Preons, Gravity with Torsion and Black Holes

Risto Raitio*
02230 Espoo, Finland

April 4, 2017

Abstract

A previous preon model for the substructure of the the standard model quarks and leptons is complemented to provide a model of black holes. Gravity theory with torsion is applied to preons producing an axial-vector field coupled to preons. The mass of the axial-vector particle is estimated to be near the GUT scale. The boson can materialize above this scale and gain further mass to become the center of a black hole at Planck energy while massless preons form the horizon. A particle-black hole duality is defined.

PACS 12.60.Rc Composite models; 04.70.-s Physics of black holes

Keywords: Preons, Black Holes, Gravity, Torsion

*E-mail: risto.raitio@gmail.com

Contents

1 Introduction

The purpose of this brief note is to develop further a spin 1/2 preon model in order to give group theoretic structure to it. The model should fulfill four requirements: (i) provide a global group structure for preons, quarks and leptons, (ii) introduce preon properties so that they endorse the standard model (SM) local gauge group structure $SU(3) \times SU2) \times U(1)$, (iii) provide a basis for introducing an applicable formulation of gravity into the model, and (iv) introduce tentatively a corpuscular structure for black holes. Briefly, this note is a proposal for beyond standard model physics including gravity and black holes.

These goals are approached as follows. The preon model [1, 2, 3, 4] is supported by the work of Finkelstein [5] using the global knot algebra $SLq(2)$ structure for preons, quarks and leptons. Secondly, the construction of the preon model directly suggests the gauge group structures $SU(2)$ and $SU(3)$ for the weak and strong interactions, respectively. Thirdly, fermion fields in Einstein-Cartan [6], or Einstein-Kibble-Sciama (EKS) [7, 8] gravity have been shown by Fabbri to yield interesting results for torsion coupling to the spin of Dirac fields [9]. This interaction is expressed as a massive axial-vector field coupling to preons. It originates from translation symmetry of the full Poincaré gauge group in the action. A model for Gedanken gravity phenomenology is in this way introduceded for energy scales, say approximately 10^{16} Gev $\leq E \leq 10^{19}$ GeV. At these energies the axial-vector boson may materialize due to preon-antipreon annihilation in stellar collisions or in similar energy density thermal environment. At $E \geq 10^{19}$ GeV the axial-vector bosons are seeds for black hole formation. Near and above Planck scale the effects of curvature are assumed to be comparatively small, while at astronomical scales the effects of curvature dominate.

There is duality between standard model matter particles and black holes. In principle, one is calculable from the other.

The organization of this note is the following. The preon model is described in section 2. The particle classification group $SLq(2)$ is discussed in appendix A. The model for black hole structure using the torsion field is described in section 3. Torsion in EKS gravity is summarized in appendix B. In section 4 some interesting thoughts on the nature of spinor fields are briefly quoted. Finally, conclusion are made in section 5. The appendices are included to make the presentation self-contained.

2 Preons

The constituents of quarks and leptons must include an odd number of spin 1/2 particles. I consider the case of three constituents, preons. Requiring charge quantization {0, 1/3, 2/3, 1} and fermionic permutation antisymmetry for same charge preons, four bound states of three light preons have been defined. These form the first generation quarks and leptons [1, 2]

$$\begin{aligned} u_k &= \epsilon_{ijk} m_i^+ m_j^+ m^0 \\ \bar{d}_k &= \epsilon_{ijk} m^+ m_i^0 m_j^0 \\ e &= \epsilon_{ijk} m_i^- m_j^- m_k^- \\ \bar{\nu} &= \epsilon_{ijk} \bar{m}_i^0 \bar{m}_j^0 \bar{m}_k^0 \end{aligned} \tag{2.1}$$

A feature in (2.1) with two same charge preons is that the construction provides a three-valued index for quark $SU(3)$ color, as it was originally discovered [10]. The corresponding gauge bosons are in the adjoint representation. The weak $SU(2)$ left handed doublets can be read from the first two and last two lines in (2.1). The standard model gauge structure $SU(N), N = 1, 2$ is emergent in this sense from the present preon model. In the same way quark-lepton transitions between lines 1↔3 and 2↔4 in (2.1) are possible. The preon and SM fermion group structure is better illuminated using the representations of the $SLq(2)$ group in the next appendix A.

The above gauge picture is supposed to hold in the present scheme up to the energy of about 10^{16} GeV. The electroweak interaction is in the spontaneously broken symmetry phase below energies of the order of 100 GeV and in the symmetric phase above it. The electromagnetic and weak forces take separate ways at higher energies (100 GeV$\ll E \ll 10^{16}$ GeV). The weak interaction restores its symmetry but melts away due to ionization of quarks and leptons into preons. The electromagnetic interaction, in turn, stays strong towards Planck scale, $M_{\rm Pl} \sim 1.22 \times 10^{19}$ GeV. Likewise, the quark color and leptoquark interactions suffer the same destiny as the weak force. One is left with the electromagnetic and gravitational forces only at Planck scale.

3 Black Hole Structure

The preons interact by coupling to an axial-vector boson W [1] arising in Einstein-Kibble-Sciama theory of gravity. The preon-preon interaction is attractive [9] providing the binding for three preon states. The mass of the axial-vector boson is estimated to be of the order of the GUT scale 10^{16} GeV (see below in this section). This makes the torsion interaction range very short. At all scales the W couples to preons relatively strongly but to the standard model particles always weakly. The role of curvature, or gravitons, is postponed to later study.

The field equation for torsion axial-vector is (B.22), from appendix B.2

$$\nabla_\rho(\partial W)^{\rho\mu} + M^2 W^\mu = X\overline{\psi}\boldsymbol{\gamma}^\mu \boldsymbol{\pi}\psi \tag{3.1}$$

where M is the axial-vector mass, X the preon–axial-vector coupling and ψ the preon wave function. The coupling X must be larger than the electromagnetic coupling α to keep the charged preons bound. In EKS gravity, X is independent of the gravitational coupling [9]. The key point of this note is that (3.1) depends only on the axial-vector W and preon ψ fields, not on gauge and metric factors.

Couplings in GUT theory are of the order 0.02 at the GUT scale. With a Yukawa potential in the Schrödinger equation $V(r) = -V_0\ exp(-ar)/r$ [19], or in our notation $-Xexp(r/M)/r$ with the physicality condition $n + l + 1 \leq \sqrt{XmM}$, one may estimate that large M correlates with small preon mass $m \ll m_{proton}$. These matters deserve naturally quantitative attention.

The axial-vector field is expected to appear as a physical particle whenever its production is energetically possible. At Planck scale energy the axial-vector boson serves as a seed for black hole formation causing a black hole to appear. With the growing black hole mass the fermion spins average out towards zero and torsion vanishes but the physical boson remains.

The horizon is a shell of massless preon-antipreon pairs. The number of pairs is correlated with the mass of the black hole, and they may form Cooper pairs. A prototype for the lightest black hole is a preon-antipreon-W excited bound state. It is a physical state which couples to quark-antiquark and lepton-antilepton pairs. This state was called gravon in [11].

There is a particle-black hole duality in this model. On the particle side the fermions - i.e. preons, quarks and leptons - dominate and the graviton axial-vector, and the graviton, is hidden. On the black hole side the axial-vector is the physical black particle and the preons are 'hidden' forming the horizon. In principle, one is calculable from the other.

One may now propose that, as far as there is an ultimate unified field theory, it is a preon theory with only gravitational and electromagnetic interactions.

[1] Weak interaction bosons are not considered in this note.

In the early universe, the strong and weak forces are generated only after massless preons combine into quarks and leptons at lower temperature. These two forces function only with short range within nuclei making atoms, molecules and chemistry possible. In a contracting phase of the universe the same processes take place in the reverse order.

4 Considerations of Spinors Fields

The incompatibility of gravity and second quantization, as well as the problem of radiative corrections, are discussed from a novel point of view in [9]. A major point is that, with gravity included in the theory, plane wave solutions do not exist. Instead, localized fields can be derived by analyzing the self-interactions of the chiral components of the spinor fields. Secondly, I quote Fabbri [9]:

"In the theory of quantum fields, electrons are point-like with quantum effects giving an electronic self-interaction in terms of radiative processes involving loops, while here the self-interaction of the spinor should be regarded as a mutual interaction of its two chiral parts giving internal dynamics for extended fields, and consequently allowing the Zitterbewegung to actually influence the particles. The Zitterbewegung of classical fields and quantum effects for structureless particles might coincide."

From this point of view, we may be closer to quantum gravity than commonly believed.

5 Conclusions

The preon model with spin 1/2 and charge 0 and 1/3 constituents discussed above has a sound group theoretical basis. Both the preons and the quarks and leptons belong to two lowest representations of the global SLq(2) group, shown in the tables 1 and 2 of the appendix. With four preons the standard model local gauge groups $SU(3) \times SU2) \times U(1)$ become visible. Preons, as Dirac spinors, are the fundamental building blocks of matter which couple to gravity predominantly by axial-vector boson coupling. Above the Planck scale the formation of black holes becomes possible with the axial-vector boson forming a seed for it and the chiral phase preon-antipreon pairs form the horizon. The preons coupled to the axial-vector may make the singularity of the hole softer or fade away. All the basic equations, the standard model and the torsion field equation B.22, are relativistic quantum equations. Therefore quantum gravity may be within reach.

The role of curvature needs to be quantitatively evaluated. It is assumed here to be a small correction in this torsional model. It is remarkable that the equations for torsion (B.22) and curvature (B.24) are so different. Thermodynamics is another area to be studied in detail. Zitterbewegung at Planck energy should provide a scale of length/area for calculations.

A dual relationship was found between matter and black holes. One is, in principle, calculable from the other.

It is hoped that the preon scheme [4] would provide a way towards a better understanding of the roles of all interactions. For that goal the weak and strong interactions are treated in this scenario in a specific way. They are emergent from the very basic fermion structure of the model (2.1). Gravity and electromagnetism are the 'original' long range interactions in the big bang of cyclic cosmology. The translation symmetry of the full Poicaré group implies axial-vector interactions which introduce a new Gedanken phenomenology for preons between the GUT scale and Planck scale. The axial-vector particle is expected to have a large mass, $M \sim 10^{16}$ GeV. Within accelerator energies axial-vector particle couplings to standard model particles are very small.

Of matters not discussed in this note I refer to [9] where substantial amount of phenomenological success is obtained beyond the standard model of cosmology, like dark matter, cosmological constant and inflation.

More work is needed to clarify the issues and gain consensus in the questions of field quantization, gravity and its full quantum version, and possible unification with electromagnetism.

Acknowledgement

I thank Luca Fabbri for correspondence.

Appendices

A Knot Theory: Preons, Quarks and Leptons

Early work on knots in physics goes back in time to 19th and 20th century [12, 13]. On the 21st century Finkelstein has proposed a model based on the group $SLq(2)$ [5]. This group actualizes the needs of the model of the previous section 2.

Let us consider the simple case of two dimensional representation of the group $SLq(2)$ which is defined by the matrix

$$T = D^{1/2}_{mm'} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{A.1}$$

where (a, b, c, d) satisfy the knot algebra

$$\begin{aligned} ab = \;& qba \quad bd = \; qdb \quad ad - qbc = 1 \quad bc = cb \\ ac = \;& qca \quad cd = \; qdc \quad da - q_1 cb = 1 \quad q_1 \equiv q^{-1} \end{aligned} \tag{A.2}$$

where q is defined as follows from the matrix ϵ

$$\epsilon = \begin{pmatrix} 0 & \alpha_2 \\ -\alpha_1 & 0 \end{pmatrix} \tag{A.3}$$

The matrix ϵ is invariant under the transformation

$$T\epsilon T^t = T^t \epsilon T = \epsilon \tag{A.4}$$

where T^t is T transposed and $q = \alpha_1/\alpha_2$.

Higher representations of $SLq(2)$ are obtained by transforming the $(2j+1)$ monomials

$$\Psi^j_m = N^j_m x_1^{n_+} x_2^{n_-}, -j \leq m \leq j \tag{A.5}$$

by

$$x_1^{'} = ax_1 + bx_2 \tag{A.6}$$

$$x_2^{'} = cx_1 + dx_2 \tag{A.7}$$

where (a, b, c, d) satisfy the knot algebra (A.2) but x_1 and x_2 commute and $n_\pm = j \pm m$, and

$$N^j_m = \left[\langle n_+ \rangle_{q_1}! \langle n_- \rangle_{q_1}! \right]^{-1/2} \tag{A.8}$$

and $\langle n \rangle_q = \frac{q^{n-1}}{q-1}$. It is found that

$$\left(\Psi^j_m \right)^{'} = \sum D^j_{mm'} \Psi^j_{m'} \tag{A.9}$$

where

$$D^j_{mm'}(q|a,b,c,d) = \sum_{\substack{\delta(n_a+n_b,n_+) \\ \delta(n_c+n_d,n_-)}} A^j_{mm'}(q, n_a, n_c) \delta(n_a + n_b, n_+^{'}) a^{n_a} b^{n_b} c^{n_c} d^{n_d} \tag{A.10}$$

where $n_\pm^{'} = j \pm m^{'}$, $D^j_{mm'}$ is a 2j+1 dimensional representation of the $SLq(2)$ algebra and the $A^j_{mm'}$ is

$$A^j_{mm'}(q, n_a, n_c) = \left[\frac{\langle n_+^{'} \rangle_1 \langle n_-^{'} \rangle_1}{\langle n_+ \rangle_1 \langle n_- \rangle_1} \right]^{1/2} \frac{\langle n_+ \rangle_1 !}{\langle n_a \rangle_1 ! \langle n_b \rangle_1 !} \frac{\langle n_- \rangle_1 !}{\langle n_c \rangle_1 ! \langle n_d \rangle_1 !} \tag{A.11}$$

The oriented 2-dimensional projection of a 3-dimensional knot can be assigned three coordinates (N, w, r) where N is the number of crossings, w is the writhe and r the rotation. One can transform to new coordinates (j, m, m'). These indices label the irreducible representations of $D^j_{mm'}$ of the symmetry algebra of the knot, $SLq(2)$ by setting

$$j = N/2, \quad m = w/2, \quad m' = (r + o)/2 \tag{A.12}$$

This linear transformations makes half-integer representations possible. The knot constraints require w and r to be of opposite parity, therefore o is an odd integer. The knot (N, w, r) may be labeled by $D^{N/2}_{w/2,(r+o)/2}(a, b, c, d)$.

Table 1: The $D^{1/2}$ representation of the four preons.

m	m'	preon
1/2	1/2	a
1/2	-1/2	b
-1/2	1/2	c
-1/2	-1/2	d

Table 2: The $D^{3/2}$ representation of the standard model particles

m	m'	particle	preons
3/2	3/2	electron	aaa
3/2	3/2	neutrino	ccc
3/2	-1/2	d-quark	abb
-3/2	-1/2	u-quark	cdd

One assigns physical meaning to the $D^j_{mm'}$ in (A.10) by interpreting the a, b, c, and d as creation operators for spin 1/2 preons. These are the four elements of the fundamental $j = 1/2$ representation $D^{1/2}_{mm'}$ as indicated in Table 1. For notational clarity, I use in Tables 1. and 2. the preon names of [5]. The preon dictionary from the notation of [1] is the following:

$$\begin{aligned} m^+ &\mapsto a, \quad m^0 \mapsto c \\ m^- &\mapsto d, \quad \bar{m}^0 \mapsto b \end{aligned} \tag{A.13}$$

The standard model particles are the following elements of the $D^{3/2}_{mm'}$ representation as indicated in Table 2.

All details of the $SLq(2)$ extended standard model are discussed in the review article [5], including the gauge and Higgs bosons and a candidate for dark matter. I do not, however, see much advantage for introducing composite gauge bosons in the model. Introduction of color from preons is done slightly differently in [5]. In the early universe developments there is similarity between the knot model and the present preon model. Therefore, apart from the differences in color interpretation, the model of [1] and the knot algebra of [5] are equivalent in the fermion sector.

In summary, knots having odd number of crossings are fermions and knots with even number of crossings are correspondingly bosons. The leptons and quarks are the simplest quantum knots, the quantum trefoils with three crossings and $j = 3/2$. At each crossing there is a preon. The free preons are twisted loops with one crossing and $j = 1/2$. The $j = 0$ states are simple loops with zero crossings.

B Einstein-Kibble-Sciama Gravity

B.1 Introduction

To build a full Poincaré group gauge theory for gravity one has boosts, rotations and translations to consider: the rotations lead to curvature and the translations to torsion in spacetime. From a different point of view, curvature arises in the form of metric from energy and torsion in the form of a connection from spin. Torsion is therefore defined on microscopic scales. Torsion requires extension of the Riemann geometry to Riemann-Cartan (RC) geometry [6]. RC gravity, or Einstein–Sciama-Kibble (ESK) [7, 8] gravity can be reduced to Einstein gravity plus torsional contributions. A theory has been developed by Fabbri [9] for gravity with torsion and spinor matter fields, which yields a massive axial-vector coupled to spinors. His goal is to explain most of the open problems in the standard model of particles (and cosmology) as well as to analyze the nature of spinor fields. Here I apply the axial-vector coupling of [9] to preon interactions.

In general relativity metric is used to measure distances and angles. Connections are used to define covariant derivatives. In general form, a covariant derivative of a vector is defined by

$$D_\alpha V^\mu = \partial_\alpha V^\mu + V^\rho \Gamma^\mu_{\rho\alpha} \tag{B.1}$$

The connection $\Gamma^\mu_{\rho\alpha}$ has three indices: μ and ρ shuffle, or transform, the components of the vector V^ρ and α indicates the coordinate in the partial derivative.

Metric and connection should be unrelated. This is implemented by demanding that the covariant derivative of the metric vanishes. In this case the connection is metric-compatible. Metric-compatible connections can be divided into antisymmetric part, given by the torsion tensor, and symmetric part which includes a combination of torsion tensors plus a symmetric, metric dependent connection called the Levi-Civita connection.

In a general Riemannian spacetime $\mathbf{R}$, at each point p with coordinates x^μ, there is a Minkowski tangent space $\mathbf{M} = T_p\mathbf{R}$, the fiber, on which the local gauge transformation of the $T_{x^\mu}\mathbf{R}$ coordinates x^a takes place

$$x'^a = x^a + \epsilon^a(x^\mu) \tag{B.2}$$

where ϵ^a are the transformation parameters, μ is a spacetime index and a a fiber frame index.

The dynamics of the theory is based on vierbeins (tetrads) $e^a{}_\mu$, not on the metric tensor $g_{\mu\nu}$. The Cartan connection has a primary role and it is

$$\Gamma_{\mu\lambda\nu} = e^a{}_\mu \partial_\lambda e_{a\nu} \tag{B.3}$$

The tensor associated with this connection is torsion tensor

$$T^\mu_{\ \lambda\nu} = e_a{}^\mu(\partial_\lambda e^a{}_\nu - \partial_\nu e^a{}_\lambda) \tag{B.4}$$

Unfortunate for the development of gravitation theory, spin was not discovered in the laboratory before 1916. Spinors were introduced in mathematics by Cartan in the 1920's and spinor wave equation was found by Dirac in 1928.

B.2 Torsion as Axial-Vector Massive Field

Torsion has the property that it can be separated from gauge and metric factors. Let us start from the metric connection

$$\Lambda^{\rho}_{\alpha\beta} = \tfrac{1}{2} g^{\rho\mu} \left(\partial_\beta g_{\alpha\mu} + \partial_\alpha g_{\mu\beta} - \partial_\mu g_{\alpha\beta} \right) \tag{B.5}$$

The torsion tensor is completely antisymmetric only if some restrictions are imposed, called the metric-hypercompatibility conditions [14, 15, 16, 17, 18]. Then it can be written in the form

$$Q_{\alpha\sigma\nu} = \tfrac{1}{6} W^{\mu} \varepsilon_{\mu\alpha\sigma\nu} \tag{B.6}$$

where W^{μ} is torsion pseudo-vector, obtained from the torsion tensor after a Hodge dual. With the metric connection and the torsion pseudo-vector the most general connection can be written as a sum of $\Lambda^{\rho}_{\alpha\beta}$ and $Q_{\alpha\sigma\nu}$ as follows

$$\Gamma^{\rho}_{\alpha\beta} = \tfrac{1}{2} g^{\rho\mu} \left[\left(\partial_\beta g_{\alpha\mu} + \partial_\alpha g_{\mu\beta} - \partial_\mu g_{\alpha\beta} \right) + \tfrac{1}{6} W^{\nu} \varepsilon_{\nu\mu\alpha\beta} \right] \tag{B.7}$$

Functions $\Omega^{a}_{b\mu}$ that transform under a general coordinate transformation like a lower Greek index vector and under a Lorentz transformation as

$$\Omega'^{a'}_{b'\nu} = \Lambda^{a'}_{a} \left[\Omega^{a}_{b\nu} - (\Lambda^{-1})^{a}_{k} (\partial_\nu \Lambda)^{k}_{b} \right] (\Lambda^{-1})^{b}_{b'} \tag{B.8}$$

are called a spin connection. The torsion in coordinate formalism is defined as follows

$$Q^{a}{}_{\mu\nu} = -\left(\partial_\mu e^{a}_{\nu} - \partial_\nu e^{a}_{\mu} + e^{b}_{\nu} \Omega^{a}_{b\mu} - e^{b}_{\mu} \Omega^{a}_{b\nu} \right) \tag{B.9}$$

and the spin connection is given by

$$\Omega^{a}_{b\mu} = e^{\nu}_{b} e^{a}_{\rho} \left(\Gamma^{\rho}_{\nu\mu} - e^{\rho}_{k} \partial_\mu e^{k}_{\nu} \right) \tag{B.10}$$

which is antisymmetric in the two Lorentz indices after both of them are brought in the same upper or lower position. The most general spinorial connection is

$$\boldsymbol{\Omega}_{\mu} = \tfrac{1}{2} \Omega_{ab\mu} \boldsymbol{\sigma}^{ab} + i q A_{\mu} \mathbb{I} \tag{B.11}$$

where A_{μ} is the gauge potential. The spinorial curvature is using the spinorial connection

$$\boldsymbol{F}_{\alpha\beta} = \partial_\alpha \boldsymbol{\Omega}_{\beta} - \partial_\beta \boldsymbol{\Omega}_{\alpha} + [\boldsymbol{\Omega}_{\alpha}, \boldsymbol{\Omega}_{\beta}] \tag{B.12}$$

Let us define the decomposition of the spinor field in its left and right parts

$$\boldsymbol{\pi}_{L} \psi = \psi_{L} \qquad \overline{\psi} \boldsymbol{\pi}_{R} = \overline{\psi}_{L} \tag{B.13}$$

$$\boldsymbol{\pi}_{R} \psi = \psi_{R} \qquad \overline{\psi} \boldsymbol{\pi}_{L} = \overline{\psi}_{R} \tag{B.14}$$

so that

$$\overline{\psi}_L+\overline{\psi}_R=\overline{\psi} \quad \psi_L+\psi_R=\psi \tag{B.15}$$

Now one has 16 linearly-independent bi-linear spinorial quantities

$$2\overline{\psi}\boldsymbol{\sigma}^{ab}\boldsymbol{\pi}\psi=\Sigma^{ab} \tag{B.16}$$
$$2i\overline{\psi}\boldsymbol{\sigma}^{ab}\psi=S^{ab} \tag{B.17}$$
$$\overline{\psi}\boldsymbol{\gamma}^{a}\boldsymbol{\pi}\psi=V^{a} \tag{B.18}$$
$$\overline{\psi}\boldsymbol{\gamma}^{a}\psi=U^{a} \tag{B.19}$$
$$i\overline{\psi}\boldsymbol{\pi}\psi=\Theta \tag{B.20}$$
$$\overline{\psi}\psi=\Phi \tag{B.21}$$

To have the most general connection decomposed into the simplest symmetric connection plus torsion terms we substitute (B.7) in (B.10) and this in (B.11). The field equations reduce to the following

$$\nabla_{\rho}(\partial W)^{\rho\mu}+M^{2}W^{\mu}=X\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\pi}\psi \tag{B.22}$$

for torsion axial-vector and

$$\begin{aligned} &R^{\rho\sigma}-\tfrac{1}{2}Rg^{\rho\sigma}-\Lambda g^{\rho\sigma}= \\ &=\tfrac{k}{2}[\tfrac{1}{4}F^{2}g^{\rho\sigma}-F^{\rho\alpha}F^{\sigma}{}_{\alpha}+ \qquad (B.23)\\ &+\tfrac{1}{4}(\partial W)^{2}g^{\rho\sigma}-(\partial W)^{\sigma\alpha}(\partial W)^{\rho}{}_{\alpha}+ \\ &+M^{2}(W^{\rho}W^{\sigma}-\tfrac{1}{2}W^{2}g^{\rho\sigma})+ \\ &+\tfrac{i}{4}(\overline{\psi}\boldsymbol{\gamma}^{\rho}\boldsymbol{\nabla}^{\sigma}\psi-\boldsymbol{\nabla}^{\sigma}\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi+\overline{\psi}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\nabla}^{\rho}\psi-\boldsymbol{\nabla}^{\rho}\overline{\psi}\boldsymbol{\gamma}^{\sigma}\psi)- \\ &-\tfrac{1}{2}X(W^{\sigma}\overline{\psi}\boldsymbol{\gamma}^{\rho}\boldsymbol{\pi}\psi+W^{\rho}\overline{\psi}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\pi}\psi)] \qquad (B.24)\end{aligned}$$

for the torsion-spin and curvature-energy coupling, and

$$\nabla_{\sigma}F^{\sigma\mu}=q\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi \tag{B.25}$$

for the gauge-current coupling; and finally

$$i\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-XW_{\sigma}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\pi}\psi-m\psi=0 \tag{B.26}$$

for the spinor field equations which again can be split as

$$\tfrac{i}{2}(\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-\boldsymbol{\nabla}_{\mu}\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi)-XW_{\sigma}V^{\sigma}-m\Phi=0$$

$$\nabla_{\mu}U^{\mu}=0$$

$$\tfrac{i}{2}(\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\pi}\boldsymbol{\nabla}_{\mu}\psi-\boldsymbol{\nabla}_{\mu}\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\pi}\psi)-XW_{\sigma}U^{\sigma}=0$$

$$\nabla_{\mu}V^{\mu}-2m\Theta=0$$

$$i(\overline{\psi}\boldsymbol{\nabla}^{\alpha}\psi-\boldsymbol{\nabla}^{\alpha}\overline{\psi}\psi)-\nabla_{\mu}S^{\mu\alpha}+$$
$$+2XW_{\sigma}\Sigma^{\sigma\alpha}-2mU^{\alpha}=0$$

$$\nabla_{\alpha}\Phi-2(\overline{\psi}\boldsymbol{\sigma}_{\mu\alpha}\boldsymbol{\nabla}^{\mu}\psi-\boldsymbol{\nabla}^{\mu}\overline{\psi}\boldsymbol{\sigma}_{\mu\alpha}\psi)+2X\Theta W_{\alpha}=0$$

$$\nabla_{\nu}\Theta-2i(\overline{\psi}\boldsymbol{\sigma}_{\mu\nu}\boldsymbol{\pi}\boldsymbol{\nabla}^{\mu}\psi-\boldsymbol{\nabla}^{\mu}\overline{\psi}\boldsymbol{\sigma}_{\mu\nu}\boldsymbol{\pi}\psi)-$$
$$-2X\Phi W_{\nu}+2mV_{\nu}=0$$

$$(\boldsymbol{\nabla}_{\alpha}\overline{\psi}\boldsymbol{\pi}\psi-\overline{\psi}\boldsymbol{\pi}\boldsymbol{\nabla}_{\alpha}\psi)+\nabla^{\mu}\Sigma_{\mu\alpha}+2XW^{\mu}S_{\mu\alpha}=0$$

$$\nabla^{\mu}V^{\rho}\varepsilon_{\mu\rho\alpha\nu}+i(\overline{\psi}\boldsymbol{\gamma}_{[\alpha}\boldsymbol{\nabla}_{\nu]}\psi-\boldsymbol{\nabla}_{[\nu}\overline{\psi}\boldsymbol{\gamma}_{\alpha]}\psi)+$$
$$+2XW_{[\alpha}V_{\nu]}=0$$

$$\nabla^{[\alpha}U^{\nu]}+i\varepsilon^{\alpha\nu\mu\rho}(\overline{\psi}\boldsymbol{\gamma}_{\rho}\boldsymbol{\pi}\boldsymbol{\nabla}_{\mu}\psi-\boldsymbol{\nabla}_{\mu}\overline{\psi}\boldsymbol{\gamma}_{\rho}\boldsymbol{\pi}\psi)-$$
$$-2XW_{\sigma}U_{\rho}\varepsilon^{\alpha\nu\sigma\rho}-2mS^{\alpha\nu}=0$$

together equivalent to the spinor field equations above. From (B.22) one sees that torsion behaves like a massive axial-vector field satisfying Proca field equations. It is noted that torsion does not couple to gauge fields. Torsion and gravitation seem to have the same coupling constant. However, in [9] it is shown that using the Einstein-Sciama-Kibble field equations these two independent fields with independent sources can have independent coupling constants.

The preon-preon interaction is attractive and of short range due to the mass of the axial-vector field. The interaction includes two free parameters, the coupling constant X and the mass M of the axial-vector. Therefore, bound states of preons may be formed by the axial-vector interaction.

References

[1] Raitio, R. (1980) A Model of Lepton and Quark Structure. Physica Scripta, 22, 197. http://dx.doi.org/10.1088/0031-8949/22/3/002

[2] Raitio, R. (2016) Combinatorial Preon Model for Matter and Unification, Open Access Library Journal, 3: e3032. doi: http://dx.doi.org/10.4236/oalib.1103032

[3] Raitio, R. (2017) On the Conformal Unity between Quantum Particles and General Relativity. Open Access Library Journal , 4: e3342. http://dx.doi.org/10.4236/oalib.1103342

[4] Raitio, R. (2017) Preons, Standard Model and Gravity with Torsion. http://vixra.org/pdf/1703.0247v1.pdf

[5] Finkelstein, R. (2016) On the SLq(2) Extension of the Standard Model and the Measure of Charge, Int. J. Mod. Phys. A, 32, 1730001 [arXiv:1511.07919].

[6] Cartan, E. (1980) in NATO ASIB Proc. 58: Cosmology and Gravitation: Spin, Torsion, Rotation, and Supergravity, edited by P. G. Bergmann and V. de Sabbata, pp. 489–491.

[7] Kibble, T. (1961) J. Math. Phys. 2, 212.

[8] Sciama, D. (1962) in Recent Developments in General Relativity (Oxford).

[9] Fabbri, L. (2017) Foundations Quadrilogy [arXiv:1703.02287].

[10] Greenberg, O. (2009) The Color Charge Degree of Freedom in Particle Physics, Compendium of Quantum Physics, ed. D. Greenberger, K. Hentschel and F. Weinert, (Springer-Verlag, Berlin Heidelberg) pp. 109-111 [arXiv:0805.0289].

[11] Raitio, R. (2015) The Decay of a Black Hole in a GUT Model. Open Access Library Journal, 2: e2031. http://dx.doi.org/10.4236/oalib.1102031

[12] Thomson, W. (1868) Trans. Roy. Soc. Edinburgh 25, 217.

[13] Faddeev, L. and Niemi, A. (1997) Nature (London) 387, 58 [arXiv:hep-th/9610193].

[14] Fabbri, L. (2007) Annales Fond. Broglie.32, 215 [arXiv:gr-qc/0608090].

[15] Hayashi, K. (1976) Phys. Lett. B65, 437.

[16] Xin Yu (1989) Astrophysical and Space Science 154, 321.

[17] Audretsch, J. and Lämmerzahl, C. (1988) Class. Quant. Grav. 5, 1285.

[18] Macias, A. and Lämmerzahl, C. (1993) J. Math. Phys. 34, 4540.

[19] Hamzavi1, M., Movahedi, M., Thylwe, K.-E. and Rajabi, A. (2012) Chin. Phys. Lett. Vol. 29, No. 8, 080302.