

Conjecture on the Poulet numbers of the form $(4^n + 1)/5$ where n is prime

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Abstract. In this paper I conjecture that any Poulet number of the form $(4^n + 1)/5$ where n is prime is either 2-Poulet number either a product of primes $p(1)*p(2)*\dots*p(k)$ such that all the semiprimes $p(i)*p(j)$, where $1 \leq i < j \leq k$, are 2-Poulet numbers.

Conjecture:

Any Poulet number of the form $(4^n + 1)/5$ where n is prime is either 2-Poulet number either a product of primes $p(1)*p(2)*\dots*p(k)$ such that all the semiprimes $p(i)*p(j)$, where $1 \leq i < j \leq k$, are 2-Poulet numbers.

Verifying the conjecture:

(for the first few such numbers)

- : $a(3) = 13$ and $a(5) = 205$ are not Poulet numbers;
- : $a(7) = 3277 = 29*113$ is a 2-Poulet number;
- : $a(11) = 838861 = 397*2113$ is a 2-Poulet number;
- : $a(13) = 13421773 = 53*157*1613$ is a Poulet number and indeed $8321 = 53*157$, $85489 = 53*1613$ and $253241 = 157*1613$ are all three 2-Poulet numbers;
- : $a(17) = 3435973837 = 137*953*26317$ is a Poulet number and indeed $130561 = 137*953$, $3605429 = 137*26317$ and $25080101 = 953*26317$ are all three 2-Poulet numbers;
- : $a(19) = 54975581389 = 229*457*525313$ is a Poulet number and indeed $104653 = 229*457$, $120296677 = 229*525313$ and $240068041 = 457*525313$ are all three 2-Poulet numbers;
- : $a(23) = 14073748835533 = 277*1013*1657*30269$ and indeed $280601 = 277*1013$, $458989 = 277*1657$, $8384513 = 277*30269$, $1678541 = 1013*1657$, $30662497 = 1013*30269$ and $50155733 = 1657*30269$ are all six 2-Poulet numbers.