\hat{c},\lambda_{n+i})] \tag{32}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\hat{a},\lambda_i)A(\hat{b},\lambda_i)[A(\hat{a},\lambda_i)A(\hat{b},\lambda_i)A(\hat{a},\lambda_{n+i})A(\hat{c},\lambda_{n+i}) - 1] \tag{33}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\hat{a},\lambda_i)A(\hat{b},\lambda_i)[A(\hat{b},\lambda_i)A(\hat{c},\lambda_i) - 1]\ \textbf{(after using } \lambda_i = \lambda_{n+i}\ \textbf{[sic])} = \text{Bell 1964:(14b)}:\ \blacktriangle \tag{34}$$

$$= \text{absurd; for, under CLR per (4), } P(\lambda_i = \lambda_{n+i} \,|\, E) = 0. \text{ So (34) joins (26) and (30) under } \blacktriangle. \tag{35}$$

2.26. Thus, under his generalization $[A(\hat{b},\lambda)]^2 = 1$ (¶2.24), Bell has a generalization [$\lambda_i = \lambda_{n+i}$per (34)] akin to an ordered sample of n classical objects subject to repetitive non-destructive testing. To eliminate such absurdities, we now avoid Bell's error and derive the consequences. Since the average of $|\,A(\hat{a},\lambda_i)A(\hat{b},\lambda_i)\,|$ is ≤ 1, valid (33) reduces to:

$$\text{Bell 1964:(14a)} = |\langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle| \leq 1 - \frac{1}{n}\sum_{i=1}^{n} A(\hat{a},\lambda_i)A(\hat{b},\lambda_i)A(\hat{a},\lambda_{n+i})A(\hat{c},\lambda_{n+i}). \quad (36)$$

2.27. Now: (i) From (4), the independent and uncorrelated random variables λ_i and λ_{n+i} generate independent and uncorrelated random variables (± 1) ; (ii) the expectation over the product of two independent and uncorrelated random variables is the product of their individual expectations; (iii) so (36) reduces to:

$$\text{Bell 1964:(14a)} = |\,\langle AB\,|\,E\rangle - \langle AC\,|\,E\rangle\,| \leq\ 1 - \langle AB\,|\,E\rangle\langle AC\,|\,E\rangle \neq \text{Bell 1964:(14b)}; \quad (37)$$

$$\text{ie, } |\,(\hat{a}\cdot\hat{b}) - (\hat{a}\cdot\hat{c})\,| \leq 1 - (\hat{a}\cdot\hat{b})(\hat{a}\cdot\hat{c}) \neq \text{ RHS Bell 1964:(15) unless } \hat{a} = \hat{b} \vee \hat{c}.\ \blacksquare \quad (38)$$

2.28. In short, given that LHS (38) is a fact: Bell's 1964:(15) is absurd and false under EPRB. In passing, the CHSH (1969) inequality – eg, Peres (1995:164) – falls similarly to another fact:

$$|\,(\hat{a}\cdot\hat{b}) + (\hat{b}\cdot\hat{c}) + (\hat{c}\cdot\hat{d}) - (\hat{d}\cdot\hat{a})\,| \leq 2\sqrt{2}.\ \therefore |\,(\hat{a}\cdot\hat{b}) + (\hat{b}\cdot\hat{c}) + (\hat{c}\cdot\hat{d}) - (\hat{d}\cdot\hat{a})\,| \nleq 2.\ \blacksquare \quad (39)$$

2.29. Finally, to complete our analysis, we consider experiment M, Mermin's (1990) 3-particle variant of GHZ – the widely-regarded quintessential version of Bell's theorem – with its (alleged) always-vs-nothing refutation of EPR. Respectively, hereafter: three spin-$\frac{1}{2}$ particles with spin-related properties λ, μ, ν emerge from a spin-conserving decay such that:

$$\lambda + \mu + \nu = \pi.\ \therefore \nu = \pi - \lambda - \mu.$$

2.30. The particles separate in the y-z plane and interact with spin-$\frac{1}{2}$ polarizers that are orthogonal to the related line of flight. Let a, b, c denote the angle of each polarizer's principal-axis relative to the positive x-axis; let the test results be A, B, C. Then, based on LHS (15)-(16) in short-form, let

$$A^{\pm} = \cos(a{-}\lambda\,|\,\lambda{\sim}a^{\pm}) = \pm 1.\ B^{\pm} = \cos(b{-}\mu\,|\,\mu{\sim}b^{\pm}) = \pm 1.\ C^{\pm} = \cos(c{-}\nu\,|\,\nu{\sim}c^{\pm}) = \pm 1. \quad (41)$$

2.31. Via the principles in (1)-(22) – and nothing more – we now derive $\langle ABC\,|\,M\rangle$, the expectation for experiment M. (Explanatory notes follow.)

$$\big\langle A^{+}B^{+}C^{+}\,|\,M\big\rangle$$

$$\equiv P(\lambda{\sim}a^{+}\,|\,M)\cos(a{-}\lambda\,|\,\lambda{\sim}a^{+})\cdot P(\mu{\sim}b^{+}\,|\,M)\cos(b{-}\mu\,|\,\mu{\sim}b^{+})$$

$$\cdot P(\nu{\sim}c^{+}\,|\,M, \lambda{\sim}a^{+}, \mu{\sim}b^{+})\cos(c{-}\nu\,|\,\nu{\sim}c^{+}) \quad (42)$$

$$= \tfrac{1}{4}P(\nu{\sim}c^{+}\,|\,M, \lambda{\sim}a^{+}, \mu{\sim}b^{+}) = \tfrac{1}{4}P((\pi - \lambda - \mu){\sim}c^{+}\,|\,M, \lambda{\sim}a^{+}, \mu{\sim}b^{+}) \quad (43)$$

$$= \tfrac{1}{4}P((\pi - a^{+} - b^{+}){\sim}c^{+}\,|\,M) = \tfrac{1}{4}\cos^2\tfrac{1}{2}(\pi - a^{+} - b^{+} - c^{+}) = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a + b + c). \quad (44)$$

$$\text{Similarly: } \big\langle A^{+}B^{-}C^{-}\,|\,M\big\rangle = \big\langle A^{-}B^{+}C^{-}\,|\,M\big\rangle = \big\langle A^{-}B^{-}C^{+}\,|\,M\big\rangle = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a + b + c), \text{ and} \quad (45)$$

$$\big\langle A^{+}B^{+}C^{-}\,|\,M\big\rangle = \big\langle A^{+}B^{-}C^{+}\,|\,M\big\rangle = \big\langle A^{-}B^{+}C^{+}\,|\,M\big\rangle = \big\langle A^{-}B^{-}C^{-}\,|\,M\big\rangle = -\tfrac{1}{4}\cos^2\tfrac{1}{2}(a{+}b{+}c). \quad (46)$$

$$\therefore \langle ABC\,|\,M\rangle \equiv \Sigma\,\big\langle A^{\pm}B^{\pm}C^{\pm}\,|\,M\big\rangle = \sin^2\tfrac{1}{2}(a{+}b{+}c) - \cos^2\tfrac{1}{2}(a{+}b{+}c) = -\cos(a{+}b{+}c).\ QED.\blacksquare \quad (47)$$

2.32. (43) follows (42) by reduction using (15)-(17). (44) follows from (43) by allocating the equivalence relations in the conditioning space to the related variables. Thus, in words, LHS (44) is one-quarter the probability that ν — ie, $\nu{\sim}(\pi - a^{+} - b^{+})$ — will be equivalent to c^{+} under $\delta_c^{\pm}$. In other words: LHS (44) $= \frac{1}{4}P(\delta_c^{\pm}q(\nu \sim \pi - a^{+} - b^{+}) \to q(c^{+})\,|\,M) =$ RHS (44) via Malus' Law; ie, (44) is the three-particle variant of (21) in the two-particle EPRB experiment sketched in (1)-(4).

2.33. Delivering Mermin's (1990:11) *crucial minus sign*, (47) is the correct result for experiment M; ie, from (47): $\langle ABC\,|\,M\rangle = -1$ when $(a + b + c) = 0$. Thus, consistent with CLR and its rules for operators and functions in 3-space, we again deliver classically-intelligible EPR correlations.

2.34. With (22), (39), and now (47) – as one with CLR, EPR and QM, but so clearly in conflict with the Bellian conclusions cited at ¶1.1 – we rest our case: Bell's theorem is refuted as Einstein's locally-causal Lorentz-invariant worldview prevails.

3 Conclusions

> 3.0. CLR delivers Bell's (2004:167) hope: 'Let us hope that these analyses [impossibility proofs concerned with local causality] also may one day be illuminated, perhaps harshly, by a simple constructive model.' For CLR validates Bell's sentiment (2004:167) – inspired by Louis de Broglie – that 'what is proved by impossibility proofs is lack of imagination.'

3.1. In the terms of our Abstract: Accepting Bell's principles at ¶1.2 – but not his false inference in (34) – we've refuted Bell's theorem and variants at (22), (39) and (47). Thus justified in supporting Einstein's world-view: we've completed the QM account of EPR correlations in a classical way; delivered Bell's hope for a simple constructive model of EPRB; justified EPR's belief that additional variables would bring locality and causality to QM's completion. We've also refuted claims that such variables are impossible — including Mermin (1990) at (47) – and this next in general: in the context of Bell's theorem 'it's a proven scientific fact that a violation of local realism has been demonstrated theoretically and experimentally,' *(Annals of Physics* Editors, 2016).

> EPR argued that additional variables would restore causality and locality to QM, after Bell (1964:195). We agree; QED.

3.2. Against Bell/d'Espagnat equality-relations, our weaker more-general $\sim$ in (11) relates new (to us) quantum things $q(\lambda)$ to more familiar things $q(\hat{a}^{\pm})$; etc. Without such weaker relations, science would hardly be possible. At ¶2.8, d'Espagnat (1979:158) – endorsed by Bell (1980:7) – claims to draw legitimate conclusions from consistent observations under *induction*. Alas, ignoring consistent observations re QM's validity and Bohr's insight, he draws illegitimate conclusions; see ¶2.10. At ¶2.22, Bell's first error arises similarly and leads to Bell's (1980:13-15) claim that the factoring of joint EPRB probabilities in accord with stochastic independence is impossible under local causality. NB: from (15)-(16) and (22), outcomes A and B would be independent and uncorrelated if, given

$$\langle A\,|\,E\rangle = \langle B\,|\,E\rangle = 0, \text{ from (15)-(16)}: \langle AB\,|\,E\rangle = \langle A\,|\,E\rangle\langle B\,|\,E\rangle = 0. \text{ But } \langle AB\,|\,E\rangle \neq 0, \text{ from (22)}: \quad (48)$$

so A and B are independent *but* correlated; a big difference. Lets' see.

3.3. Given: (i) Bell (2004:240) and "a full specification of local beables in a given space-time region," though such a specification is likely unknowable; (ii) Bell (2014:243) referring to correlations which permit symmetric factorizations as 'locally explicable;' (iii) (Bell (2014:243 again), taking such 'factorizations to be a consequence of local causality and not a *formulation* thereof'; then:

$$P(A^+B^+|E,\hat{a},q(\lambda\sim\hat{a}^+),\hat{b},q(\mu\sim\hat{b}^+),...) = P(A^+|E,\hat{a},q(\lambda\sim\hat{a}^+),...)P(B^+|E,\hat{b},q(\mu\sim\hat{b}^+),...) = 1. \quad (49)$$

$$\therefore \text{ – eliminating unknowable equivalence relations from (49) –: } P(A^+B^+\,|\,E,\hat{a},q(\lambda),\hat{b},q(\mu)) \quad (50)$$

$$= P(A^+\,|\,E,\hat{a},q(\lambda))P(B^+\,|\,E,\hat{b},q(\mu)) = P(q(\lambda) \overset{\delta^{\pm}_{\hat{a}}}{\sim} q(\hat{a}^+)\,|\,E)P(q(-\lambda) \overset{\delta^{\pm}_{\hat{b}}}{\sim} q(b^+)\,|\,E) \quad (51)$$

$$= \tfrac{1}{2}P(q(\hat{a}^-) \overset{\delta^{\pm}_{\hat{b}}}{\sim} q(b^+)\,|\,E) = \tfrac{1}{2}\sin^2\tfrac{1}{2}(\hat{a},\hat{b}) = P(A^+|\,E)P(B^+|\,EA^+) = P(A^+B^+\,|\,E) \text{ via (17)-(20).} \quad (52)$$

3.4. (49)-(52) illustrates the fact that stochastic independence at the micro-level may reduce to Bayes' Rule at the macro-level. This fact also holds if source $\triangleleft\, E \,\triangleright$ in (1)-(2) is replaced by a source $\triangleleft\, W \,\triangleright$ that emits oppositely-polarized spin-$\frac{1}{2}$ particle-pairs. (Then $\langle AB\,|\,W\rangle = -\frac{1}{2}\hat{a}\cdot\hat{b}$.)

3.5. Many agree with du Sautoy (2016:170), "Bell's theorem is as mathematically robust as they come." However, Bell's use of $[A(\hat{b}, \lambda)]^2 = 1$ (see ¶2.24), renders Bell's theorem unphysical under EPRB, mathematically false at (22), absurd at (26) and (30). For, per ¶2.25, it's certain that Bell's use of $[A(\hat{b}, \lambda)]^2 = 1$ is invalid under EPRB due to multiple pairing/matching problems; ie, under $i \neq j$, the product of uncorrelated scalars is $A(\hat{b}, \lambda_i)A(\hat{b}, \lambda_j) = \pm 1$. Further: Bayes' Rule for the probability of joint outcomes is never false (neither mathematically nor experimentally) – confirming Bell's (2004:239) 'utmost suspicion' – he did throw the baby out with the bathwater. NB: Bayes' Rule is certainly relevant under EPRB; for A and B are independent but correlated per (48).

3.6. Re ¶2.19-20, considerations for a wholesale reinterpretation of QM remain: 'collapse' as the Bayesian updating of an equivalence class via prior correlations; 'states' as multivectors in 3-space; 'measurements' as the outcomes of interactions; 'wave-particle duality' as an equivalence relation.

3.7. Further, our Lorentz-invariant analysis resolves Bell's dilemma re action-at-a-distance (AAD hereafter) and nonlocality: (i) we've dispensed with AAD; (ii) we've validated Einstein's program; (iii) we do get away with locality:

> 'I cannot say that AAD is required in physics. *I can say that you cannot get away with no* AAD. You cannot separate off what happens in one place and what happens in another. *Somehow they have to be described and explained jointly.* That's the fact of the situation; Einstein's program fails ... *Maybe we have to learn to accept not so much* AAD*, but the inadequacy of no* AAD. ... *That's the dilemma.* We are led by analyzing this situation to admit that, somehow, distant things are connected, or at least not disconnected. ... I step back from asserting that there is AAD and *I say only that you cannot get away with locality*. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is AAD,' after Bell (1990:5-13); *emphasis added.*

3.8. We thus justify Bell's belief:

> "It's my feeling that all this AAD and no AAD business will [pass; like ether-based theories]. But someone will come up with the answer, with a reasonable way of looking at these things. If we are lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, and it won't lead to a big new development. But anyway, I believe the questions will be resolved," Bell (1990:9).

3.9. For, in line with the quotations at ¶2.19, our common purpose carries the day:

> "Now nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary. It is hard for me to envisage intelligible discourse about a world with no classical part -- no base of given events, be they only mental events in a single consciousness, to be correlated. On the other hand, it is easy to imagine that the classical domain could be extended to cover the whole. The wave functions – not beables in our terms; see Bell (2004:53) – would prove to be a provisional or incomplete description of the quantum-mechanical part, of which an objective account would become possible. It is this possibility, of a homogeneous account of the world, which is for me the chief motivation of the study of the so-called 'hidden variable' possibility," Bell (2004:28-29).

3.10. In summary: Bell's theorem is refuted – see (22), (39), (47) – under Einstein's locally-causal Lorentz-invariant worldview. At peace with QM, and special and general relativity, a consistent account of the world beckons.

4 Appendix

4.1. From ¶2.18, we now provide an analogous local-realistic QM-based calculation of the expectation $\langle AB|E\rangle$. We allow that the pairwise conservation of total angular momentum in (4) $[\lambda_i + \mu_i = 0]$ reduces to the pairwise conservation of intrinsic spin $[\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 = 0]$ during each particle's interaction with the polarizing-field. Based on Dirac (1982:149), let the intrinsic spin of a spin-half particle be $\boldsymbol{s} = \frac{\hbar}{2}\boldsymbol{\sigma}$.

$$\therefore \langle AB|E\rangle \equiv \left\langle (\hat{a}\cdot\boldsymbol{\sigma}_1)(-\boldsymbol{\sigma}_2\cdot\hat{b})\right\rangle = -\left\langle (\hat{a}\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_1\cdot\hat{b})\right\rangle = -\left\langle (\hat{a}\cdot\boldsymbol{\sigma})(\boldsymbol{\sigma}\cdot\hat{b})\right\rangle \text{ (for convenience)} \tag{53}$$

$$= -(\hat{a}_x\,\hat{a}_y\,\hat{a}_z)\left\langle \begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{pmatrix}(\sigma_x\,\sigma_y\,\sigma_z)\right\rangle \begin{pmatrix}\hat{b}_x\\ \hat{b}_y\\ \hat{b}_z\end{pmatrix} = -(\hat{a}_x\,\hat{a}_y\,\hat{a}_z)\left\langle \begin{pmatrix}\sigma_x^2 & \sigma_x\sigma_y & \sigma_x\sigma_z\\ \sigma_y\sigma_x & \sigma_y^2 & \sigma_y\sigma_z\\ \sigma_z\sigma_x & \sigma_z\sigma_y & \sigma_z^2\end{pmatrix}\right\rangle \begin{pmatrix}\hat{b}_x\\ \hat{b}_y\\ \hat{b}_z\end{pmatrix} \tag{54}$$

$$-(\hat{a}_x\hat{a}_y\hat{a}_z)\begin{pmatrix}\langle 1\rangle & \langle i\sigma_z\rangle & \langle -i\sigma_y\rangle\\ \langle -i\sigma_z\rangle & \langle 1\rangle & \langle i\sigma_x\rangle\\ \langle i\sigma_y\rangle & \langle -i\sigma_x\rangle & \langle 1\rangle\end{pmatrix}\begin{pmatrix}\hat{b}_x\\ \hat{b}_y\\ \hat{b}_z\end{pmatrix} = -(\hat{a}_x\hat{a}_y\hat{a}_z)\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\hat{b}_x\\ \hat{b}_y\\ \hat{b}_z\end{pmatrix} = -\hat{a}\cdot\hat{b}.\ QED.\blacksquare \tag{55}$$

4.2. The unitary diagonal follows from $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$. Off-diagonal elements have expectation zero via this fact: $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are correlated random variables so their components take positive and negative values equiprobably.

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