

=====
Key point of the proof (Global in Time Solvability of Incompressive NSIVP in Periodic Space)

Global in Time Solvability is confirmed, based on nonincreasing of L^2 -norm of the solution $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$ gotten as a priori estimation, by means of upper limit estimation of solution based on integral equation with regard to the solution.

However, estimation by means of simple combination of following integral equation (1.3) and Hölder inequality and expanded Young's inequality has limitation and can't give result to be proved.

$$(1.3) \quad \mathbf{u}_t = K_t * \mathbf{a} - \int_0^t d\tau \mathcal{P}(\partial K_{t-\tau} * \mathbf{u}_\tau \mathbf{u}_\tau)$$

For example, simply using $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$ and equation above, following pro forma inequality is confirmed.

$$\|\mathbf{u}_t\|_q \leq \|\mathbf{a}\|_q + C \|\mathbf{a}\|_2^2 \int_0^t d\tau \|\partial K_{t-\tau}\|_q$$

However, considering $\|\partial K_{t-\tau}\|_q = c(t-\tau)^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$, for $n \geq 3, q \in (n, \infty]$, this factor diverges in case of integrating with regard to τ .

In the paper, avoiding divergence like the above by means of time transferred integral equation (1.4), useful estimation is confirmed and global in time solvability is proved.

$$(1.4) \quad \mathbf{u}_t^\Phi = K_t^\Phi * \mathbf{a} - \int_0^t d\tau \mathcal{P}(\partial K_{t-\tau}^\Phi * \varphi_\tau \cdot \mathbf{u}_\tau^\Phi \mathbf{u}_\tau^\Phi)$$

By means of this relation and $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$, following relation is confirmed.

$$\|\mathbf{u}_t^\Phi\|_q \leq \|\mathbf{a}\|_q + C \|\mathbf{a}\|_2^2 \int_0^t d\tau \varphi_\tau \|\partial K_{t-\tau}^\Phi\|_q$$

Based on $\|\partial K_{t-\tau}^\Phi\|_q = c(\Phi(t-\tau))^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$ and using appropriate time transformation function $\Phi = \Phi(t)$, even though for $n \geq 3, q \in (n, \infty]$, factor $\|\partial K_{t-\tau}^\Phi\|_q$ doesn't diverge for integrating with regard to τ and with integral factor φ integral keeps finite, useful estimation can be confirmed.

$$\|\mathbf{u}_t^\Phi\|_q \leq \|\mathbf{a}\|_q + Cc \|\mathbf{a}\|_2^2 \int_0^t d\tau \varphi_\tau \Phi_{t-\tau}^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$$

=====