A condition on a non-collatz number at the boundary of a successive collatz numbers set

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Abstract

We give a condition that an odd number in the neighborhood of a successive collatz numbers set must verify to be a non-collatz number, and we use the result for odd numbers of the form $6k-1$ at the boundary of a successive collatz numbers set.

1 Introduction

Since 1937, mathematicians tried to prove the conjecture of Collatz also known as the *3x+1* problem and Syracuse conjecture. In this paper, we prove a theorem that give another criteria on odd numbers to verify the conjecture under some conditions.

2 Definitions

We will use the modified form of Collatz's sequence defined by : $U : \mathbb{N}^* \longrightarrow \mathbb{N}^*$

$$U(n) = \frac{n}{2} \text{ if n is even}$$
$$U(n) = \frac{3n+1}{2} \text{ if n is odd}$$

The conjecture assumes that for every (non-zero) integer n there exists d such that : $U_{\boldsymbol{d}}(n) = 1$.
In this paper, we mean by:

$*$ An odd (respectively even) collatz number: an odd (respectively even) number of $\mathbb{N}^*$ verifying the Collatz conjecture.
$*$ A non-collatz number (assumption): a number of $\mathbb{N}^*$ that does not verify the Collatz conjecture.

3 Lemma

Let $\mathcal{A}_n = \{1, 2, 3, ..., E(\frac{2n-1}{3})\}$ be the set of all successive collatz numbers $\leqslant E(\frac{2n-1}{3})$ where n is an odd number $\geqslant 3$.
$\mathcal{A}_n = \{t \in \mathbb{N}^* : t \text{ a collatz number} \leqslant E(\frac{2n-1}{3})\}$.
Let $\mathcal{B}_n$ be the set of all successive collatz numbers inferior or equal to E($\frac{n+1}{6}$).
$\mathcal{B}_n = \{t \in \mathbb{N}^* : t \text{ a collatz number} \leqslant E(\frac{n+1}{6})\}$.

Lemma 1 *Let $n = 2p + 1$ be an odd number ($p \in \mathbb{N}^*$) with $\mathcal{A}_n \neq \varnothing$*

$$n \text{ is an odd non-collatz number} \Rightarrow \forall\ t \text{ odd} \in \mathcal{A}_n, \frac{3t+1}{2} \neq n$$

Proof 1 *Let $t \in \mathcal{A}_n$, where t is odd $\Rightarrow$ t is an odd collatz number $\Rightarrow U_{\mathbf{1}}(t) = \frac{3t+1}{2}$ is a collatz number $\Rightarrow$ if $n = \frac{3t+1}{2}$ then $n = U_{\mathbf{1}}(t)$ is also an odd collatz number.*

Then we can now state the theorem:

4 Theorem

Theorem 1 *Let $n = 2p + 1$ be an odd number ($p \in \mathbb{N}^*$) with $\mathcal{B}_n \neq \varnothing$*

$$\text{If } n \text{ is an odd non-collatz number then } n \neq 6k - 1 \ (\text{where } k = 1, 2, 3, \cdots \in \mathcal{B}_n)$$

Proof 2 *We consider $n = 2p + 1$ (where $p = 1, 2, 3, \cdots \in \mathbb{N}^*$) as an odd non-collatz number. From Lemma 1, we can write that,*

$$\forall\ t \in \mathcal{A}_n : \frac{3n+1}{2} \neq n \longrightarrow \frac{3t+1}{2} \neq 2p + 1 \longrightarrow t \neq \frac{4p+1}{3}$$

The resolution of the equation $t = \frac{4p+1}{3}$ giving odd numbers t, shows that it is satisfied by values of $p = 2, 5, 8, 11, 14, \cdots$
Therefore p is of the form $p = 2 + 3\alpha$ (where $\alpha = 0, 1, 2, 3, \cdots$)
t will have values like $t = 3, 7, 11, 15, 19 \cdots$, and n will have the form:

$$n = 2p + 1 = 2(2 + 3\alpha) + 1 = 6\alpha + 5 = 6k - 1 \ (where \ k = 1, 2, 3, \cdots \in \mathcal{B}_n)$$

5 Conclusion

We can generalize this result on a given set of successive collatz numbers already verified $\mathcal{C}_n = \{1, 2, 3, ..., n = 2p\}$. Therefore if the next odd element $n + 1$ is of the form $6k-1$ then it is a collatz number.