

An Elementary Proof of Goldbach's Conjecture

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Human is very easily cheated by 2 words, infinite and anything, because we never see infinite and anything, so we always make simple thing complex. Goldbach's conjecture is about all very simple numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, we divided any even numbers into 5 groups and primes into 4 groups, Goldbach's conjecture becomes much simpler. Here we give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic, the prime number theorem, and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes, the prime number theorem

Introduction

Prime numbers¹ are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. Two of the basic rules are the "fundamental theorem of arithmetic" and Euclid's proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic,^[1] which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.^[2] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof^[2] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just n primes, then the biggest prime will be labeled P_n . Now we can form the number Q by multiplying together all these primes and adding 1, so

$$Q = (P_1 \times P_2 \times P_3 \times P_4 \dots \times P_n) + 1$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n .

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach's conjecture is about all numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, if we divide all even numbers into 5 groups and primes into 4 groups, Goldbach's conjecture will be much simpler.

If a number ($N > 13$) is not divisible by 3 or any prime which is smaller or equal to $N/3$, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: $1/3, 1/7 \times 2/3, 1/11 \times 2/3 \times 6/7, 1/13 \times 2/3 \times 6/7 \times 10/11 \dots$,

Let N_o represent any odd number, the chance of N_o to be a non-prime is: $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + \dots]$ -----Formula 1

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let \sum represent the sum of the infinite terms and $\Delta = 1 - \sum$, according to Euclid's proof^[3] that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. \sum may be very close to 1 when N is growing to ∞ , but always less than 1. Let $\Delta = 1 - \sum$, when N is growing to ∞ , Δ may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If Δ is 0, then there is no prime, that is not true.

The sum of first 20 terms = $[(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47) +$

$$\begin{aligned}
& (1/59 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53) + \\
& (1/61 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59) + \\
& (1/67 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61) + \\
& (1/71 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67) + \\
& (1/73 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67 \times 70/71) + \\
& (1/79 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 58/59 \times 60/61 \times 66/67 \times 70/71 \times 72/73) \\
& = [0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009069 + \\
& 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003989] = 0.688872
\end{aligned}$$

For the first 20 term: $\Sigma = 0.688872$, $\Delta = 1 - \Sigma = 0.311128$

The chance of N_0 to be a prime is: $\Delta = 1 - [(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/19 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19) + (1/29 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 36/37 \times 40/41 \times 42/43 + \dots)]$ -----Formula 2

Let \$1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191, ...; \$3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193, ...; \$7 represents a prime with 7 as its last digit, such as 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197, ...; and \$9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199, ...

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71, ...; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73, ...; O7 represents an odd number with 7 as its last digit, such as 7, 17, 27, 37, 47, 57, 67, 77, ...; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79, ...

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number (O1) with 1 as its last digit is a product of \$1 x \$1, \$3 x \$7, and \$9 x \$9. The first \$9 is 19, but the odd number 9 = 3 x 3, so 3 is the smallest prime for \$9.

The chance of any odd number O1 to be a prime is: $\Delta_1 = 1 - \sum_{i=1}^{\infty} [(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31)] + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + \dots]$ -----

Formula 3.

The sum of first 20 terms of $\sum_{i=1}^{\infty} [(1/3) + (1/11 \times 2/3) + (1/13 \times 2/3 \times 10/11) + (1/19 \times 2/3 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 10/11 \times 12/13 \times 18/19) + (1/29 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23) + (1/31 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29) + (1/41 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31) + (1/43 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41) + (1/53 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43) + (1/59 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53) + (1/61 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59) + (1/71 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61) + (1/73 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71) + (1/79 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73) + (1/83 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79) + (1/89 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83) + (1/101 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89) + (1/103 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89 \times 100/101) + (1/109 \times 2/3 \times 10/11 \times 12/13 \times 18/19 \times 22/23 \times 28/29 \times 30/31 \times 40/41 \times 42/43 \times 52/53 \times 58/59 \times 60/61 \times 70/71 \times 72/73 \times 78/79 \times 82/83 \times 88/89 \times 100/101 \times 102/103)] = 0.333333 + 0.060606 + 0.046620 + 0.029444 + 0.046086 + 0.01748 + 0.01568 + 0.011966 + 0.010747 + 0.008517 + 0.007506 + 0.006484 + 0.006032 + 0.005783 + 0.00529 + 0.004953 + 0.004564 + 0.003976 + 0.003861 + 0.003612 = 0.63254$

For the first 20 term: $\sum_{i=1}^{\infty} = 0.63254$, $\Delta_1 = 1 - \sum_{i=1}^{\infty} = 0.36746$, for any number N, $\Delta_1 > 0$.

Every odd number with 3 as its last digit is a product of \$3x\$1 or \$7x\$9; \$1 is decided by \$3 or \$3 is decided by \$1 and \$9 is decided by \$7 or \$7 is decided by 9, so we need to consider only \$3 and \$7, \$1 and \$9, \$3 and \$9, or \$1 and \$7.

The chance of any odd number O3 (\$3 x \$1 or \$7 x \$9) to be a prime is: $\Delta_3=1-\sum_3=1-[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23 \times 36/37) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23 \times 36/37 \times 42/43) + \dots]$ -----Formula 4

The chance of any odd number O7 (\$7 x \$1, or \$3 x \$9) to be a prime is: $\Delta_7=1-\sum_7=1-[(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + \dots]$ -----Formula 5

The formula 4 is same as formula 5, so there is almost same number of primes with 3 or 7 as their last digit.

The sum of first 20 terms of $\sum_3 = [(1/3) + (1/7 \times 2/3) + (1/13 \times 2/3 \times 6/7) + (1/17 \times 2/3 \times 6/7 \times 12/13) + (1/23 \times 2/3 \times 12/13 \times 6/7 \times 16/17) + (1/37 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 22/23) + (1/43 \times 2/3 \times 6/7 \times 16/17 \times 36/37 \times 12/13 \times 22/23) + (1/47 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47) + (1/67 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53) + (1/73 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67) + (1/83 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83) + (1/103 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97) + (1/107 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103) + (1/113 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107) + (1/127 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113) + (1/137 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127) + (1/157 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137) + (1/163 \times 2/3 \times 6/7 \times 12/13 \times 16/17 \times 36/37 \times 22/23 \times 42/43 \times 46/47 \times 52/53 \times 66/67 \times 72/73 \times 82/83 \times 96/97 \times 102/103 \times 106/107 \times 112/113 \times 126/127 \times 136/137 \times 156/157) = N/10[1/3+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.80+1/317.27+1/344.97+1/398.24+1/416.11+]=[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.006075+0.005848+0.0050773+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331] =0.6102883$

For the first 20 term: $\sum_3 = \sum_7 = 0.6103$, $\Delta_3 = \Delta_7 = 1 - \sum_3 = 0.3897$, $\Delta_3 = \Delta_7$,

For a large number N, Δ_3 should be very close to Δ_7 . For or any number N, Δ_3 or $\Delta_7 > 0$

Every odd number (O9) with 9 as its last digit is a product terms of \$1 x\$9, \$3x\$3 or \$7x\$7. We need to consider only \$1, \$3, and \$7.

The chance of any odd number O9 to be a prime is: $\Delta_9 = 1 - \sum_9 = 1 - [(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13) + (1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + \dots]$ -----Formula 6

The sum of first 20 terms of $\sum_9 = [(1/3) + (1/7 \times 2/3) + (1/11 \times 2/3 \times 6/7) + (1/13 \times 2/3 \times 6/7 \times 10/11) + (1/17 \times 2/3 \times 6/7 \times 10/11 \times 12/13)] + [1/2(1/23 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17) + (1/31 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23) + (1/37 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31) + (1/41 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37) + (1/43 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41) + (1/47 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43) + (1/53 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47) + (1/61 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53) + (1/67 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61) + (1/71 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67) + (1/73 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71) + (1/83 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73) + (1/97 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83) + (1/101 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83 \times 96/97) + 1/103 \times 2/3 \times 6/7 \times 10/11 \times 12/13 \times 16/17 \times 22/23 \times 30/31 \times 36/37 \times 40/41 \times 42/43 \times 46/47 \times 52/53 \times 60/61 \times 66/67 \times 70/71 \times 72/73 \times 82/83 \times 96/97 \times 100/101] = 0.333333 + 0.095238 + 0.051948 + 0.039960 + 0.028208 + 0.019622 + 0.013926 + 0.01129 + 0.009914 + 0.009222 + 0.008242 + 0.007152 + 0.006097 + 0.005462 + 0.005004 + 0.004868 + 0.004222 + 0.003424 + 0.003393 + 0.003294 = 0.660395$

For the first 20 term: $\sum_9 = \sum_9 = 0.660395$, $\Delta_9 = 1 - \sum_9 = 0.339605$, for any number N, $\Delta_9 > 0$

Euclid's proof that the set of prime numbers is endless, from these formulas, the number of primes \$1, \$3, \$7, \$9 are very close and they are endless.

For N=600, the smallest prime \$1 is 11, it decided the possible largest prime \$3 is 53, the smallest prime \$9 is 19, but 3 x 3 is 9, so the possible largest prime \$7 is 47, 47 x 3 x3=423 (the next will 67 x 3 x3=603>600), so we have: Prime number with 3 as its last digit=600/10 - { 600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)] = 600/10-600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 = 60-60x0.566654 = 60-34 = 26, it is same as the real number. For N=600, we have $\Delta_3=1-\sum 3=1-0.566654 = 0.433346$, every odd number with 3 as its last digit has almost 43% chance to be bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime. a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime.

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120,...

N-\$3= O7, if we can prove at least one of these O7 is \$7, then N is the sum of 2 primes; or N-\$7= O3, if we can prove at least one of these O3 is \$3, then N is the sum of 2 primes as table 1.

Table 1.

\$7	...	137	127	107	97	67	47	37	17	7
N-\$7	...	N-137	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7
\$3	...	113	103	83	73	53	43	23	13	3
N-\$3	...	N-113	N-103	N-83	N-73	N-53	N-43	N-23	N-13	N-3

From Formula 3, 4, 5, and 6, we know there are similar numbers of primes with 1, 3, 7, or 9 as their last digit.

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 7 as their last digit (the first line of table 1, $4x$ is due to there are only $1/4$ of total primes with 7 as its last digit), correspondingly, we have $N/[4x\ln(N)]$ of $O3(N-7$, the second line of table 1) in which there should be approximately $\{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\}$. For any number $N > 100$, $\{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\} > 4$, so for any number $N > 100$, there are more than 4 pairs of primes in which one prime (7) has 7 as its last digit and another prime ($N-7$) has 3 as its last digit. When N with 0 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For 3 and $N-3$ pairs (the bottom 2 lines of table 1) is also true.

Table 2.

\$1	...	151	131	101	71	61	41	31	11
$N-1$...	$N-151$	$N-131$	$N-101$	$N-71$	$N-61$	$N-41$	$N-31$	$N-11$
\$9	...	149	139	109	89	79	59	29	19
$N-9$...	$N-149$	$N-139$	$N-109$	$N-89$	$N-79$	$N-59$	$N-29$	$N-19$

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 1 as their last digit (the first line of table 2), correspondingly, we have

$N/[4x\ln(N)]$ of $O9(N-\$1$, the second line of table 2) in which there should be approximately $\{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\}$. For any number $N>100$, $\{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\}>4$, so for any number N , there are more than 4 pairs of primes in which one prime ($\$1$) has 1 as its last digit and another prime ($N-\$1$) has 9 as its last digit. When N with 0 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $\$9$ and $N-\$9$ pairs (the bottom 2 lines of table 2) is also true.

For $N = 600$ (see table 3), $\{N/[4x\ln(N)]\}/\ln\{N/[4x\ln(N)]\}=\{600/[4x\ln(600)]\}/\ln\{600/[4x\ln(600)]\} =23.449/\ln(23.449) =23.449/3.155 =7.43$, this is for only $\$7$ and $N-\$7$ pairs, plus $\$3$ and $N-\$3$ pairs, there are $7.43x2=14.86$, in fact, 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit. That should be true for $\$1$ and $N-\$1$ pairs and $\$9$ and $N-\$9$ pairs, in fact, 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime 9 as its last digit.

Table 3.

7	17	27	37	47	57	67	77	87	97	107	117	127	137	147	157	167	177	187
Prime	Prime	3x9	Prime	prime	3x19	Prime	7x11	3x29	Prime	prime	3x3x13	Prime	prime	3x7x7	prime	prime	3x59	11x17
593	583	573	563	553	543	533	523	513	503	493	483	473	463	453	443	433	423	413
Prime	11x53	3x191	Prime	7x79	3x181	13x41	prime	3x3x3x1 9	Prime	17x29	3x7x23	11x43	prime	3x151	Prime	prime	3x3x4 7	7x59
223	233	243	253	263	273	283	293	303	313	323	333	343	353	363	373	383	393	403
prime	prime	3x3x3 x3x3	11x23	prime	3x7x13	prime	prime	3x101	Prime	17x19	3x111	7x7x7	Prime	3x11x 11	Prime	Prime	3x131	13x31
377	367	357	347	337	327	317	307	297	287	277	267	257	247	237	227	217	207	197
13x29	prime	3x7x1 7	prime	prime	3x109	Prime	prime	3x3x33	7x41	Prime	3x89	Prime	13x19	3x79	Prime	7x31	3x3x2 3	Prime
387	397	407	417	427	437	447	457	467	477	487	497	507	517	527	537	547	557	567
3x3x43	prime	11x37	3x139	7x61	23x19	3x149	prime	prime	3x3x5 3	prime	7x71	3x13x 13	11x47	17x31	3x179	prime	prime	3x3x3 x3x7

213	203	193	183	173	163	153	143	133	123	113	103	93	83	73	63	53	43	33
3x71	7x29	prime	3x61	Prime	prime	3x3x17	11x13	7x19	3x41	3x3x13	prime	3 x31	prime	7x11	3x3x7	prime	prime	3x11
																3	13	23
																Prime	Prime	prime
																597	587	577
																3x199	prime	prime
589	579	569	559	549	539	529	519	509	499	489	479	469	459	449	439	429	419	409
19x31	3x193	Prime	13x43	3x3x61	7x7x11	23x23	3x178	Prime	Prime	3x163	Prime	7x67	3x3x3x17	Prime	Prime	3x11x13	Prime	Prime
11	21	31	41	51	61	71	81	91	101	111	121	131	141	151	161	171	181	191
Prime	3x7	Prime	Prime	3x17	Prime	Prime	3x3x3	7x13	Prime	3x37	11x11	Prime	3x47	Prime	7x23	3x3x19	Prime	Prime
381	371	361	351	341	331	321	311	301	291	281	271	261	251	241	231	221	211	201
3x127	7x53	19x19	3x3x3x13	11x31	Prime	3x107	Prime	7x43	3x97	Prime	Prime	3x3x29	Prime	Prime	3x7x11	13x17	Prime	3x67
219	229	239	249	259	269	279	289	299	309	319	329	339	349	359	369	379	389	399
3x73	Prime	Prime	3x83	7x37	Prime	3x3x31	17x17	13x23	3x103	11x29	7x47	3x113	Prime	Prime	3x3x41	prime	Prime	3x7x19
209	199	189	179	169	159	149	139	129	119	109	99	89	79	69	59	49	39	29
11x19	Prime	3x3x7	Prime	13x13	3x53	Prime	Prime	3x43	7x17	Prime	3x3x11	Prime	Prime	3x23	Prime	7x7	3x13	Prime

391	401	411	421	431	441	451	461	471	481	491	501	511	521	531	541	551	561	571
17x23	Prime	3x137	Prime	Prime	3x3x7x7	11x41	Prime	3x157	13x37	Prime	3x167	7x73	Prime	3x3x5x9	Prime	19x29	3x11x17	Prime
																	591	581
																	3x197	7x81
																	9	19
																	3x3	Prime

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 52, 62, 72, 82, 92, 102, 112, 122, ...,

Table 4.

\$3	...	103	83	73	53	43	23	13	3
N-\$3	...	N-103	N-83	N-73	N-53	N-43	N-23	N-13	N-3
\$9	...	149	139	109	89	79	59	29	19
N-\$9	...	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 3 as their last digit (the first line of table 4), correspondingly, we have $N/[4x\ln(N)]$ of $O9(N-3$, the second line of table 4) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}>1$, so for any number N , there are more than 1 pair of primes in which one prime (3) has 3 as its last digit and another prime ($N-3$) has 9 as its last digit. When N with 2 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For 9 and $N-9$ pairs (the bottom 2 lines of table 4) is also true.

Table 5.

\$1	...	151	131	101	71	61	41	31	11
$N-1$...	$N-151$	$N-131$	$N-101$	$N-71$	$N-61$	$N-41$	$N-31$	$N-11$

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 1 as their last digit (the first line of table 5), correspondingly, we have $N/[4x\ln(N)]$ of $O1(N-1$, the second line of table 5) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}>1$, so for any number N , there are more than 1 pair of primes in which one prime (1) has 1 as its last digit and another prime ($N-1$) has 1 as its last digit. When N with 2 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

3. When any even integer (N) has 4 as its last digit, such as 14, 24, 34, 44, 54, 64, 74, 84, 94, 104, 114,....

Table 6.

\$7	...	127	107	97	67	47	37	17	7
N-\$7	...	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 7 as their last digit (the first line of table 6), correspondingly, we have $N/[4x\ln(N)]$ of $O7(N-7)$, the second line of table 6) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\} > 1$, so for any number N , there are more than 1 pair of primes in which one prime ($\$7$) has 7 as its last digit and another prime ($N-7$) has 7 as its last digit. When N with 4 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 7.

\$3	...	153	113	103	83	53	43	23	13	3
N-\$3	...	N-153	N-113	N-103	N-83	N-53	N-43	N-23	N-13	N-3
\$1	...	181	151	131	101	71	61	41	31	11
N-\$1	...	N-181	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 3 as their last digit (the first line of table 7), correspondingly, we have $N/[4x\ln(N)]$ of $O1(N-3$, the second line of table 7) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}>1$, so for any number N , there are more than 1 pair of primes in which one prime (3) has 3 as its last digit and another prime ($N-3$) has 1 as its last digit. When N with 4 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For 1 and $N-1$ pairs (the bottom 2 lines of table 7) is also true.

3. When any even integer (N) has 6 as its last digit, such as 6,

4. 16, 26, 36, 46, 56, 66, 76, 86, 96, 106, 116,....

Table 8.

3	...	153	113	103	83	73	53	43	23	13	3
$N-3$...	$N-153$	$N-113$	$N-103$	$N-83$	$N-73$	$N-53$	$N-43$	$N-23$	$N-13$	$N-3$

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 3 as their last digit (the first line of table 8), correspondingly, we have $N/[4x\ln(N)]$ of $O3(N-3$, the second line of table 8) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}>1$, so for any number N , there are more than 1 pair of primes in which one prime (3) has 3 as its last digit and another prime ($N-3$) has 3 as its last digit. When N with 6 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 9.

\$9	...	149	139	109	89	79	59	29	19
N-\$9	...	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19
\$7	...	127	107	97	67	47	37	17	7
N-\$7	...	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 9 as their last digit (the first line of table 9), correspondingly, we have $N/[4x\ln(N)]$ of $O7(N-\$9$, the second line of table 9) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\} > 1$, so for any number N , there are more than 1 pair of primes in which one prime ($\$9$) has 9 as its last digit and another prime ($N-\$9$) has 7 as its last digit. When N with 6 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $\$7$ and $N-\$7$ pairs (the bottom 2 lines of table 9) is also true.

5. When any even integer (N) has 8 as its last digit, such as 8, 18, 28, 38, 48, 58, 68, 78, 88, 98, 108, 118,....

Table 10.

\$9	...	149	139	109	89	79	59	29	19
N-\$9	...	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19

According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 9 as their last digit (the first line of table 10), correspondingly, we have $N/[4x\ln(N)]$ of $O9(N-\$9$, the second line of table 10) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\} > 1$, so for any number N , there are more than 1 pair of primes in which one prime ($\$9$) has 9 as its last digit and another prime ($N-\$9$) has 9 as its last digit. When N with 8 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 11.

\$7	...	127	107	97	67	47	37	17	7
N-\$7	...	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7
\$1	...	151	131	101	71	61	41	31	11

N-1	...	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11
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According to the prime number theorem, the number of prime numbers less than N is approximately given by $N/\ln(N)$, so for any number N , there is approximately $N/[4x\ln(N)]$ of primes with 7 as their last digit (the first line of table 11), correspondingly, we have $N/[4x\ln(N)]$ of $O1(N-7$, the second line of table 11) in which there should be approximately $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\}$, For any number N , $\{N/[4x\ln(N)]\}/4\ln\{N/[4x\ln(N)]\} > 1$, so for any number N , there are more than 1 pair of primes in which one prime (7) has 7 as its last digit and another prime ($N-7$) has 1 as its last digit. When N with 8 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For 1 and $N-1$ pairs (the bottom 2 lines of table 11) is also true.

So we have proved that Goldbach's Conjecture is true if the prime number theorem is true for any even number > 4 .