Density Problem of Périat

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$$x \in [0; I] \subset \mathbb{R}_+$$

$$G \ni f(x)$$

$$f(0) = 0$$

$$f \in C^\infty([0; I])$$

$$\frac{df}{dx}(0) = 0$$

$$\left\{ \exists!\, \alpha \in \mathbb{R}_+ \;\middle|\; -\alpha \le \frac{df}{dx}(x) \le \alpha \right\}$$

$$F \subset G$$

$$A \in [-\alpha I; \alpha I]$$

$$\{f \in F \mid f(I) = A\}$$

$$P(A) = \frac{|F|}{|G|} = ?$$

Conjectures :

$$P(A) = P(-A)$$

$$P(\pm \alpha I) = 0$$

$$\frac{dP}{dA}(\pm \alpha I) = 0$$

$$\frac{dP}{dA}(0) = 0$$

$$\int_{-\alpha I}^{\alpha I} P(A)\, dA = 1$$

If resolved, expand to dimension 3 with :

$$A \in [-\alpha I; \alpha I]^3$$

Conjectures :

$$\left\|\overrightarrow{OA}\right\| = \left\|\overrightarrow{OB}\right\| <=> P(A) = P(B)$$

$$\left\|\overrightarrow{OC}\right\| = \alpha I <=> P(C) = 0 => \frac{dP}{dA}(C) = 0$$

$$\left\|\overrightarrow{OD}\right\| = 0 <=> \frac{dP}{dA}(D) = 0$$

$$\iiint_{-\alpha I}^{\alpha I} P\big((x; y; z)\big)\, dx\, dy\, dz = 1$$