The last slice of cake

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Abstract

This letter is the short continuation of the previous paper titled "The infinitesimal error", available for free at the internet address `http://vixra.org/abs/1703.0280`. This letter is written just to further clarify the subject of "The infinitesimal error".

Consider a nice circular cake of volume 1. I can slice it diametrally in half. Each of two slices has volume 2^{-1}. I can slice these 2^1 slices the second time in half. I end up with 2^2 slices, each of volume 2^{-2}. And so on. I can slice all the slices in half the nth time, and each of the 2^n slices have volume 2^{-n}. This can go on forever. What is the volume of the "final" slice? Nowadays mathematics would say that $\lim_{n\to\infty} 2^{-n} = 0$. This would be the volume of the "final" slice. This is what we teach kids in schools. We say that the volume of each slice tends to zero, which is obviously true, and that this result can be collected symbolically as $\lim_{n\to\infty} 2^{-n} = 0$. We then go on and use this result as written.

But here's the problem with this. First, we say that each slice is smaller, and that the volume of each slice tends to zero. This is fine and obvious. But then we say that there is the "final" slice, whose volume actually is zero, and express this fact by writing $\lim_{n\to\infty} 2^{-n} = 0$. This last statement, the statement $\lim_{n\to\infty} 2^{-n} = 0$, is in the sharp contrast to the statement that each slice tends to have volume zero. It's in contrast, because the result $\lim_{n\to\infty} 2^{-n} = 0$ says that there is a concrete fixed number one could call "infinity" and denote it by $\aleph_0$, resulting in the fixed result $2^{-\aleph_0} = 0$ for the volume of the "final" slice. But there is no "final" slice! Suppose there is the final slice. I can slice it in half again. The volume of each of the new two slices is not the same as the volume of the original "final" slice. It's only half of it. I can slice it further as well. I can go on until the result becomes so small that is beyond any measuring. In other words, if we introduce infinitesimals and denote them collectively by, say,

ϵ, then instead of $\lim_{n\to\infty} 2^{-n} = 0$, one should really have $\lim_{n\to\infty} 2^{-n} = \epsilon$. Or, even more precisely, $\lim_{n\to\infty} 2^{-n} = \epsilon_n$, or even $\lim_{n\to\infty} 2^{-n} = \epsilon(2^{-n})$.

Do notice that there are two conflicting ideas we have about limits nowadays. Limit as a process of the result tending to some accumulation point, and the notion of the limit being a fixed number. The consequences of this ambiguity of the notion of a limit can be scary and faulty, as illustrated in the paper "The infinitesimal error". For instance, the existence of cardinals is demonstrated there to lead to a contradicting and faulty result that $\mathbb{R}$ is not dense! But one can always slice the slice in half, obviously. The error disappears as soon as one admits there are no cardinals.

In my opinion, Kurt Gödel politely said to the world that there is an error in cardinals, and that we are unable to prove it, when he introduced his two now legendary theorems on incompleteness and inconsistency. In other words – remove the errors, and the result shall be error free. If we introduce faulty axioms in our theory that contradict other axioms – what do we expect then?

So in conclusion, by the fact that there is no fixed "infinity", one concludes that there are no cardinals, and that the process of limiting never ends. In some cases this means that we must re-introduce the notion of an "infinitesimal".