Quantum of canonical electromagnetic angular momentum = $\hbar/2$

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Abstract

It is analytically determined that the smallest theoretically possible nonzero canonical electromagnetic angular momentum $\hbar/2$ arises when an electron is inserted into one magnetic flux quantum. The analysis further reveals how magnetic flux quantization is inherently linked up with angular momentum quantization. Bohr's correspondence principle is satisfied.

1 Introduction

Magnetostatic fields are devoid of angular momentum but can acquire canonical angular momentum if they include an electrostatic charge. [1] It can thus be expected that canonical angular momentum is subjected to angular momentum quantization and that the smallest theoretically possible canonical angular momentum defines its quantum. The task thus is to determine the smallest QED-permitted canonical angular momentum being generally given by $\mathcal{L} = q\,\Phi/2\,\pi$ where a charge q immersed in a static electromagnetic field of flux Φ. Obviously the minimum of $\mathcal{L}$ results when the smallest possible charge - e- is located within the smallest possible magnetic flux - $\Phi_0 = h/2e$ - a magnetic flux quantum. It can thus be anticipated that $\hbar/2 = e\,\Phi_0/2\,\pi$ defines the quantum of canonical electromagnetic angular momentum. [2]
For the sake of clarity and briefness the electrons own spin angular momentum will not be considered here.

2 Canonical electromagnetic angular momentum

Consider a point-like charge q of mass m fixed at a location $\vec{r}$ in the x-y plane where a magnetostatic field with vector-potential $\vec{A}$ prevails. Its Lagrangian:

$$\mathcal{L} = \frac{m}{2}\dot{\vec{r}}^2 + q\,\vec{A}\,\dot{\vec{r}} \tag{1}$$

can be transformed into canonical momentum $\vec{p}_0$:

$$\vec{p}_c = \frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} = m\,\dot{\vec{r}} + q\,\vec{A} \tag{2}$$

For a motionless charge ($\dot{\vec{r}} = 0$):

$$\vec{p}_0 = q\,\vec{A} \tag{3}$$

Canonical angular momentum $\vec{L}_0$ attributed to $\vec{p}_0$ at radius $\vec{r}$ generally is: [3]

$$\vec{L}_0 = \vec{r} \times \vec{p}_0 = q(\vec{r} \times \vec{A}_{(r)}) \tag{4}$$

The next step is to determine $\vec{A}_r$ at the location $\vec{r}$ of charge q within a uniform magnetic field $\vec{\Phi}$ of radius $\vec{r}$:

$$\vec{A}_{(r)} = \vec{r} \times \vec{\Phi}/2\pi r^2 \tag{5}$$

By substitution with (5) in (6) canonical angular momentum is:

$$|\vec{L}_0| = q\,|r|\,(|\vec{\Phi}|/2\pi r) = q\,\Phi/2\pi \tag{6}$$

3 Lowest limit of angular momentum

(6) indicates that the lowest QED-permitted canonical angular momentum is determined by the lowest QED-permitted nonzero values of q and Φ which in this case are $q = e$ and $\Phi = \Phi_0$. Thus

$$L_{0e} = e\,r\,\Phi_0/2\pi r = e\,\Phi_0/2\pi = \hbar/2 \tag{7}$$

is the lowest QED-permitted limit of angular momentum carried by a fluxon charged with e, in accordance with Bohr's correspondence principle. [4] In addition, (7) makes possible to redefine a fluxon based on angular momentum quantization, without any reference to superconductivity: [2]

$$\vec{\Phi_0} = \vec{\hbar}\,\pi/e \tag{8}$$

Note that in (7) and (8) the following parallelism applies: $\vec{L} \parallel \vec{\hbar} \parallel \vec{\Phi}$

4 Synopsis/conclusion

a) It is analytically proven that the smallest theoretically possible canonical electromagnetic angular momentum $\hbar/2$ is realized by a fluxon charged with e.

b) Magnetic flux-quantization is inherently linked up with angular momentum quantization.

c) Canonical electromagnetic angular momentum satisfies Bohr's correspondence principle.

References

[1] A.R. Edmonds, Angular momentum in quantum mechanics, Princeton University Press, 1968.

[2] F. London, Superfluids, Macroscopic Theory of Superconductivity, Structure of Matter Vol. 1 (Wiley, New York, 1960), p.152)

[3] http://isites.harvard.edu/fs/docs/icb.topic1327064.files/143bF2013Sec10.pdf

[4] https://plato.stanford.edu/entries/bohr-correspondence/