The world as emergent from pure entropy

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We propose a framework to understand the world by an ensemble of theorems rather than by a set of axioms. We prove meta-logically that the theorems of the ensemble must have *feasible* proofs and must recover *universality*. The ensemble is axiomatized when it is constructed as a partition function, in which case its axioms are, up to an error rate, the bits of Ω (the halting probability of a prefix-free universal Turing machine).

As a consequence of the axiomatization, the ensemble adopts the mathematical structure of an ensemble of statistical physics. It is from this context that the laws of physics are derived. It is shown that the Lagrange multipliers of the partition function are the fundamental Planck units and that the background, a thermal spacetime, emerges as a consequence of the limits applicable to the conjugate pairs. The background obeys the relations of special and general relativity, dark energy, the arrow of time, the Schrödinger equation, the Dirac equation and it embeds the holographic principle. In this context, the laws of physics connect to the limits of feasible mathematics.

The framework is so fundamental that informational-equivalents to length, time and mass (assumed as axioms in most physical theories) are here formally derivable. It can prove that no alternative framework can contain less bits of axioms than it contains (hence it is necessarily the simplest theory). Furthermore, it can prove that, for all worlds amenable to this framework, the laws of physics will be the same (hence there can be no alternatives).

0.1 Notation

We will use the following notations: The double vertical lines $|X|$ means the length of the string X. The suffix b, for example in 110_b, refers to the binary notation. The symbol Σ refers to an alphabet for strings. The notation Σ_b refers to the binary alphabet. In this case $X \in \Sigma_b$ means that X is a binary string, for example $0001_b \in \Sigma_b$.

1 Introduction

It is generally understood that a final and ultimate theory of everything (ToE) in physics should be elegant (in the mathematical sense). Indeed, it is hoped that a ToE could be formulated as a relatively small set of axioms, which explains all of the physics of the universe. The size of the set could (hopefully?) be between 10 to 30 axioms which would make the theory particularly elegant. Finding such a final theory is a major unsolved problem of physics.

Pondering on the difficulty of finding such a theory, I asked myself; what would happen if we were to eliminate the requirement that it should be elegant. Would the problem become simpler, and if so by how much? In the first part of this paper, if you allow me the indulgence, we will formulate a mathematically inelegant theory to describe the universe. This exercise will be justified under the thesis that any theory is better than no theory. Then in the second part, we will see that the exercise was well worth the effort as we will be able to improve the elegance of the theory quite significantly.

Relaxing the requirement on elegance allows us to define the universe using as many statements as we want (even possibly infinitely many statements). We can think of this method as a fool-proof way to produce a theory capable of describing all of the universe. To formulate it, we simply keep adding statements until we eventually explain all of the facts. As we will be speaking generally, the explicit formulation of the statements is not the primary interest. Therefore, we can discuss them abstractly by naming them Statement1, Statement2, Statement3, Statement4, etc. As these statements would not all be logically independent, they could presumably be axiomatized to a set of logically-independent axioms: Axiom1, Axiom2, Axiom3, etc..

I noticed that a set of abstract axioms could be connected to the arrow of time when I realized that any set of n logically independent axioms contained every axioms in its subsets but cannot by itself determine the axioms found in its supersets. For example, the set comprised of Axiom1, Axiom2, Axiom3, Axiom4 contains at least the information that is contained in the subset of Axiom1, Axiom2, Axiom3. But, as the axioms are logically independent, the smaller set cannot determine what Axiom5 is. All that was left to do was to define each instant of time as a specific set of a certain size. As time moves forward, the size of the set would increase to accommodate more axioms. The result is that the information of the past would be encoded in the set associated with the present, but the future would be undecidable until it occurs - yielding the arrow of time. At that point I was hooked to the idea: Time may have an arrow because the number of axioms grows with it. Under this initial intuition, I decided to further investigate the abstract connections between axioms and theorems with the goal of deriving more notions of physics.

After some research, I was able to show that the laws of physics are a meta-logical consequence of the connection between axioms and theorems. As long as the theorems have the property[1] that they recover *universality* and that they have *feasible* proofs, the laws of physics will be derivable and neither the axioms nor the theorems need to be explicitly specified. In this context, the limits of feasible mathematics and the laws of physics are the same.

[1] Discussed rigorously in section 4

The fundamental equation describing the universe in this interpretation is a discrete partition function describing a statistical ensemble of theorems. The equation, stated here, will be formally derived and justified in this paper;

$$Z = \sum_{i=1}^{\infty} 2^{-Af_i - |p_i|} \tag{1.1}$$

, where

$Z \in (\mathbb{R} \cap [0,1])$	numerical value of the sum	(1.2)
$\left(2^{-Af_i - \lvert p_i \rvert}\right)$	micro-state representing a theorem	(1.3)
$p_i \in \Sigma_b$	prefix code of theorem (binary string)	(1.4)
$\lvert p_i \rvert \in \mathbb{N}_{\geq 0}$	length of code	(1.5)
$f_i \in \mathbb{N}_{\geq 0}$	size of proof	(1.6)
$A \in \mathbb{R}_{\geq 0}$	conjugate to size of proof	(1.7)

let us also introduce:

$\Omega \in (\mathbb{R} \cap [0,1])$	halting probability of a prefix-free universal Turing machine	(1.8)

This equation connects a system of arbitrary logical complexity (but otherwise bounded by a proof-size cutoff) to a binary expansion of the axioms able to decide it's theorems. Each term of the sum can be interpreted as a micro-state describing a theorem. The micro-states are aware of two core properties of the theorems of the system; the length of their encodings (p_i) and the length of their proofs (f_i). The numerical value Z monotonically converges towards Ω as the conjugate A vanishes to zero. Z encodes (as bits) the axioms required to decide all statements of the system up to a proof-size cutoff enforced by A. In this way the interpretation of Z is similar to Ω but is only valid up to an error bound which goes to zero as A goes to zero.

This meta-logical equation provides an axiomatization procedure for feasible mathematics. It further recovers universal mathematics when the feasibility bound is lifted. With some interpretational subtleties[2], it holds for all possible formal systems. As a result, any successful and formal theory of everything in physics, as it would presumably contain axioms and theorems, must be meta-logically bounded by this equation.

This previously unknown yet relatively simple equation yields a surprising amount of physics provided that we are willing to adopt *information* and *computation* as the backbones of physics. From this equation, we show that the Lagrange multipliers of the partition

[2] The interpretation that Z converges towards an algorithmically-random number (Ω) only holds for ensemble of theorems sufficiently universal to embed the description of a universal Turing machine. For example, it holds for Peano's axioms and ZFC but not for Presburger arithmetic. In the later case, the equation would still produce a number for Z but it would not necessarily describe incompressible axioms.

function are the Planck units and that the background, a thermal spacetime, is emergent. We can show that this background obeys the laws of special relativity, general relativity, dark energy, the Schrödinger equation, the Dirac equation and that it embeds the holographic principle.

These laws are derived from pure reason with no appeal to physical observations as the limits of feasible mathematics. As the ensemble of theorems embeds a proof-size cutoff we can think of it as describing the limits of ideal computation connecting theorems to axioms.

1.1 *Outline*

The proof that we present in this paper is outlined as follows;

1. We start from the standard construction of Ω, the halting probability of a prefix-free universal Turing machine.

$$\Omega = \sum_{i=1}^{\infty} 2^{-H(p_i)-|p_i|} \qquad \text{where } H(p_i) = \begin{cases} 0 & p_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{1.9}$$

2. We augment Ω with knowledge of the size of the proof of each theorem (f_i). With this, we obtain the definition of Z.

$$Z = \sum_{i=1}^{\infty} 2^{-Af_i-|p_i|} \tag{1.10}$$

3. With knowledge of proof-size, Z decides feasible mathematics. The bound on feasibility is set by the value of A. (Of note, when the bound on feasibility is lifted ($A \to 0^+$), then $Z \to \Omega$. In this context, Ω would be interpreted as deciding universal mathematics.)

4. The limits of feasible mathematics and the laws of physics are the same.

1.2 *Avoiding the errors of the past*

Many philosophers, notably Leibniz, Spinoza and Bertrand Russel, have attempted to construct a description of all of reality as true/false statements. Specifically, Leibniz tried to create a language that would be able to decide any statements. As expected by the Gödel incompleteness theorem, the project ultimately failed. This is often and incorrectly interpreted as a failure of the true/false project. To understand why the failure interpretation is incorrect, an important nuance must be understood. It is not the project of encoding a system as true/false statements per se that as failed. In fact, this form of encoding is always possible when no restrictions are applied to the decidability of the statements. The failure occurs when one tries to formalize the project into a set of axioms of lesser complexity than the statements it encodes so as to improve its elegance. In the most complex systems, such as with the universe, many statements are undecidable and must remain so for the system to be consistent.

Having some statements undecidable might appear to be quite the limitation to the purest of Platonists, but as we will see it is what actually gives complexity to the universe.

2 *Preliminaries*

We must be careful with our use of language from this point on. Referring to the encoding as true/false statements will not suffice. Indeed, we can show that some problematic statements, known as logical paradoxes, do not have a true or a false answer. For example, the barber paradox can neither be true nor false without introducing a contradiction. Another example is the note-on-a-wall statement. Imagine a note on a wall with a single statement that reads "All statement on this note are false" - is the statement on the note true or false? Let's see. First, assume it is false. Then there must be one true statement on the note. But, we just assumed that its only statement is false. Hence the assumption must be wrong. Okay, now assume it is true. Then as per the statement, all statements must be false. But, we just assumed that it is true. A contradiction is obtained in both cases. This is an undecidable statements which contradicts the idea that all statements have a yes or no answer.

To get rid of these problematic statements, we simply re-align the scope of the true/false encoding to a more modern formulation. To properly define the encoding, we will introduce the notion of sentences, theorems, provable and non-provable. First, we refrain from using the word statement in favour of the word *sentence*. If a sentence of a language is provable within a logical system, we will say that the sentence is a theorem of the logical system. In the general case, it is undecidable if a sentence is or is not a *theorem* of a formal system. Reprising the note-on-a-wall example, we would ask: "is the sentence on the note a theorem" - the answer is simply no.

As a result, we no longer use the word *true* or *false* to describe sentences but will rather use *theorem* or *not-a-theorem* and the problem evaporates. For example, we will not refer to a sentence as being 'true in the universe' but instead as being a 'theorem of the universe'. Explicitly, the expression 'a theorem of the universe' means a sentence which is a theorem of the theory which explains the universe.

2.1 Formalism

Let us consider a specific language, say binary, and associate to each of its sentences a boolean value to be interpreted as related to the provability of the sentence within a formal logic system. The boolean value will be *true* if its associated sentence is a theorem and will be *false* otherwise. We list all sentences of binary in shortlex order (sorted both alphabetically and by length). Then, we associate a boolean value to each sentence. As the sentences are enumerated in shortlex, it is easy to see that all sentences are associated with a boolean value. As an illustrative example, consider the following case;

n	sentence (all)	boolean value (examples)	is a theorem?	
1	0_b	1	yes	(2.1)
2	1_b	0	no	(2.2)
3	00_b	1	yes	(2.3)
4	01_b	1	yes	(2.4)
5	10_b	1	yes	(2.5)
6	11_b	0	no	(2.6)
7	000_b	0	no	(2.7)
⋮	⋮	⋮	⋮	

As we can see, all sentences of binary are part of the list. If the sentence is a theorem of the logic system, its associated boolean value is 1, otherwise it is 0.

Historically, Émile Borel[3] suggested a know-it-all number which would encode the answer to all yes/no questions of a language. Here, we revisit this concept but using the modern terms. In his definition, we replace the word *question* with *sentence* and the *yes/no answer* with *theorem/not-a-theorem*.

[3] Gregory J. Chaitin. How real are real numbers? International Journal of Bifurcation and Chaos, 16(06):1841–1848, 2006a. DOI: 10.1142/S0218127406015726. URL http://www.worldscientific.com/doi/abs/10.1142/S0218127406015726

Definition 2.8 (Borel number)**.** *A Borel number is a real number between* 0 *and* 1*. It starts with* 0 *followed by a period and followed by an infinite expansion of binary digits. The digits of the expansion are obtained by concatenating all boolean values back to back. Each digit corresponds to the boolean value associated with its corresponding sentence. The* i^{th} *digit after the decimal corresponds to the boolean value associated with the* i^{th} *sentence. The Borel number of the above example would be* $0.1011100..._b$*. A Borel number encodes the provability of each sentence of a language as a single real number. A Borel number is an example of a real number that is non-computable in the general case.*

The Chaitin construction[4], also called an Omega number, is a

[4] Gregory J. Chaitin. A theory of program size formally identical to information theory. J. ACM, 22(3): 329–340, July 1975. ISSN 0004-5411. DOI: 10.1145/321892.321894. URL http://doi.acm.org/10.1145/321892.321894

generalization of the above modern formulation of the Borel number. Instead of associating each sentence to a natural number, they are instead associated to a *prefix-free code*.

Definition 2.9 (Prefix-free code)**.** *A prefix-free code is a set of sentences with the property that no member of the set is the prefix of another. For example, the sentence* 0_b *is a prefix of* 00_b *hence the set of these two sentences would not be a prefix-free code. But the set of* 0_b, 10_b, 110_b, 1110_b, 11110_b, *... would.*

If each sentence of a language $(s_1, s_2, s_3, ...)$ are associated to a prefix-free code $(p_1, p_2, p_3, ...)$ then the kraft inequality[5] holds.

[5] L. G. Kraft. A device for quanitizing, grouping and coding amplitude modulated pulses. Master's thesis, Mater's Thesis, Department of Electrical Engineering, MIT, Cambridge, MA, 1949

$$1 \geq \sum_{i=1}^{n} 2^{-|p_i|} \geq 0 \qquad (2.10)$$

, where

$$n \in (\mathbb{N} \cup \{\infty\}) \qquad \text{total number of codes} \qquad (2.11)$$

$$p_i \in \Sigma_b \qquad \text{prefix code string (in binary)} \qquad (2.12)$$

$$|p_i| \in \mathbb{N}_{\geq 0} \qquad \text{length of code} \qquad (2.13)$$

The inequality guarantees that the sum over the exponential decay of the length of the codes will be between 0 and 1 inclusively. In the case where the sentences are encoded with the unary code (a certain prefix-free code defined as : 0_b, 10_b, 110_b, 1110_b, 11110_b, ...), the modern formulation of the Borel number is recovered.

The Chaitin construction is defined for all sentences of a language encoded with a prefix-free code p. In this construction, each sentence is considered to be a program which either halts (if the sentence is provable), or doesn't (if it is non-provable).

$$\Omega = \sum_{i=1}^{n} 2^{-H(p_i)-|p_i|} \qquad \text{where } H(p_i) = \begin{cases} 0 & p_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \qquad (2.14)$$

, where

$$\Omega \in \mathbb{R} \cap [0, 1] \qquad \text{halting probability} \qquad (2.15)$$

When this construction applies to the programs that are executed by universal Turing machine (UTM), it is possible to prove that Ω is normal, algorithmically random, non-computable and non-compressible. Like a Borel number, Ω encodes which sentences of a language are theorems. Unlike the Borel number however, the digits of Ω do not necessarily have a one-to-one correspondence to a specific sentence. As a result, the interpretation of Ω is different:

The Chaitin construction connects to the halting problem of computer science. Consider a universal Turing machine running a program that is searching for a proof of p_1. If a proof is found, the program halts and $H(p_1)$ is equal to 0. Consequently, the term of the sum associated to p_1 does not vanish. In the case where p_2 is not a theorem, then the program will search forever and will never halt. In this case $H(p_2)$ is equal to ∞ and its contribution to the sum vanishes.

Knowledge of n bits of Ω allows an observer to count the total number of programs of length less than or equal to n that halts. The observer can then use this information to solve the halting problem for programs of length less than or equal to n. Hence, as the halting problem is unsolvable, the infinite expansion of the bits of Ω must be non-computable. As it is normalized between 0 and 1, Ω is often interpreted as the halting probability of a random program for a prefix-free universal Turing machine.

2.2 *Statistical physics*

Before continuing to the next section, we will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy

$$S = -k_B \sum_{x \in X} p(x) \ln p(x) \tag{2.16}$$

, where

$$S \in \mathbb{R}_{\geq 0} \quad \text{entropy} \tag{2.17}$$

$$k_B \approx 1.38 \times 10^{-23} \frac{m^2 kg}{s^2 K} \quad \text{Boltzmann constant} \tag{2.18}$$

$$X \quad \text{ensemble of micro-states} \tag{2.19}$$

$$x \in X \quad \text{micro-state} \tag{2.20}$$

$$p(x) \in \mathbb{R} \cap [0,1] \quad \text{probability of the system being in micro-state } x \tag{2.21}$$

Observable	Conjugate	Relation
Energy E	Temperature T	$\beta = 1/(k_b T)$
Volume V	Pressure p	$\gamma = p/(k_b T)$
Number of particles N	Chemical potential μ	$\delta = -\mu/(k_b T)$

Table 1: Typical observables of statistical mechanics.

subject to the fixed macroscopic observables. The solution for this is the Gibbs ensemble. Taking the observables listed in Table 1 as examples, the partition function becomes

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{2.22}$$

, where

$$Z \in \mathbb{R}_{>0} \qquad \text{normalization constant} \tag{2.23}$$

The probability of occupation of a micro-state is;

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{2.24}$$

The average values and their variance for the observables are;

$$\overline{E} = \sum_{x \in X} p(x)E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{2.25}$$

$$\overline{V} = \sum_{x \in X} p(x)V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{2.26}$$

$$\overline{N} = \sum_{x \in X} p(x)N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{2.27}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{E,V} = -\frac{\mu}{T} \tag{2.28}$$

which can be summarized as

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{2.29}$$

This is known as the equation of state of the thermodynamic system. The entropy can be recovered from the partition function. It is given by

$$S = k_B \left(\ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{2.30}$$

2.3 *Algorithmic thermodynamics*

Many authors (Bennett et al., 1998, Chaitin, 1975, Fredkin and Toffoli, 1982, Kolmogorov, 1965, Zvonkin and Levin, 1970, Solomonoff, 1964, Szilard, 1964, Tadaki, 2002, 2008) have discussed the similarity between physical entropy $S = -k_B \sum p_i \ln p_i$ and the entropy in information theory $S = -\sum p_i \log_2 p_i$. Furthermore, the similarity between the halting probability Ω and the Gibbs ensemble of statistical physics has also been studied (Li and Vitanyi, 2008, Calude and Stay, 2006, Baez and Stay, 2012, Tadaki, 2002). Indeed, the Gibbs ensemble compares to the halting probability as follows;

$$\begin{array}{cc} \text{Gibbs ensemble} & \text{Halting probability} \\ Z = \sum_{x \in X} e^{-\beta(E+pV+Fx)} & \Omega = \sum_{p \text{ halts}} 2^{-|p|} \end{array} \tag{2.31}$$

Interpreted as a Gibbs ensemble, the halting probability forms a statistical ensemble where each program corresponds to one of its micro-state. It maximizes the entropy subject to constraints on its observables. The halting probability admits a single observable; the prefix code length $|p_i|$. As a result, it describes the partition function of a system which maximizes the entropy subject to the constraint that the average length of the codes is a constant $\overline{|p|}$;

$$\overline{|p|} = \sum_{p \text{ halts}} |p| 2^{-|p|} \qquad \text{from 2.25} \tag{2.32}$$

, where

$$\overline{|p|} \in (\mathbb{R} \cap [0, 2]) \qquad \text{Average prefix code length of } \Omega \tag{2.33}$$

In this interpretation, the halting probability will have an entropy which corresponds to the choice of prefix-free codes available to encode the programs.

$$S = k_B \left(\ln \Omega + \overline{|p|} \ln 2 \right) \qquad \text{from 2.30} \tag{2.34}$$

where the constant $\ln 2$ comes from the base 2 of the halting probability function instead of base e of the Gibbs ensemble.

John C. Baez and Mike Stay[6] take the analogy further by suggesting an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be observables. Starting from Gregory Chaitin's Ω number, the halting probability

[6] John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL http://dx.doi.org/10.1017/S0960129511000521

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{2.35}$$

is extended with algorithmic observables to obtain

$$\Omega' = \sum_{p \text{ halts}} 2^{-\beta E(p) - \gamma V(p) - \delta N(p)} \tag{2.36}$$

Noting the similarity between the Gibbs ensemble of statistical physics (2.22) and (2.36), these authors suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the tradeoff between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.
3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (2.36), they derive analogues of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*.

Other authors have suggested other somewhat arbitrary mappings[7].

[7] Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986; and Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic information theory. In Local Proceedings of the Computability in Europe 2008 (CiE 2008), pages 425–434. University of Athens, Greece, Jun 2008. URL http://arxiv.org/abs/0801.4194

2.4 *Existence of a preferred mapping*

We have found that there exists a preferred mapping such that the laws of physics can be seen as having an origin in algorithmic information theory (AIT). The mapping will be introduced starting from section 4 onwards. As we introduce physical quantities into the discourse, we will initially prefix them with the word *algorithmic* so as to make their AIT-origin explicit. For example, the *action* will be introduced from AIT as the *algorithmic-action*. Introducing a quantity prefixed with the word algorithmic, as in *algorithmic-"quantity"*, should be understood as posing a mapping between the AIT-derived quantity and the physical quantity. The units of these algorithmic quantities will also follow the pattern. For example, the *algorithmic-action* has the units of *algorithmic-Joules* times *algorithmic-seconds*, etc.

3 *Philosophical justifications*

Developing the philosophy behind the methodology will be of the utmost importance as it will allow us to define the scope of the theorems that are part of the universe. Without this part, we would not know if the universe comprises all possible theorems, or a fraction of such and if so which fraction specifically. We want to avoid the situation where we over-scoped the set of theorems as that would mean that we would recover properties not applicable to the universe. For example, should the theorems encode the positions of particles? Or should they encode the solutions to mathematical problems? etc. This is what we want to answer in this section. But, before we dive in, let us do a brief recap of select philosophical results that will be of use to us.

3.1 *The cogito ergo sum*

To understand how the scoping of the theorems will be achieved, we have to recall the philosophy of René Descartes (1596-1650), the famous 17th century french philosopher most well-known for his derivation of *cogito ergo sum* - I think, therefore I am. As we will see, the proper scoping is naturally obtained when we modernize his universal doubt method into a formal logic system such as first order logic. But first, let us recall what the universal doubt method is and how it applies to the derivation of the cogito.

Descartes' main idea was to come up with a test that every statement must pass before it will be accepted as true. The test will be the strictest test imaginable. Any reason to doubt a statement will be a sufficient reason to reject it. Then, any statement which survives the test will be considered irrefutable.

Using this test, and for a few years, Descartes rejected every statement he considered. The laws and customs of society, as they have no logical justifications, are obviously the first to be rejected. Then, he rejects any information that he collects with his senses; vision, taste, hearing, etc, on the grounds that a "demon" (think hallucinogens) could trick his senses without him knowing. He also rejects the theorems of mathematics on the grounds that axioms are required to derive them, and such axioms could be wrong. For a while, his efforts were fruitless and he doubted if he would ever find an irrefutable statement.

But, eureka! He finally found one which he published in 1641. He doubts of things! The logic goes that if he doubts of everything, then it must be true that he doubts. Furthermore, to doubt he must think and to think, he must exist (at least as a thinking being). Hence, cogito ergo sum, or I think, therefore I am.

3.2 Miniversal logic

We now refocus our interest to Descartes' universal doubt method itself and not so much in the cogito. To identify the theorems of the universe, we will repeat Descartes' universal doubt method within the context of a formal logic system. The method will produce a *minimal* set of rules whose theorems are the theorems of the *universe* - hence the name *Miniversal logic*.

Miniversal logic is, in many ways, similar to the constructivist project in mathematics but taken to the extreme. We select first-order logic as our starting point. Then, as we do not know which axioms are the true axioms of the universe, we remove all formal axioms from first-order logic on the ground that they carry doubt. Then, we further remove all *rules of inference* with the exception of the *rule of deduction*. This method parallels Descartes' universal doubt method within first order logic. The main argument is that if we remove from first order logic all formal axioms and all rules of inference which could potentially be controversial, then whatever theorem is left will surely be irrefutable. The result is a system of logic which, essentially, does not deceive its user.

Like Descartes with the cogito, we will also obtain statements that cannot be doubted of, but since we have formalized Descartes' method within first-order logic, the irrefutable statements obtained will be logic statements and are therefore mathematically usable. Specifically, the irrefutable statements obtained are the theorems of Miniversal logic.

To write sentences in a clear and unambiguous manner, Miniversal logic preserves the syntax of first order logic but does not retain its rules of inference (with the exception of the rule of deduction). As only the *rule of deduction* remains, let us recall its definition.

Definition 3.1 (Rule of deduction)**.** *The rule of deduction formalize the idea that proving a theorem using a set of assumptions is valid within these assumptions. It shows that if by assuming A one can show that* $A \vdash B$*, then* $A \to B$ *is a theorem of the logical system. It is often considered one of the most obvious rule of inference of logic, as without it we cannot extend a* *logical system with new axioms/assumptions. Using the rule of deduction, we can start from seemingly nothing and rebuild any of the familiar logic systems such as Peano's axioms (PA) or set theory (ZFC) by assuming their axioms.*

Why keep the rule of deduction? For the simple reason that using it does not introduce doubt but removing it would. It is the only standard rule that has this property. For example lets consider another rule, say the rule of excluded middle. Adopting this rule in the foundation of the theory would increase doubt as it is impossible to determine a-priori if this is a valid rule of the universe or not. However, introducing it by first appealing to the rule of deduction would be fine. Indeed, in the latter case we would say "if we *assume the rule of excluded middle* (via the rule of deduction), then we can prove by contradiction, for example, that $\sqrt{2}$ is irrational". It only affects the branch of the tree under which it is assumed and not the whole system.

In Miniversal logic, no theorem stands on its own. Any theorem must include, within its description, the list of assumptions that are required to prove it. The user of Miniversal logic is always reminded that the theorems that he proves are of the form 'Assume A, then A proves C'. Hence, by the rule of deduction, $A \rightarrow C$ is a theorem, but C by itself never is. Miniversal logic can be interpreted as the starting point of all logical work - it is the state of mind a logician is in before having morning coffee and selecting a specific system of axioms to work with. As a result and compared to other logic systems, it more accurately represents reality as it reflects the full freedom available to the logician to select any set of axioms prior to working.

3.3 Discussion on metaphysics

Le me apologize for injecting metaphysics into a physics paper, but if you will allow, we will see that it will be useful. In this paper we are only interested in deriving the following:

Definition 3.2 (Bridge from metaphysics to physics)**.** *A method to remove elements from the set of all possible universes until only one element is left. The derivation should rely only on the application of pure reason. It should not rely on empirical observations.*

The goal of this section is to construct a bridge from metaphysics to physics. The completion of the exercise will identify all sentences that are theorems of the universe. To iron out the subtleties we will present, in the long standing tradition of philosophy, an hypothetical dialogue about the thesis. The dialogue is based on a number of real conversations[8] which has been edited and combined to remove

[8] Specifically, when Alice's dialogue is taken verbatim from a conversation with Toid Boigler, it will be side-noted with the initials TB.

repetitions, to accelerate the flow and to help illustrate the points being made.

Alice: - *I believe in empiricism. To derive the laws of physics, one must make observations. Without these observations, there is no way to know which of many possible worlds is the actual world. For example, is the geometry of the universe euclidian or hyperbolic? Is the speed of light maximal? Does the microscopic world obeys the Schrödinger equation? Etc. Pure reason alone cannot prove these to be actual. Only continual observations followed by refinements or falsifications can improve our degree of confidence in a scientific theory.*

I understand your point of view, but I believe I have found a bridge between metaphysics and physics that allows one to obtain irrefutable knowledge about the universe. I will try to explain the bridge from the following angle. First, lets assume that the cogito is true: *I think, therefore I exist*. Do you agree with the cogito?

Alice: - *Yes.*

Then, for I to exist, the universe must be *restricted* in some way. At the very least, it must be such that it does not contradict the existence of thought. We have now essentially reformulated the anthropic principle as an extension to the cogito. I think, therefore I exist - *and to exist, I must actually exist in a universe capable of supporting such existence.* Would you agree that this argument rules out some universes?

Alice: - *Fair enough, yes - it rules out the [...] universes incompatible with the existence of thought.[...]*[9]

[9] TB

OK. From that, we already have a slight connection between metaphysics and physics. An argument from pure reason, the cogito extended with the anthropic principle, can be used to place restrictions on what the universe can be. As it contains very little information, the restriction is very loose, but it is nonetheless a restriction.

Alice: - *I agree that the anthropic principle rules out universes that are not capable of producing an observer. But, a scientific theory should make precise and falsifiable predictions and the anthropic principle is not sufficiently specific for that.*

Now we enter the core of the argument. We will use Miniversal logic to improve the specificity of the anthropic principle. Each theorem of Miniversal logic that we can provide will serve to further restrict what the universe is. For example, using my mind I can prove the sentence "PA implies that two plus two equals four", and since my mind is in the universe, then the sentence must be a theorem of both my mind and the universe. This is how we find the theorems applicable to the universe. We have now restricted the universe by two statements instead of one. So the previously poorly defined bound is now slightly better defined. Agree, or disagree?

> Alice: *Well, you want the phrase "theorem of the universe" to be telling us something about the physical world; to put it in your own words, "...this is how we bridge metaphysics to physics." But how does this work? If "true in the universe" just means provable in PA or ZFC or whatever (as you seem to have just said), how does this provide any link with physical reality at all?* [10]

[10] TB

Hold on, it appears that you have missed a subtlety. "Provable in the universe" means provable in the Miniversal logic system I defined earlier. If you use another logic system than Miniversal logic (such as PA) then the argument does not work. If you use PA or ZFC, then the theorems rely on the axioms PA or ZFC. As the universe might have other axioms than PA or ZFC, we cannot prove that PA's or ZFC's theorems are indeed the theorems of the universe. However, Miniversal logic teaches us that the theorems of the universe are not "two plus two equals four", they are "Assume PA, then two plus two equals four". The "Assume PA" prefix is what the subtlety is all about. "Assume PA, then 2+2=4" is a theorem of the universe because, it is true that in the universe, if you assume PA you can prove (within PA) that 2+2=4. You can easily do the exercise in your head to prove that it can be done in the universe.

> - Alice: *OK, so you want to think of all mathematical proofs as conditional - if certain axioms hold, then certain consequences follow. Fine. How does that provide any connection with physics or the physical world?*[11]

[11] TB

Well yes, mathematical proofs that are explicitly conditional on assumptions derived exclusively from the rule of deduction are theorems of the universe. Whereas those that do not meet this condition are theorems of their respective logic system. For example, "2+2=4" is a theorem of Peano's axioms. But, "Assume PA, then 2+2=4" is a theorem of the universe. So all worlds where "Assume PA, then 2+2=4" is not true are ruled out.

> - Alice: *This is one point where I am a little confused. Pure logic (call it [Miniversal] logic if you want) guarantees that PA implies 2+2=4. So it's hard to see what worlds it rules out - unless you mean worlds in which there is a mind, but that [this] mind is too [primitive] to realize that PA implies 2+2=4. Is that the kind of world that you take to have been ruled out? If so, I am OK with what you have said.*[12]

[12] TB

Yes - that is part of what I am ruling out. Generally speaking, I am ruling out any world which does not embed *universal reason*. I also rule out worlds for which logic would be incomplete and worlds which would contradict logic by say, letting you prove any theorems regardless of the axioms that you assume.

Since our mind is able, in principle, to explore all branches of Miniversal logic and since the universe must embed our mind, we can precisely identify all the theorems of the universe: The ultimate theory which describes the universe must have, as its theorems, all theorems of Miniversal logic.

> Alice: *Here I really don't know what you mean, unless you are just saying that there are no 'violations' of [Miniversal] logic in the world. If that's what you mean, I'm happy with that claim.*[13]

[13] TB

I am indeed claiming that there are no violations of Miniversal logic in the universe, but I am also claiming something additional. What I am claiming is that we can use Miniversal logic and the anthropic principle to completely restrict the universe to a single solution.

Consider the following; each theorem of Miniversal logic that we supply can be used to restrict the universe further. In principle, we can supply arbitrarily many theorems. PA has "2+2=4" as a theorem, but it also has "2+3=5", etc. Then, ZFC also has infinitely many theorems as well. If we keep supplying theorems, we will eventually supply all theorems for all branches of Miniversal logic[14]. Furthermore as Miniversal logic is universal, all possible theorems for all possible sets of assumptions will eventually be supplied. No patches of theorems will be left out by the process.

[14] To avoid hanging on non-provable sentences, we will have to work in dovetail. We will return to the notion of dovetailing in the next section.

As a result, we will have maximally restricted what the universe can be. Indeed, the universe cannot be simpler than Miniversal logic because that would mean leaving a theorem out (but we already said the work will eventually supply all possible theorems so none can be left out). What about complexity - can the universe be more complex then Miniversal logic? The universe cannot be more complex than Miniversal logic either because that would mean the universe has theorems that Miniversal logic hasn't (but this cannot be the case because Miniversal logic already embeds all possible theorems within its branches).

Therefore, as the universe is restricted both from the perspective of increasing its complexity as well as reducing it, the bound cannot be improved furthermore. The method herein described fully restricts the universe to a single solution.

> Alice: *I am not sure [I see where you are going with this]. I'm happy to say that the universe must allow for the possibility of a mind that, in principle, can verify all the theorems of [Miniversal] logic. But what follows from that?*[15]

[15] TB

Usually a theory is first defined by a set of axioms, then the theorems follows from them. In our present situation, we have a list of theorems but we do not have the theory which explains such theorems. The theory is *inside-out*. The next step will be to *axiomatize* the theory into a short list of axioms instead of infinitely many theorems.

> Alice: *I don't understand this at all. What we have now are all the tautologies of [Miniversal] logic. What connection is there between that and a physical theory?*[16]

[16] TB

The connection is that, for the reasons stated, the theorems of Miniversal logic are the theorems of the universe. Hence Miniversal logic, as its theorems are identical to those of the universe, must fully describe the universe.

> Alice: *You say "The theorems of [Miniversal] logic are theorems of the universe.". If by this you just mean that the universe obeys the laws of [Miniversal] logic, then yes, I agree. Then you say "Hence Miniversal logic, as its theorems are identical to those of the universe, must fully describe the universe." This seems clearly wrong. It is true in the universe that there [is the law of gravity]. That there [is the law of gravity] is, however, not a theorem of [Miniversal] logic. Thus, the theorems of [Miniversal] logic do not fully describe the universe.*[17]

[17] TB

There is a misunderstanding. I am not claiming that the laws of the universe can be found within Miniversal logic under a certain set of assumptions. What I am claiming is that Miniversal logic is itself isomorphic to the universe. Miniversal logic along with the anthropic principle has allowed us to establish that whatever theory of physics we construct to explain the universe, it must exactly recover the theorems of Miniversal logic - it cannot do more and it cannot do less.

Understanding a theory from infinitely many theorems is neither convenient nor elegant. To improve the elegance, we will axiomatize Miniversal logic to its most elegant form. The limits of this ideal axiomatization will be found to have the same behaviour as the universe. It is from those limits that the laws of physics are derived.

> Alice: *Can you spell out the [axiomatization that] you have in mind?*[18]

[18] TB

Yes, we are now ready to return to our interpretation of the universe as an ensemble of theorems.

3.4 Summary

The main argument of this section can be broken down and summarized in a few points.

1. As Miniversal logic is a reproduction of Descartes' universal doubt method, its theorems are 'irrefutable' for the same reasons and to the same degree as the cogito is 'irrefutable'.

2. The anthropic principle guarantees that each theorem of Miniversal logic is a 'synthetic a-priori' statement. Hence, each theorem

restricts what the universe can be. The implication is that the ultimate theory which explains the universe must have, as its theorems, all theorems of Miniversal logic.

3. Miniversal logic is universal - it embeds all possible theorems within its branches. Hence the ultimate theory which explains the universe cannot have more theorems than Miniversal logic.

4. As a result, the 'truth content' of the universe is identical to the 'truth content' of Miniversal logic - no more no less. The universe is reduced to a single solution.

5. The conjecture is that by axiomatizing the theorems of Miniversal logic to its most elegant form, the laws of physics will be recoverable from the limits of this axiomatization.

4 *The universe as an ensemble of theorems*

Definition 4.1 (Universal ensemble)**.** *An ensemble of theorems is universal if it includes all possible theorems for all possible assumptions.*

The ensemble of theorems corresponding to the universe must include all theorems of Miniversal logic. Furthermore, as Miniversal logic contains all theorems, the ensemble is universal.

We are looking for a construction of the ensemble of theorems that will connect it to its axiomatic representation. This can be done by making use of Chaitin's construction. To be able to construct Ω, we first adopt first-order Peano's axioms (PA) as the meta-language. As per the standard construction of Ω in PA, we begin by listing in shortlex all sentences of Miniversal logic. Each sentence is either a theorem or a non-theorem - this defines $H(p_i)$. We then encode the sentences with a prefix-free code. The degree of contribution of each encoded sentence to the sum is exponentially proportional to the negative of the length of the code. This produces:

$$\Omega = \sum_{i=1}^{\infty} 2^{-H(p_i)-|p_i|} \qquad \text{where } H(p_i) = \begin{cases} 0 & p_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{4.2}$$

The sum is performed over all theorems of Miniversal logic and it produces a single real number (Ω) as the result. To help fix the idea, consider the following example using a unary prefix-free code;

$$= 2^{-\infty}2^{-1} + 2^{-0}2^{-2} + 2^{-0}2^{-3} + 2^{-0}2^{-4} + 2^{-\infty}2^{-5} + ... \tag{4.3}$$

The presence of the negative infinity in the term of the exponential causes some terms to vanish to zero. Note that the suffix b indicates the binary notation.

$$= 0_b + 0.01_b + 0.001_b + 0.0001_b + 0_b + ... \tag{4.4}$$
$$= 0.01110..._b \tag{4.5}$$

which recovers Ω for the example values.

As the bits of Ω are algorithmically random, they can be given an interpretation as axioms. This is explained by Gregory Chaitin[19]. He explains that the first N bits of Ω can decide the halting problem for programs of length less than or equal to 2^N bits. As the halting problem is unsolvable, the infinite expansion of Ω must be algorithmically random. Hence, the value of the bits of Ω are, at best, set axiomatically.

[19] Gregory J. Chaitin. The limits of reason. Scientific American, 294(3): 74–81, 2006b. URL https://www.cs.auckland.ac.nz/~chaitin/sciamer3.html; and G. J. Chaitin. Foundations of Mathematics. ArXiv Mathematics e-prints, February 2002

As the infinite expansion of Ω is algorithmically random, the representation is not yet mathematically elegant (as it contains infinitely many bits/axioms). The elegance will be soon be improved in the next sections on entropy 4.2 and on feasibility 4.3.

- Alice: *I have reservations about using a meta-language to describe a universal ensemble of theorems. Since PA is a branch of Miniversal logic, are we not inadvertently restricting the universality of the ensemble by describing it using a system which is only a branch of it?*

To address this reservation, I will argue that Miniversal logic repeats itself within some of its branches. Any branches which is rigorous and sufficiently flexible to repeat Miniversal logic can potentially be used as a meta-language to describe the universal ensemble. One such system is first-order PA.

To visualize why this is possible, consider a famous example, the hyperwebster dictionary, proposed by mathematician Ian Stewart. In this example, Ian Stewart considers the case of a publishing company attempting to create a book which contains all words producible by the permutations of the letters A to Z. As there are infinitely many such words, the book would be never end (hence it is meant to be understood as an abstract object). The book will contain every possible permutation of letters including; garbled words likes ADEERFKG, valid words such as OBJECT, etc. The publisher arranges the words in the following way:

$$A, \quad AA, \quad AAA, \quad ... \quad AB, \quad ABA, \quad ABAA, \quad ... \quad AC, \quad ... \quad AZ, \quad AZA, \quad ... \tag{4.6}$$
$$B, \quad BA, \quad BAA, \quad ... \quad BB, \quad BBA, \quad BBAA, \quad ... \quad BC, \quad ... \quad BZ, \quad BZA, \quad ... \tag{4.7}$$
$$C, \quad CA, \quad CAA, \quad ... \quad CB, \quad CBA, \quad CBAA, \quad ... \quad CC, \quad ... \quad CZ, \quad CZA, \quad ... \tag{4.8}$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$
$$Z, \quad ZA, \quad ZAA, \quad ... \quad ZB, \quad ZBA, \quad ZBAA, \quad ... \quad ZC, \quad ... \quad ZZ, \quad ZZA, \quad ... \tag{4.9}$$

Then, as the example goes, the publisher decides to split the dictionary in 26 volumes. One volume for each letter. The volume A contains the first line (4.6). The volume B contains the second line, and so on. But then, an editor notices that there is no need to print the 25 other volumes as their content can be recovered in volume A by simply removing the leading A from each word.

$$(A), \quad (A)A, \quad (A)AA, \quad ... \quad (A)B, \quad (A)BA, \quad (A)BAA, \quad ... \quad (A)C, \quad ... \quad (A)Z, \quad (A)ZA, \quad ... \tag{4.10}$$

removing the leading A produces the content of all of the volumes:

$$A, \quad AA, \quad ... \quad B, \quad BA, \quad BAA, \quad ... \quad C, \quad ... \quad Z, \quad ZA, \quad ... \tag{4.11}$$

The implication is that the volume contains the same content as the book. This example is often used to illustrate the non-intuitiveness of infinities. It is for the similar reasons that we can recover the full content of Miniversal logic using english, french, first-order PA, etc. - even in the case where such are branches of Miniversal logic.

Starting from PA, we can recover the branches of Miniversal logic by making use of Gödel's numbering method. We first list all sentences of the english language (or the binary language) in shortlex. Then, to each sentence we associate a prime number. Finally, we define a set of equations which defines the allowable operations to be done of the prime numbers - this is the set of axioms. As PA contains both addition and multiplication, all sets of axioms can be encoded in such a way. As a result, the description of Miniversal logic is recovered within PA. Hence, a universal ensemble described using PA will not have its universality restricted.

4.1 *Language of algorithmic information theory*

We will now be working with Ω extensively. Consequently, we will adopt the language of algorithmic information theory (AIT) and we will replace the following expressions by their AIT-equivalents.:

$$
\begin{aligned}
&\textit{Proof-theoretic} && \textit{AIT} \\
&\text{sentence} &\implies& \text{program} \quad (4.12)\\
&\text{the sentence is a theorem} &\implies& \text{the program halts} \quad (4.13)\\
&\text{the sentence is not a theorem} &\implies& \text{the program does not halt} \quad (4.14)
\end{aligned}
$$

This language will be more in line with the subject matter.

4.2 *Entropy*

As we have previously stated, Miniversal logic restricts the universe to a single solution. However, the formulation of Miniversal logic itself is not unique. Should we express Miniversal logic in first order logic, or second order logic? Should we use english, french, binary, etc.? Since the choice of formulation of Miniversal logic itself is not fixed by Miniversal logic, then to accurately represent reality, the Chaitin construction must somehow embed these choices. Let us consider this in more detail.

First, we consider that all choices of formulation of Miniversal logic are logically equivalent such that none are preferred. Then, we also consider that the construction of Ω imposes that the sentences of Miniversal logic ultimately be encoded as a prefix-free code. These considerations implies the presence of an entropy applicable to the choice of prefix-free codes. Is the Chaitin construction compatible with this assessment? Yes, the Chaitin construction does carry an entropy and it is given by the standard relation :

$$S = \left(\ln \Omega + \overline{|p|} \ln 2\right) \qquad \text{from 2.30} \tag{4.15}$$

and its equation of state is

$$dS = (\ln 2) d\overline{|p|} \qquad \text{from 2.29} \tag{4.16}$$

As we can see, the Chaitin construction is able to carry an entropy applicable to the choice of prefix-free code. Indeed, each available choice for the formulation of Miniversal logic corresponds to a specific numerical value of Ω. Hence, each possible formulation of Miniversal logic contains the same universal information.

4.3 *Feasibility cutoff*

In the previous section regarding bridging metaphysics to physics, we have argued that an observer can produce, in principle, the proof of any theorem of Miniversal logic. Furthermore, we have argued that as the universe embeds the observer, then any produced proof is also a theorem of the universe (anthropic principle). Furthermore, as Miniversal logic is universal, then the process will eventually enumerate all possible theorems. Hence the methodology can completely identify the theorems of the theory which explains the universe. We note that for the argument to work we must allow the notion 'in principle', which in this context means that an observer can submit proofs of arbitrary size for theorems of arbitrary size.

In reality however an observer will hit a size limit to any proof or theorem he can produce. What if a theorem is true but its proof requires $TREE(9999)$ bits? Alternatively, what if a theorem has a short proof but the theorem itself requires a googolplex amount of bits to express? To accurately represent what the observer can feasibly do, we must add a feasibility cutoff to the ensemble. With such a cutoff, the ensemble will only contain the size-bounded proofs to size-bounded theorems. It describes all 'synthetic a-priori' statement that an observer can *feasibly* produce to restrict the universe to a solution. The resulting blob, which is no longer universal, will recover its universality as the feasibility bound grows to infinity.

To adjust the equation such that it describes a *feasible* blob, we must first introduce the concept of dovetailing. Consider an observer attempting to prove the theorems of Miniversal logic. The observer could pick a sentence at random and work at it until the proof is found. The problem is that if the sentence is non-provable, the observer will hang attempting to prove it. So instead, the observer might try to write one bit of proof for each sentence, then the second bit for each sentence, etc. However, since there are infinitely many sentences, the observer will never return to write the second bit. The solution is to dovetail the work.

Definition 4.17 (Dovetailing)**.** *Dovetailing is a proof-finding strategy for a system of logic to guarantee that progress will be made on arbitrarily-many theorems even in the presence of non-provable sentences.*

Definition 4.18 (Simple dovetailing)**.** *Consider the case of simple dovetailing. First, an observer write one bit of the proof for the first sentence. Then, the observer write a bit of the proof for both the first and the second sentence. Then, the observer write a bit of the proof for the first, the second and the third sentence. And so on. The observer stops writing bits for sentences whose proof is completed. In the case of a non-provable sentence, the observer would write a bit for it for all future iterations. Using this method, progress will eventually be made on every sentence and no sentence will cause the observer to hang indefinitely.*

There are a few additional concerns and pitfalls that we want to avoid when introducing the feasibility cutoff.

- Consider that an observer can attempt to calculate Ω by working on each theorem using simple dovetailing. As proofs are found, the observer adds their contributions to Ω. After an infinite amount of time, Ω will indeed be recovered. However, the calculation does not converge towards Ω as it progresses and discontinuously yields Ω only at infinity. To see why, consider the case where the first non-halting program has a length of i. Since the general non-halting problem is unsolvable, at most the calculation of Ω differs from the real value of Ω by 2^{-i}. The error rate does not decrease during the calculation and only vanishes at infinity when all halting programs are known. As each bit of Ω is interpreted as an axiom of the system, the lack of convergence via simple dovetailing causes the system to hit a complexity limit quite soon into the calculation.

- When we define the state of the universe by a construction of Ω which includes a proof-size cutoff (let's use the symbol Z to denote the new construction), we want to avoid the situation where the bits are overwritten as the calculation progresses. For example, say at some point in the calculation $Z(t_1) = 0.001100..._b$ and at some other point $Z(t_2) = 0.101100..._b$ (In this example the proof associated with the first sentence is completed and the first bit flips to a 1). As the bits are interpreted as axioms, the unfortunate result is that the axioms are rewritten during the calculation. As this calculation will be connected to time in a future section (section 6.1), this would essentially imply that the future can rewrite the past. To avoid this, we would require that Z monotonically converges towards Ω as $t \to \infty$.

- As the concept of *feasibility* is the last metaphysical argument in our toolbox, introducing it into the ensemble must be sufficient to recover the familiar laws of physics such as; the arrow of time, general relativity, dark energy, the Dirac equation, etc.

- Finally, as we have chosen to construct the framework as an ensemble of theorems, then the dovetailing algorithm that we introduce must not dissolve the construction. For that to happen we will have to introduce it as a conjugate pair.

With all of these requirements in mind, we might expect the feasibility term to be quite complicated. However this is not so at all. Simply adding; the *algorithmic-frequency*, represented by f, along with its conjugate the *algorithmic-action*, represented by A, as a conjugate pair is enough to meet all of these requirements! In the proof-theoretic paradigm, A can be interpreted as the "effort" required to complete a proof and the algorithmic-frequency can be interpreted as the rate at which steps of the proof must be made to complete the proof within the allocated effort. Adding this conjugate pair replaces $H(p_i)$ and gives this equation:

$$Z = \sum_{i=1}^{\infty} 2^{-Af_i - |p_i|} \tag{4.19}$$

, where

$$\begin{aligned}
&Z \in (\mathbb{R} \cap [0,1]) && \text{numerical value of the sum} && (4.20)\\
&\left(2^{-Af_i - |p_i|}\right) && \text{micro-state representing a theorem} && (4.21)\\
&p_i \in \Sigma_b && \text{prefix code} && (4.22)\\
&|p_i| \in \mathbb{N}_{\geq 0} && \text{length of the prefix-code} && (4.23)\\
&f_i \in \mathbb{N}_{\geq 0} && \textit{algorithmic-frequency} && (4.24)\\
&A \in \mathbb{R}_{\geq 0} && \textit{algorithmic-action} && (4.25)
\end{aligned}$$

As with any conjugate, the value of A is the same for all micro-states. With this addition, Z is now aware of the size of the proof of each theorem of Miniversal logic (via f_i). With knowledge of proof size, a feasibility bound is now introduced by the effect of the *algorithmic-action* A. To see exactly how all of this pans out, we will now study in more detail the equation. First, lets prove this theorem.

Theorem 4.26. *Af_i recovers $H(p_i)$ when $A \to 0^+$.*

Proof. An arbitrary sentence i is associated to a corresponding $f_i \in [0, \infty]$. If the sentence is a theorem the size of its proof will be finite $f_i \in [0, \infty[$. If the sentence is not a theorem, there will be no finite proof of it $f_i = \infty$. When the effort required to prove a theorem goes to zero (e.i. $A \to 0^+$), all theorems are proven effortlessly. In this case,

$$\lim_{A \to 0^+} Af_i = \begin{cases} 0 & p_i \text{ is a theorem} \\ \infty & \text{otherwise} \end{cases} \tag{4.27}$$

This is the definition of $H(p_i)$. We recall it here;

$$H(p_i) = \begin{cases} 0 & p_i \text{ is a theorem} \\ \infty & \text{otherwise} \end{cases} \tag{4.28}$$

Therefore,

$$\lim_{A \to 0^+} A f_i = H(p_i) \tag{4.29}$$

The theorem is proven. Replacing $H(p_i)$ by Af_i in the construction still allows us to recover Ω when the effort required to perform computation vanishes to 0^+.

□

Let us now expand Z explicitly with an example. Assume a system comprised of three micro-states with prefix code-length $|p_1| = 1$, $|p_2| = 2$ and $|p_3| = 3$ and with the *algorithmic-frequencies* $f_1 = 5$, $f_2 = \infty$ and $f_3 = 5$. In this example, f_1 and f_2 are theorems and f_3 is a non-theorem. For the purposes of simplicity we can assume that all other sentences are non-theorems. In this case the system is not universal but let us nonetheless use it as a simplified numerical example. The sum Z becomes;

$$Z(A) = 2^{-1-5A} + 2^{-2-\infty A} + 2^{-3-5A} \tag{4.30}$$

We will now produce a series of numerical calculations with progressively smaller values of A and we will look at the evolution of the error rate $\xi(A) = \Omega - Z(A)$. For this system, $\Omega = 0.101\bar{0}_b$.

A	$Z(A)$	$\xi(A)$	error	
∞	0	Ω	max	(4.31)
1	$0.000000101..._b$	0.10011011_b	$\approx 2^{-1}$	(4.32)
0.1	$0.011100010..._b$	$0.00101110..._b$	$\approx 2^{-3}$	(4.33)
0.01	$0.100110101..._b$	$0.00000010..._b$	$\approx 2^{-6}$	(4.34)
0.001	$0.011100010..._b$	$0.00000000..._b$	$\approx 2^{-9}$	(4.35)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
0	Ω	0	none	(4.36)

As we can see, reducing the effort to perform computational steps causes the system Z to monotonically converges towards Ω. The error rate decreases as more valid bits of Ω are obtained. The error rate is the axiomatic cutoff. This yields an asymmetric arrow connected to the non-computability of Z. It will be presented in section 6.1 as a possible solution to the problem of the arrow of time in physics.

4.4 Summary

1. We have shown that a universal ensemble of theorems can be axiomatized in PA by constructing it as the halting probability Ω. In this case each bit of Ω can be interpreted as an algorithmically random axiom.

2. A universal theory, such as Miniversal logic, requires infinitely many bits of Ω to be fully axiomatized. Hence, it cannot be made finitely elegant. However, the same theory but now with a feasibility cutoff on proof-sizes introduced as a conjugate pair can be made finitely elegant. We name the resulting ensemble Z.

3. We have defined Z as an ensemble constructed from Ω but which is also aware of the proof-size of each theorem of Miniversal logic. In this case, the conjugate A acts as a cutoff which bounds the proofs to a *feasible* size. The numerical value of Z converges towards Ω with a monotonically decreasing error rate.

4. The ensemble is therefore finitely axiomatized by the first N random bits of Ω. Furthermore, as Ω is non-compressible, then for any value of A, the system is reduced to the smallest amount of bits of Ω which is able to decide the theorems of the ensemble and such that Ω is recovered at $A \to 0^+$. For $A > 0$, the amount is always finite. Mathematically, the theory is now elegant.

5 Thesis

We are now in a position to define the following thesis;

Definition 5.1 (Thermal UTM thesis)**.** *It is the thesis that a prefix-free universal Turing machine which maximizes the entropy during the calculation of its own halting probability Ω, is isomorphic to the universe. To maximize the entropy, the calculation is performed via thermal dovetailing. Like other forms of dovetailing, it allows work to be done on arbitrarily many sentences even in the presence of non-provable sentences. The calculation is expressed as the following sum*

$$Z = \sum_{i=1}^{\infty} 2^{-A\frac{1}{t_i}-|p|} \qquad \textit{where } t_i = \frac{1}{f_i} \tag{5.2}$$

and occurs when $\bar{t}$ goes from 0 to ∞ and as Z goes to Ω.

Definition 5.3 (Thermal dovetailing)**.** *Thermal dovetailing is an algorithm according to which the work done on programs is scheduled so as to maximize the entropy of the system during the computation. To guarantee that the entropy of maximized, thermal dovetailing is introduced into* *the Ω construction as a conjugate-pair. Thermal dovetailing exponentially suppresses the contribution to Ω of programs based on their halting time.*

5.1 Thermodynamics

How will this equation connect to physics? It has been said that statistical physics is perhaps the most general of all the disciplines of physics. Hence, the reader has perhaps already intuited that it would be first in line.

> "A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts."
>
> - - A. Einstein

> "But although, as a matter of history, statistical mechanics owes its origin to investigations in thermodynamics, it seems eminently worthy of an independent development, both on account of the elegance and simplicity of its principles, and because it yields new results and places old truths in a new light in departments quite outside of thermodynamics."
>
> - - J.W. Gibbs

Indeed, the equation for Z embeds all the familiar concepts of statistical physics. It carries an entropy, its mathematical formulation is the same as that of a Gibbs ensemble, it has multiple microstates applicable to the formulation of Miniversal logic and now, with a feasibility cutoff, the microstates extends to the size of proofs as well. The entropy and equation of state of Z are;

$$S = \ln Z + (\ln 2)\left(A\overline{f} + \overline{|p|}\right) \qquad \text{entropy} \qquad (5.4)$$

$$dS = (\ln 2)\left(Ad\overline{f} + d\overline{|p|}\right) \qquad \text{algorithmic equation of state} \qquad (5.5)$$

These relations are similar to the ones obtained for Ω (at 4.16) with the exception that they have the additional term $Ad\overline{f}$. Let's interpret this term. In statistical physics, the equation for the average frequency $\overline{f}$ is (from 2.25):

$$\overline{f} = \sum_{i=1}^{\infty} p(x) f_i \qquad \text{where } p(x) = \frac{1}{Z} 2^{-Af_i - |p|} \qquad (5.6)$$

The interpretation then is that 5.2 is the probability measure that maximizes entropy subject to the constraint that the mean algorithmic-frequency is some constant $\overline{f}$, and the algorithmic-action A is its conjugate variable. The term $\overline{|p|}$ is given a similar interpretation:

$$\overline{|p|} = \sum_{i=1}^{\infty} p(x)|p_i| \qquad \text{where } p(x) = \frac{1}{Z}2^{-Af_i-|p|} \tag{5.7}$$

Because of $\overline{|p|}$, equation 5.2 has the further constraint that the mean length of the prefix-free codes are some constant $|p|$. But where is its missing conjugate variable? Just like a physical system can have a fixed temperature ($\beta = 1$), we can imagine that its conjugate variable would be a variable D which is permanently fixed to 1.

- Alice: *This equation is still missing some of the more physical elements of the Gibbs ensemble. For example, there is no temperature term and none of its quantities are physical. The Gibbs ensemble refers to something physical. This equation does not.*

The principles of statistical physics are not restricted to physical systems. As we have seen, a purely information system such as Miniversal logic can meet the requirements applicable to the Gibbs ensemble such that its conventional interpretation is applicable to it. I will agree that it is not yet readily obvious how the laws of physics come out of this equation, however, simple transformations will take care of that.

We can rewrite the equation to give it a form that is associated to physical system, including an *algorithmic-temperature* and other "physically-associable" variables. To do so, we will perform a number of mathematical transformation and will rename most of its variables. Lets us first complete this purely mathematical exercise, then we will return to your question. As only legitimate mathematical operations will be applied (on both side of the equation), the transformed equation remains equivalent to 5.2 but its connection to physics will be more apparent.

Theorem 5.8. *The equation, using standard mathematical operations,*

$$dS = (\ln 2)\left(A d\overline{f} + d\overline{|p|}\right) \qquad \textit{algorithmic formulation} \tag{5.9}$$

can be rewriten to

$$\frac{1}{\ln 2} T dS = 2\pi S d\overline{f} + F d\overline{x} + k d\overline{A} + p d\overline{V} + \ldots \qquad \textit{action-frequency formulation} \tag{5.10}$$

$$\frac{1}{\ln 2} T dS = -P d\overline{t} + F d\overline{x} + k d\overline{A} + p d\overline{V} + \ldots \qquad \textit{power-time formulation} \tag{5.11}$$

where,

$$T \in \mathbb{R} \quad \textit{algorithmic-temperature} \tag{5.12}$$
$$S \in \mathbb{R}_{\geq 0} \quad \textit{entropy} \tag{5.13}$$
$$\mathcal{S} \in \mathbb{R} \quad \textit{entropic action} \tag{5.14}$$
$$\overline{f} \in \mathbb{R}_{\geq 0} \quad \textit{algorithmic-frequency (average)} \tag{5.15}$$
$$P \in \mathbb{R} \quad \textit{entropic power} \tag{5.16}$$
$$\overline{t} \in \mathbb{R}_{\geq 0} \quad \textit{algorithmic-time (average)} \tag{5.17}$$
$$F \in \mathbb{R} \quad \textit{entropic force} \tag{5.18}$$
$$\overline{x} \in \mathbb{R}_{\geq 0} \quad \textit{algorithmic-length (average)} \tag{5.19}$$
$$k \in \mathbb{R} \quad \textit{entropic viscosity} \tag{5.20}$$
$$\overline{A} \in \mathbb{R}_{\geq 0} \quad \textit{algorithmic-area (average)} \tag{5.21}$$
$$p \in \mathbb{R} \quad \textit{entropic pressure} \tag{5.22}$$
$$\overline{V} \in \mathbb{R}_{\geq 0} \quad \textit{algorithmic-volume (average)} \tag{5.23}$$

Proof. As a first step, we will be taking a Taylor expansion[20] over $d\overline{|p|}$. To do so, we first pose $L(p) := \overline{|p|}$. Then, the Taylor expansion for $L(p)$ is:

[20] Taking the Taylor expansion introduces a smoothness requirement on $L(p)$. We will talk about this in more detail after this proof.

$$L(p) = L(0) + L'(0)p + \frac{1}{2}L''(0)p^2 + \frac{1}{6}L''(0)p^3 + ... \tag{5.24}$$

then taking its derivative,

$$dL(p) = L'(0)dp + L''(0)pdp + \frac{1}{2}L'''(0)p^2dp + ... \tag{5.25}$$

then switching the notation back to $|p|$,

$$d\overline{|p|} = L'(0)d\overline{|p|} + L''(0)\overline{|p|}d\overline{|p|} + \frac{1}{2}L'''(0)\overline{|p|}^2d\overline{|p|} + ... \tag{5.26}$$

We replace $d\overline{|p|}$ in the algorithmic formulation by its Taylor expansion. We obtain:

$$\frac{1}{\ln 2}dS = Ad\overline{f} + L'(0)d\overline{|p|} + L''(0)\overline{|p|}d\overline{|p|} + L'''(0)\overline{|p|}^2d\overline{|p|} + ... \tag{5.27}$$

We multiply each side of the equation by a constant T,

$$T\frac{1}{\ln 2}dS = TAd\overline{f} + TL'(0)d\overline{|p|} + TL''(0)\overline{|p|}d\overline{|p|} + TL'''(0)\overline{|p|}^2d\overline{|p|} + ... \tag{5.28}$$

We rename each of the coefficients of the Taylor expansion as the multiplication of two variables; $L'(0) := g'(0)G'(0)$, $L''(0) := g''(0)G'(0)$ and $L'''(0) := g'''(0)G'''(0)$, we obtain

$$T\frac{1}{\ln 2}dS = TAd\overline{f} + Tg'(0)G'(0)d\overline{|p|} + Tg''(0)G''(0)\overline{|p|}d\overline{|p|} + Tg'''(0)G'''(0)\overline{|p|}^2 d\overline{|p|} + ... \tag{5.29}$$

We rename $F := Tg'(0)$, $k := Tg''(0)$, $p := Tg'''(0)$ and $2\pi\mathcal{S} := TA$. We also pose $dx := G'(0)d\overline{|p|}$, $dA := G''(0)\overline{|p|}d\overline{|p|}$ and $dV := G'''(0)\overline{|p|}^2 d\overline{|p|}$, we obtain[21] [22].

$$\frac{1}{\ln 2}TdS = 2\pi\mathcal{S}d\overline{f} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ... \tag{5.30}$$

To final step is to use the definition $t := 1/f$ to convert $\mathcal{S}$ to a power P.

$$2\pi\mathcal{S}d\overline{f} = 2\pi\mathcal{S}d\left(\frac{1}{\overline{t}}\right) \qquad t := 1/f \tag{5.31}$$

$$2\pi\mathcal{S}d\overline{f} = 2\pi\mathcal{S}\left(-\left(\overline{t}\right)^{-2}\right)dt \qquad df = -t^{-2}dt \tag{5.32}$$

$$2\pi\mathcal{S}d\overline{f} = -Pd\overline{t} \qquad \text{where } P = 2\pi\mathcal{S}\left(\overline{t}\right)^{-2} \tag{5.33}$$

We finally rewrite the algorithmic formulation to

$$\frac{1}{\ln 2}TdS = 2\pi\mathcal{S}d\overline{f} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ... \qquad \text{action-frequency formulation} \tag{5.34}$$

$$\frac{1}{\ln 2}TdS = -Pd\overline{t} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ... \qquad \text{power-time formulation} \tag{5.35}$$

This proves the theorem.

□

We take note that, as the Taylor expansion was taken, the physical equation of state is only defined for smooth functions of $L(p) = \overline{|p|}$. Therefore, a smoothness of the space of $L(p)$ is implicitly assumed. Such smoothness can be obtained by any analytical continuation technique such a spline interpolation, etc.. The smoothness approximation, as it is inexact, introduces a restriction on the domain of applicability of the action-frequency and the power-time formulation. Hence, these two formulations are less general than the algorithmic formulation. Specifically, the applicability of these two formulations occurs over sizes much greater than the minimal prefix-code step size.[23]

- Alice: *So you have rewritten the equations using basic mathematical transformations and renamed most of the variables. But these are just arbitrary variable names. There is no proof that the x in your equation corresponds to a length in nature, that the t in your equation corresponds to time in nature or that the T corresponds to a temperature, etc. Just naming them as such does not make it so.*

[21] The use of the factor 2π will become clear when we connect the action $\mathcal{S}$ to the reduced Planck constant and f to the angular frequency ω in section 10. It is added here so as to recover the definition of the Planck units in terms of $\hbar$, commonly used in physics.

[22] The p in $p := TG'''(0)$ refers to pressure, not to program.

[23] For example, we can imagine that the minimal prefix-code step size for programs is significant on the smallest scales (order of the Planck length). In this case, understanding small-scale spacetime would involve studying the algorithmic formulation itself. This would be difficult as non-computable effects would be dominant on this scale. But when the sizes involved are much greater, the power-time formulation would be an appropriate smooth approximation.

The laws of physics will be recovered from these formulations. For example, the Dirac equation will be recovered in section 9.2 and the length and time referred to by it will correspond to the x and the t in the power-time formulation. Same thing will happen for special relativity in section 7, or general relativity in section 8.2 or any of the other laws that we will derive. The x and the t of the laws will properly connect to the t and the x of the power-time formulation (and to each other). Hence, they form a coherent web of laws which corresponds to our empirical observations of what time and length are.

- Alice: *But the physical connection itself is not proven by this. It doesn't prove that the variables exist objectively.*

Normally the laws of physics are derived from empirical observations. In such a context, it is clear that the force F, the mass m and the acceleration a (in say $F = ma$) does correspond to real physical quantities. It is simply the law that best fits the data to date. Thus, no leap of faith is required to connect it to the physical world it describes as such connection is implied by the empirical origin of the theory.

However, in this paper, the derivation of the laws has an origin in pure reason and not from empirical observations. Therefore the claim that such laws corresponds to real physical quantities may appears to require a "leap of faith". This possible objection is actually incorrect for a few of reasons.

Let's first understand what Z means in this context. Z is an universal ensemble of theorems bounded by a feasibility bound. From Miniversal logic, we can prove that the universe, as it embeds the observer, is describable by such an ensemble. Therefore, the entropy of Z must also be admitted by the universe. The entropy is the physical connection. The variables (t, x, etc.) that are introduced as quantities that are emergent from this entropy. They are not parachuted from thin air. For example, an entropic force is a force that can emergence from the entropy of a system. It is defined as:

$$F := T\frac{\partial S}{\partial \overline{x}} \tag{5.36}$$

The derivation of the force F from the power-time formulation yields the standard definition of an entropic force in statistical physics. This is true for all other quantities : $\mathcal{S}$, P, k and p. Let us derive each of them explicitly here:

Theorem 5.37. *The action-frequency formulation implies an entropic action as per the statistical physics definition.*

Proof.

$$2\pi\mathcal{S} = T\left[\frac{\partial}{\partial\overline{f}}\left(2\pi\mathcal{S}d\overline{f} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ...\right)\right|_{x,A,V} \tag{5.38}$$

□

Theorem 5.39. *The power-time formulation implies an entropic power, an entropic force, an entropic viscosity and an entropic pressure as per the statistical physics definition.*

Proof.

$$-P = T\left[\frac{\partial}{\partial\overline{t}}\left(-Pd\overline{t} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ...\right)\right|_{x,A,V} \tag{5.40}$$

$$F = T\left[\frac{\partial}{\partial\overline{x}}\left(-Pd\overline{t} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ...\right)\right|_{t,A,V} \tag{5.41}$$

$$k = T\left[\frac{\partial}{\partial\overline{x}}\left(-Pd\overline{A} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ...\right)\right|_{t,x,V} \tag{5.42}$$

$$p = T\left[\frac{\partial}{\partial\overline{V}}\left(-Pd\overline{A} + Fd\overline{x} + kd\overline{A} + pd\overline{V} + ...\right)\right|_{t,x,A} \tag{5.43}$$

□

These entropic quantities are necessary consequences of the irreducible entropy intrinsic to any system describable by Z (including the universe itself).

- Alice: *Ok for the conjugate $\mathcal{S}$, P, F, k and p as they are emergent from the entropy. But, what about the observables $\overline{f}$, $\overline{t}$, $\overline{x}$, $\overline{A}$ and $\overline{V}$? Are you not making a semantic error by calling t as time and x as length? For instance, you are naming t as time because you know that, from empirical observations, this is what t represents. If you did not have the benefit of empirical observations, these would just be nameless variables with no physical meaning and with arbitrary properties.*

I suppose it depends on the semantics. If we use an empirical definition of time such as "Time is what a clock measures" then yes, it is a semantical error. However, we now have a mathematical definition of time to compete with it. Bluntly, time is the variable t in this equation:

$$Z = \sum_{i=1}^{\infty} 2^{-A\frac{1}{t_i}-|p|} \qquad \text{where } \frac{1}{t_i} = f_i \tag{5.44}$$

Likewise the length x also now has a proper mathematical definition. Bluntly, it is the first term of the Taylor expansion of the analytic continuation of the function $|p|$. The observables, just like the emergent conjugates, also have an explanation in statistical physics. They are the average values of the algorithmic quantities. The relations are

	variable	average	
algorithmic-frequency	f	$\overline{f} = \frac{-\partial \ln Z}{\partial \mathcal{S}}$	(5.45)
algorithmic-time	t	$\overline{t} = \frac{-\partial \ln Z}{\partial P}$	(5.46)
algorithmic-length	x	$\overline{x} = \frac{-\partial \ln Z}{\partial F}$	(5.47)
algorithmic-area	A	$\overline{A} = \frac{-\partial \ln Z}{\partial k}$	(5.48)
algorithmic-volume	V	$\overline{V} = \frac{-\partial \ln Z}{\partial p}$	(5.49)

Any system describable by the construction Z will have these average quantities emergent from the entropy.

5.2 *Time and length as thermal effects*

Where does time and length come from? Consider the observables t and x of the power-time formulation. As we have just discussed, their average value is given by the standard relations (from 2.25).

	variable	average	
algorithmic-time	t	$\overline{t} = \frac{-\partial \ln Z}{\partial P}$	(5.50)
algorithmic-length	x	$\overline{x} = \frac{-\partial \ln Z}{\partial F}$	(5.51)

According to statistical physics, $\overline{t}$ and $\overline{x}$ are the average of the observables t and x. The role of their respective conjugate (P and F) is to vary the ponderation over the t_i and the x_i across the system so as to meet the average value. From these relations, we now interpret time as the average $\overline{t}$ of the proof-sizes and length as the average $\overline{x}$ of theorem-sizes for the system. As they are averages over the entropy, we can think of them as a *thermal* time and a *thermal* length. Let us revisit an earlier theorem armed with our new definition of thermal time.

$$Z = \sum_{i=1}^{\infty} 2^{-A\frac{1}{t_i}-|p_i|} \qquad \text{where } f_i = \frac{1}{t_i} \tag{5.52}$$

Theorem 5.53. *As* $\overline{t} \to \infty$ *then* $A \to 0^+$ *and* $Z \to \Omega$.

Proof. The convergence of Z to Ω can now be interpreted as an increase in the age of the system (as $\bar{t}$ varies from 0^+ to ∞) instead of as a reduction in the effort required for computation (A varies from ∞ to 0^+). As a system ages, the average proof-size of the theorems comprising its ensemble increases. At $\bar{t} \to \infty$, all proofs regardless of size are included in the system, hence $Z \to \Omega$. □

After this section, we will see explicitly how this averaging effects has the properties and laws that we associate with length and time; such as special relativity, general relativity, etc. - hence do correspond to our idea of what a physical length or time is. The implication is that the time and space axis should not be understood as fundamental concepts but should instead be understood as an emergent *thermal* spacetime.

5.3 Universal Brownian motion

The primary conjugate pairs applicable to the power-time formulation are Fx and Pt. The F and P are the conjugate and are constant for all micro-states of the system. However x and t are each adjusted for their specific micro-states and are not fixed within the system. Therefore, like any statistical observable of statistical physics, x and t undergo fluctuations. The equations describing the process are (from relation 2.25).

	variable	fluctuation	
algorithmic-time	t	$\overline{(\Delta t)^2} = \dfrac{\partial^2 \ln Z}{\partial P^2}$	(5.54)
algorithmic-length	x	$\overline{(\Delta x)^2} = \dfrac{\partial^2 \ln Z}{\partial F^2}$	(5.55)

Using the original argument made by Einstein in 1905 which lead to the derivation of Brownian motion, we argue here that fluctuations of the t and x variables produce a universal Brownian motion along the axis themselves. What does a thermal spacetime with fluctuations look like? The consequences of such are nothing to be feared. Indeed, we will show in section 9.1 that Brownian motion over dx will produce the Schrödinger equation. Furthermore, we will show in section 9.2 that Brownian motion over both dx and dt will produce the Dirac equation.

- Alice: *Are you suggesting a pilot-wave interpretation where particles undergo Brownian motion until a measurement is made?*

Not at all. Rather, we are suggesting that any positional or time information undergoes a "Dirac equation-like diffusion" so as to make positional or time information perishable over time. To illustrate, we can imagine placing a mark at a position in space. After a certain time, Brownian motion will diffuse the position of the marker at any number of possible locations until its actual position is measured again. Instead of being punctual, the marker could be continuous and weighted and the same diffusion-like behaviour will be observed. This Brownian motion would universally apply to the axis themselves. This is not a claim that a particle is punctual.

5.4 Universal constants

For convenience, we repeat here the action-frequency and the power-time formulations.

$$\frac{1}{\ln 2}TdS = 2\pi\mathcal{S}df + Fdx + kdA + pdV + ... \quad \text{action-frequency formulation} \tag{5.56}$$

$$\frac{1}{\ln 2}TdS = -Pdt + Fdx + kdA + pdV + ... \quad \text{power-time formulation} \tag{5.57}$$

These formulations contain the following conjugates: The force F, the power P, the action $\mathcal{S}$, the viscosity k and the pressure p. They are the Lagrange multipliers of the partition function. For a system at maximal entropy, these values are constant throughout the system and are the primary constants defining it. We will show in section 10 from first principles that these are indeed the Planck force, the Planck power and the Planck action. This will allows us to recover the gamut of the Planck units.

The reader might wonder why we leave the derivation of the Planck units for the end. The reason is simple; the true fundamental constants (at least as far as this formulation is concerned) are the force F, the power P and the action $\mathcal{S}$. However, the Planck constant assumes that the primary fundamental constants are $\hbar$, the speed of light c and the Gravitational constant G. Although it is an equivalent construction, we cannot connect the two until we first obtain a lot of preliminary results. For example, to rewrite F in terms of G, we will first need to recover the law of gravitation to show explicitly how the two connects. This will indeed be done but not until we reach section 10. Until then, the reader is advised to keep in the back of the mind that F, P, k, p, $\mathcal{S}$, etc. refers to Planck units.

5.5 Regimes and cycles

We will study the power-time formulation as we would study any thermodynamic equation of state. A thermodynamic regime can be produced as the permutations over posing some derivatives to zero, while allowing the others to vary. The equation contains 6 derivatives (dS, dt, dx, dA and dV). As each can be set to 0 or allowed to vary, the formulation yields a total of 57 thermodynamics regimes (57 is obtained by taking the total of 64 permutations and subtracting those for which all variables would be 0). We will look at the simplest regimes and will leave the others as open problems. The list below serves as an outline for the future sections of the paper. The principal law provable from each regime is named in the rightmost column.

Regime						*Law*	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
$(\ln 2)^{-1} TdS =$	$-Pdt$	$+Fdx$	0	0	0	Limiting speed	(5.58)
$(\ln 2)^{-1} TdS =$	$-Pdt$	0	$+kdA$	0	0	Limiting viscosity	(5.59)
$(\ln 2)^{-1} TdS =$	$-Pdt$	0	0	$+pdV$	0	Limiting vol. flow rate	(5.60)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
$0 =$	$-Pdt$	$+Fdx$	0	0	0	Special relativity	(5.61)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
$(\ln 2)^{-1} TdS =$	$-Pdt$	0	0	0	0	Arrow of time	(5.62)
$(\ln 2)^{-1} TdS =$	0	$+Fdx$	0	0	0	Law of Inertia	(5.63)
$(\ln 2)^{-1} TdS =$	0	0	$+kdA$	0	0	General relativity	(5.64)
$(\ln 2)^{-1} TdS =$	0	0	0	$+pdV$	0	Dark energy	(5.65)
$(\ln 2)^{-1} TdS =$	0	0	0	0	$+c(dx)^{\geq 4}$	Darker energies?	(5.66)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
$(\ln 2)^{-1} TdS =$	$-Pdt$	$+Fdx$	$+kdA$	$+pdV$	$+\dots$	(The Universe)	(5.67)

As examples of how this tabular should be interpreted; the regime defined by $dA = 0$, $dV = 0$ and $(dx)^{\geq 4} = 0$ will allow us to prove the existence of the limiting speed. The regime defined by $dx = 0$, $dA = 0$, $dV = 0$ and $(dx)^{\geq 4} = 0$ will allow us to prove the arrow of time. Etc..

5.6 *Summary*

Since the last summary,

1. We have claimed that the the power-time formulation suggests multiple regime of physics, each associated with the provability of a specific law of physics.

2. The Lagrange multiplier $\mathcal{S}$, P, F, k and p of the ensemble are

the primary constants of the system and are emergent from the entropy of any system describable by Z.

3. The observables f, t, x, A and V are statistical averages resulting from the entropy of, again, any system describable by Z.

4. As x and t are entropic averages, a universal Brownian motion applies to all axis which encodes space or time information.

We will now derive the laws of physics themselves.

6 *Arrow of time*

Adding a time variable to a Gibbs ensemble adds a whole new dynamic to a thermodynamic system. The system now becomes aware of future, past and present entropy and can translate from time to space and from space to time for an entropic cost (provided that various limits are respected). By studying thermodynamic cycles involving space and time, I was able to investigate what happens to the entropy when a system is translated forward or backward in time and draw conclusions in regards to the arrow of time. In the model presented, space serves as an entropy sink that encourages a forward arrow of time, the future is non-computable and the past is singular.

6.1 *Negative power*

Theorem 6.1. *In the power-time formulation, increasing* $\bar{t}$, *while keeping the other variables constant, decreases the entropy.*

Proof.

$$(\ln 2)^{-1} T dS = -P dt \qquad \text{regime 5.62} \qquad (6.2)$$

$$\implies \frac{dS}{dt} = -(\ln 2)\frac{P}{T} \qquad \text{decreasing entropy} \qquad (6.3)$$

□

Definition 6.4 (Halting entropy)**.** *The halting entropy is the contribution by the following term to the entropy over time.*

$$-(\ln 2)\,\frac{P}{T} \qquad (6.5)$$

Alice: - *Why does the entropy decreases with time?*

We have to be careful about the formulation of the question. The power-time formulation admits other terms; Fdx, kdA and pdV. The term $-Pdt$, as it has a negative sign, works towards reducing the entropy, but the other variables, as their signs are positive, work in the other direction. Thus, the entropy of the system as a whole need not necessarily decrease. It is more accurate to say that increasing $\bar{t}$, while keeping the other variables constant, decreases the entropy.

In this situation, the halting entropy decreases because the system has a negative power. So a similar question would be what is a negative power and why is it negative is this system? Before we answer, let us first study its more familiar cousin: the negative temperature.

If we understand temperature as the random movement and vibrations of molecules then a temperature is always equal to or above zero. However, the Gibbs ensemble allows a generalized definition of temperature as the tradeoff between energy and entropy. Most systems cannot admit a negative temperature as their entropy will always increase at higher energies. But for some system, for example the population inversion in a laser, their entropy saturate at higher energies. Hence, they can admit a negative temperature.

A negative power has essentially the same interpretation. As $\bar{t}$ is increased, the entropy is decreased.

> Alice: - *You have explained macroscopically what happens, but how does this pans out in the partition function. Why is there less entropy at increased* $\bar{t}$

To understand why, recall the construction of the partition function (5.2) as an ensemble of theorems. Each micro-state is defined by both the theorems-size and the proof-size. At $\bar{t} = 0$, the average proof-size is 0. Hence, the provability of all sentences is unknown. As $\bar{t}$ is increased, the provability of more sentences becomes known. Eventually, $\bar{t}$ is so high that most theorems are proven. Borrowing the term from laser thermodynamics, this would be when the population is inverted. Thus, the entropy in regards to unprovability of sentences decreases with time.

> Alice: - *How does this result reconcile with the second law of thermodynamics which states that the entropy increases with time (or in some ideal cases stays constant)?*

To answer that, we need to introduce the concept of exfoliation of spacetime.

6.2 *Exfoliation*

An entropy decreasing with time would violate the second law of thermodynamics. We suggest that an entropic exfoliation along the observables dx, dA and dV occurs to offset the reduction.

Theorem 6.6. *The negative power of the power-time formulation implies an entropic exfoliation along the* dx, dA *and* dV *observables.*

Proof.

$$\frac{TdS}{\ln 2} = -Pdt + Fdx + kdA + pdV \qquad \text{regime 5.67} \qquad (6.7)$$

$$\frac{dS}{dt} = (\ln 2)\frac{1}{T}\left[\frac{Fdx}{dt} + \frac{kdA}{dt} + \frac{pdV}{dt} - P\right] \qquad \text{exfoliation} \qquad (6.8)$$

□

Definition 6.9 (Exfoliation entropy)**.** *The exfoliation entropy is the contribution by the following term to the entropy over time.*

$$(\ln 2)\frac{1}{T}\left[\frac{Fdx}{dt} + \frac{kdA}{dt} + \frac{pdV}{dt}\right] \qquad (6.10)$$

Note that P is not present in the definition as it is already associated with the halting entropy.

To investigate this result, let us look at three cases;

$$\frac{Fdx}{dt} + \frac{kdA}{dt} + \frac{pdV}{dt} < P \implies \frac{dS}{dt} < 0 \quad \text{decreasing entropy} \quad (6.11)$$

$$\frac{Fdx}{dt} + \frac{kdA}{dt} + \frac{pdV}{dt} = P \implies \frac{dS}{dt} = 0 \quad \text{constant entropy} \quad (6.12)$$

$$\frac{Fdx}{dt} + \frac{kdA}{dt} + \frac{pdV}{dt} > P \implies \frac{dS}{dt} > 0 \quad \text{increasing entropy} \quad (6.13)$$

At (6.12), a shift occurs in the direction of the production of entropy over time. It is the point at which the exfoliation entropy overtakes and exceeds the reduction in halting entropy. The second law of thermodynamics, which states that $dS/dt \geq 0$ will hold for (6.12) and (6.13), but will be violated for (6.11).

6.3 *Discussion*

In this section, we will explain why these two theorems provide us with an understand of time and its arrow. Indeed, it links the arrow of time to three concepts; 1) a reduction in halting entropy over time, 2) a non-computability of future theorems and 3) an increase in exfoliation entropy over time. This derivation more closely matches the observer's understanding of time. Indeed,

1. at the beginning of time the number of possible future alternatives is maximal. To reflect this, the halting entropy is at its maximum at $t = 0$, and the exfoliation entropy is equal to 0. This matches our current empirical beliefs in that the exfoliation entropy at the Big Bang is very low.

2. during the evolution the future becomes past which is immutable. As the past becomes immutable, the halting entropy of the bits

defining it are equal to 0. This is because we "remember" or "observe" only one past. This reduction in halting entropy is offset by a growth in exfoliation entropy, which is related to the size and complexity of the space encoded by the exfoliation observables. This growth in space entropy obeys the observed second law of thermodynamic.

3. at the end of time there is no future. The value of Ω has been calculated, and the full history of the system is now "set in stone". The halting entropy is 0 and the exfoliation entropy is at its maximum. This matches the hypothesis of the heat death.

Alice: - *I am still not clear on this. The conventional wisdom in physics is that the arrow of time is connected to an increase in entropy with time. Now you seem to be saying the reverse of that.*

Yes, I am stating that time has an arrow principally because the halting entropy decreases with time, and yes, it contradicts the conventional wisdom. But nonetheless it is correct for the following reason:

An observer who defines a statistical system without inserting a time observable will see a second law of thermodynamic. This statistical effects is heuristically explained by the H-theorem of Boltzmann. However, this is not necessarily the case when time is inserted into the ensemble as an observable. Such an ensemble then becomes aware of the past, the present and the future. Each instant represents a different micro-state distribution of the occupancy of the system. A system with a low $\bar{t}$ has a completely different micro-state distribution than a system with a high $\bar{t}$. Hence, there is no time reversal symmetry and such lack of symmetry is very explicit.

Alice: - *But why is time specifically associated with a decrease in entropy, and not an increase?*

The partition function encodes all past, present and future states as micro-states. Thus, at a specified $\bar{t}$ theorems with longer proofs will not be part of the system and the future will be undecided. The system will carry an entropy attributable to the undecidability of the future. The role of increasing time is to consume this entropy by closing possible futures. In other words, time collapses future undecidability into a singular past.

Alice: - *But, a system cannot decrease its entropy without violating the second law of thermodynamics.*

A system can decrease its entropy if it is connected to an entropy sink. For example, biological life can reduce its entropy but only at the cost of severely increasing it in its environment. This requires work and in the case of Earth, the Sun supplies excess energy to the ecological system. Thus, the power-time conjugate can decrease the entropy as long as it is connected to a sink.

Alice: - *So, there should be a sink in the universe available to offset the decrease in halting entropy over time?*

In the case of time, the sink is the universe itself. It is in this context that the laws of physics will be derived. They are the limits required to produce a sink of sufficient entropy to accommodate a forward direction of time.

Alice: - *Why do we remember the past but not the future?*

An observer cannot pre-calculate his exact future before it occurs without increasing the size of the sink. Here we make a distinction between calculating a probable future versus the exact future. Calculating a probable future does not necessarily imply a reduction of entropy within the system but calculating the exact future requires consuming the entropy of all possible alternative futures hence a sink is required.

Alice: - *How do we understand this from the algorithmic theory perspective?*

The exfoliation variable represents the entropy in the choice of available prefix-free encodings for the programs of the UTM. In the beginning of the calculation, when no bits of Ω are known, it doesn't make sense to speak of the ways to encode this information as there is nothing to encode. Hence, the entropy of the result should be 0. As more bits of Ω are known, then more ways to encode this information exist and the entropy associated with the possible encodings increases.

Alice: - *Does the second law of thermodynamic need to be corrected?*

Absolutely. To my knowledge, statistical physics has never been used with a time observable. Time was always considered to be an independent background to statistical physics. But, when we add a time observable to a Gibbs ensemble, new physics emerge.

Indeed, an observer cannot move forward into the future unless its potential future alternatives are closed. Hence, its halting entropy must decrease with time. The second law of thermodynamics is a consequence of the system increasing its present entropy to offset the reduction in future alternatives as time moves along.

7 *Special relativity*

7.1 *Limiting relations*

Let us immediately prove the first three regimes (5.58, 5.59 and 5.60). To prove that these are limits, we will consider the assumption that an observer who evolves forward temporally must see a growth in the size of its available entropy sink to offset the reduction in future alternatives. The limit occurs when the sink exactly offsets the reduction in entropy by time - in which case $dS/dt = 0$.

Theorem 7.1. *The power-time formulation of the equation of states (5.57) implies a limiting speed.*

Proof.

$$(\ln 2)^{-1} TdS = Fdx - Pdt \qquad \text{regime 5.58} \tag{7.2}$$

$$\frac{1}{\ln 2}\frac{T}{F}\frac{dS}{dt} = \frac{dx}{dt} - \frac{P}{F} \qquad \text{division by } Fdt \tag{7.3}$$

□

To see why this implies a maximum speed, first consider that the units of this equation are *length/time* hence are describing a speed. Second, consider the following three cases;

$$\frac{dx}{dt} = \frac{P}{F} \implies \frac{dS}{dt} = 0 \tag{7.4}$$

$$\frac{dx}{dt} < \frac{P}{F} \implies \frac{dS}{dt} < 0 \tag{7.5}$$

$$\frac{dx}{dt} > \frac{P}{F} \implies \frac{dS}{dt} > 0 \tag{7.6}$$

We notice a reversal in the production of entropy at the inflexion point where $dS/dt = 0$. Therefore, for an observer at rest to evolve forward in time, it must see its entropy sink grow at the speed of P/F.

Remark 7.7. *As we have already mentioned, we will show in section 10 that P is the Planck power and F is the Planck force. Indeed, when they are, we do recover c the speed of light;*

$$P\left(\frac{1}{F}\right) = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{7.8}$$

Thus, the entropy sink of an observer moving forward in time must grow at the speed of light.

Theorem 7.9. *The following relations each characterize a limiting quantity.*

approx.			
none	$\frac{1}{\ln 2}\frac{T}{F}\frac{dS}{dt} = -P$	*maximum power* (J/s)	(7.10)
$S \propto L$	$\frac{1}{\ln 2}\frac{T}{F}\frac{dS}{dt} = \frac{dx}{dt} - \frac{P}{F}$	*maximum speed* (m/s)	(7.11)
$S \propto A$	$\frac{1}{\ln 2}\frac{T}{k}\frac{dS}{dt} = \frac{xdx}{dt} - \frac{P}{k}$	*maximum viscosity* (m^2/s)	(7.12)
$S \propto V$	$\frac{1}{\ln 2}\frac{T}{p}\frac{dS}{dt} = \frac{x^2dx}{dt} - \frac{P}{p}$	*max. vol. flow rate* (m^3/s)	(7.13)

Proof. Each relation can easily be obtained from (5.57) by posing the other observables to 0. To show that the quantities are inflexion limits, it suffices to notice that they each correspond to a growth of the entropy sink that an observer at rest must see to fuel its forward translation in time. □

It is well-known that a limiting speed implies special relativity (section 7), but what about to other two? It is less well known, but nonetheless, a maximum viscosity does implies general relativity. In this context, we can interpreted space as being encoded by bits moving very slowly (like molasses) on the surface of a sphere (section 8.2). The maximum volumetric flow rate is associated with dark energy and is responsible for the Hubble horizon - beyond which the flow rate would be exceeded (section 8.3).

7.2 *Light cones as thermodynamic cycles*

In this section, we look at the thermodynamic cycle of the system transiting through time and space starting at O to A to B and back to O, as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles, such as those involving pressure and volume but also of relativistic light cones.

We select regime 5.61 (special relativity) for our cycle.

$$\frac{1}{\ln 2}TdS = -Pdt + Fdx \tag{7.14}$$

O to A: As O is translated forward in time to A while keeping the distance constant ($dx = 0$), the entropy must decrease over time to compensate.

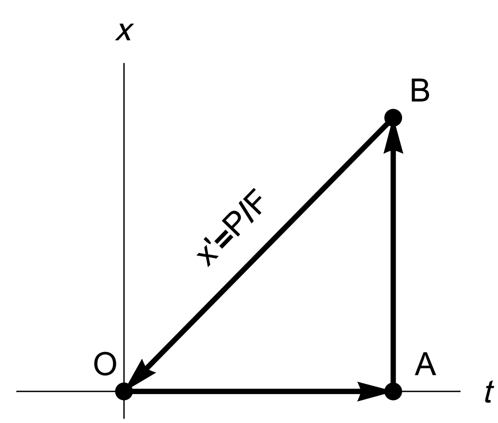

Figure 1: A thermodynamic cycle through space, time and entropy as observables.

$$\left(\frac{1}{\ln 2}TdS = -Pdt + Fdx\right|_{dx=0} \tag{7.15}$$

$$\implies \frac{dS}{dt} = -(\ln 2)\frac{P}{T} \tag{7.16}$$

A forward translation in time causes the system to know more bits of Ω. As the unknown bits of Ω carry an entropy, knowing more reduces the entropy. Conversely, a backward translation in time causes the system to erase bits from its pool of information which increases its entropy. A backward translation in time is equivalent to erasing halting information about the system's present.

A to B: As A is translated forward in space to B while keeping the time constant ($dt = 0$), the entropy must increase over space to compensate.

$$\left(\frac{1}{\ln 2}TdS = -Pdt + Fdx\right|_{dt=0} \tag{7.17}$$

$$\implies \frac{dS}{dx} = (\ln 2)\frac{F}{T} \tag{7.18}$$

We conclude that the further away from A a region is, the higher its entropy will be. Since $dt = 0$, no change in time is experienced.

O to B: As O is translated forward both in time and in space to B while keeping the entropy constant ($dS = 0$), the system has a speed c.

$$\left(\frac{1}{\ln 2}TdS = Fdx - Pdt\right|_{dS=0} \tag{7.19}$$

$$\implies \frac{dx}{dt} = \frac{P}{F} \tag{7.20}$$

We conclude that an object traveling at speed P/F is neither encouraged nor discouraged by entropy. However, the type of entropy changes. The rate P/F is the rate of conversion of time entropy to space entropy. At O, the system is comprised exclusively of time entropy as its future is not yet determined. As the system evolves towards B, its time entropy is decreased over time as the system replaces its future entropy with a singular past. Its space entropy (which encodes the singular past), however is increased to offset the reduction.

As a backward translation in time erases the most recently calculated bits of Ω, we conclude that the system "forgets its future" during the backward translation.

7.3 *Lorentz's transformation*

To recover the Lorentz's factor γ, let us consider figure 2. Two observers start at the origin S and travel in space-time respectively to O and O'. We regard O' as traveling at speed $|v|$ in the reference frame of O. From standard trigonometry, we derive the following values for the length of the segments;

Segment	Length	
$\lvert\overline{SO}\rvert$	L	(7.21)
$\lvert\overline{SO'}\rvert$	$L\cos\theta$	(7.22)
$\lvert\overline{O'O}\rvert$	$L\sin\theta$	(7.23)

We start with the Pythagorean theorem and solve for $\cos\theta$.

$$\left|\overline{SO}\right|^2 = \left|\overline{SO'}\right|^2 + \left|\overline{O'O}\right|^2 \quad (7.24)$$

$$L^2 = (L\cos\theta)^2 + (L\sin\theta)^2 \quad (7.25)$$

$$1 = (\cos\theta)^2 + (\sin\theta)^2 \quad (7.26)$$

$$\sqrt{1-(\sin\theta)^2} = \cos\theta \quad (7.27)$$

We consider that the distance between two observers moving at constant speed is simply vt. Hence, $\left|\overline{O'O}\right| = vt$. Solving for $\sin\theta$, we obtain

$$\left|\overline{O'O}\right| = vt = L\sin\theta \quad (7.28)$$

$$\implies \sin\theta = \frac{vt}{L} \quad (7.29)$$

From equation (7.27) and (7.29), we get the reciprocal of the Lorentz factor,

$$\sqrt{1-\frac{v^2t^2}{L^2}} = \cos\theta = \gamma^{-1} \quad (7.30)$$

$$\implies \gamma = \frac{1}{\sqrt{1-\frac{v^2t^2}{L^2}}} \quad (7.31)$$

Finally, we consider that L is the distance travelled in time by O in its own reference frame. This is given via the relation $dx = cdt$. Hence $L = ct$. We obtain,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad (7.32)$$

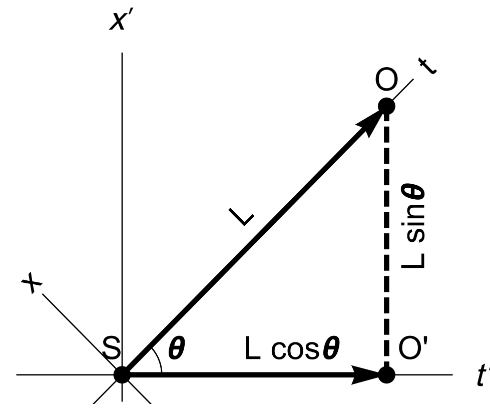

Figure 2: The spacetime intervals between two observers. Here O' travels at speed $|v|$ in O's reference frame.

which is the well-known Lorentz factor and is the multiplication constant connecting $\left|\overline{SO}\right|$ to $\left|\overline{SO'}\right|$.

8 Saturation of entropy

8.1 Entropy saturation principle

The power-time formulation suggests three competing entropic saturation rates; linear, quadratic and cubic. Each is responsible for a saturation principle of a different dimensional level. In the quadratic case, it corresponds to the well-known holographic principle[24]. In the other cases, it corresponds to an exfoliation of the holographic screen into spacetime.

[24] G. 't Hooft. Dimensional Reduction in Quantum Gravity. ArXiv General Relativity and Quantum Cosmology e-prints, October 1993; and L. Susskind. The world as a hologram. Journal of Mathematical Physics, 36:6377–6396, November 1995. DOI: 10.1063/1.531249

Theorem 8.1. *The power-time formulation implies a holographic principle in the quadratic saturation.*

Proof.

$$(\ln 2)^{-1} T dS = k dA \qquad \text{regime 5.64} \tag{8.2}$$

$$\int T dS = (\ln 2) \int k dA \tag{8.3}$$

$$TS = (\ln 2) k \frac{1}{2} A + C \tag{8.4}$$

$$\implies S \propto A \qquad \text{holographic principle} \tag{8.5}$$

□

The laws of physics that will be derived as a consequence of quadratic saturation will necessarily contain a holographic principle linking the entropy to the area enclosing the volume.

However, this need not necessarily hold at other entropic saturations, for example, when under cubic saturation. Indeed, the power-time formulation would appear to suggest three different scales, each having a saturation principle of a different dimensional size.

Dimension	Associated Term	Entropy	
$1D$	Fdx	$S \propto L$	(8.6)
$2D$	kdA	$S \propto A$	(8.7)
$3D$	pdV	$S \propto V$	(8.8)

We can further show that these saturation scales compete with each other. Recall the definitions of dx, dA and dV:

$$dx := G'(0)d|p| \tag{8.9}$$
$$dA := G''(0)|p|d|p| \tag{8.10}$$
$$dV := G'''(0)|p|^2 d|p| \tag{8.11}$$

The terms $G'(0)$, $G''(0)$ and $G'''(0)$ are associated with the coefficient of the Taylor expansion. For dx, dA and dV to vary independently, the coefficients must be varied. For example, we can stuff all the entropy into the dA variable, but to do so, we must set the coefficient $G'(0)$ and $G'''(0)$ to zero, and vice versa. Likewise, the entropy of dA can be reduced provided that $G''(0)$ is reduced and $G'(0)$ and $G'''(0)$ are increased.

In this scenario, the exfoliation variables each compete with each other for entropy. Hence, a holographic screen can be exfoliated into the linear term while preserving the total entropy of the system.

8.2 2D saturation of entropy (General relativity)

In this section, we will show how regime 5.64 suggests that general relativity is an emergent entropic phenomenon attributable to the quadratic saturation of entropy (holographic principle).

Theorem 8.12. *The 2D saturation of entropy implies general relativity.*

Proof. Our goal in this proof is to derive the Einstein field equation of general relativity starting from the holographic principle. First consider that the units of TdS are *algorithmic-Joules*, hence we can pose $dE = TdS$ where dE is an *algorithmic-energy*.

$$\frac{1}{\ln 2}TdS = kxdx \tag{8.13}$$
$$\implies S \propto A \tag{8.14}$$
$$\implies dE = \gamma dA \tag{8.15}$$

Deriving general relativity from $dE = \gamma dA$ has indeed been done before in the literature, notably by Ted Jacobson, then later (and differently) by Erik Verlinde[25]. Furthermore, Christoph Schiller argues that a maximum power (7.10) implies the Field equation[26]. Here, we will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

Jacobson, starting from $dE = TdS$, first connects dE to an arbitrary coordinate system and energy flow rates,

$$dE = \int T_{ab}k^a d\Sigma^b \tag{8.16}$$

[25] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. DOI: 10.1103/PhysRevLett.75.1260. URL https://link.aps.org/doi/10.1103/PhysRevLett.75.1260; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[26] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2

Here T_{ab} is an energy-momentum tensor, k is a killing vector field and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that, assuming that the holographic principle holds (here we have an equivalent saturation principle 5.64), the right part of (8.15) can be rewritten to

$$dA = \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{8.17}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab} k^a d\Sigma^b = \gamma \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{8.18}$$

which can only be satisfied if

$$T_{ab} = \gamma \frac{c^2}{a} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{8.19}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant).

□

8.3 3D saturation of entropy (Dark energy)

Associating dark energy to a volumetric entropy has been suggested and discussed by other authors before[27]. Here, we suggest that dark energy provides the interpretation for cubic saturation of entropy.

$$TdS = (\ln 2) p dV \qquad \text{regime 5.65} \tag{8.20}$$

To determine the value of the pressure p associated with volumetric entropy, we consider the case of an entropic force. In this case, the pressure relates to the force as

$$F = -pA \tag{8.21}$$

$$\implies p = -\frac{F}{A} = -\frac{F}{4\pi x^2} \tag{8.22}$$

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward the enclosing area.

[27] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL `http://www.sciencedirect.com/science/article/pii/S0370269310014048`; and Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12):1250066, 2012. DOI: 10.1142/S0217751X12500662. URL `http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662`

To determine x, it suffices to notice that the exfoliation variables encodes the informational content of the universe up to a boundary on size, which is common to all terms of the Taylor expansion. Physically and as argued by Easson et al., it makes sense to connect this bound to the Hubble horizon as it defines an event horizon applicable to the "instantaneous" system. As it is an event horizon, its temperature is given by De Sitter's temperature and is constant at the horizon. Therefore, an entropic force is expected. To obtain the magnitude of the force, it suffices to calculate the entropic force as per the Bekenstein-Hawking entropy and the De Sitter temperature, both applicable to event horizons.

$$dS = 2\pi \frac{k_B c^3}{G\hbar} x dx \quad \text{Bekenstein-Hawking entropy} \tag{8.23}$$

$$T = \frac{\hbar}{k_B}\frac{H}{2\pi} \quad \text{De Sitter temperature} \tag{8.24}$$

$$F = T\frac{dS}{dx} \quad \text{entropic force} \tag{8.25}$$

$$\implies F = \left(\frac{\hbar}{k_B}\frac{H}{2\pi}\right)\left(2\pi\frac{k_B c^3}{G\hbar}x\right) \tag{8.26}$$

$$= \frac{c^3}{G}Hx \quad \text{clean up} \tag{8.27}$$

As x is the radius of the Hubble horizon $x = c/H$, we obtain the final value of the force $F = c^4/G$, the Planck force. Finally, the pressure is given by;

$$F = \frac{c^4}{G} \quad \text{Planck force} \tag{8.28}$$

$$\implies p = -\frac{F}{A} = -\left(\frac{c^4}{G}\right)\left(\frac{1}{4\pi(c/H)^2}\right) \tag{8.29}$$

$$p = -\frac{c^2H^2}{4\pi G} \quad \text{(negative pressure)}$$

This is close to the current measured value for the negative pressure associated with dark energy[28]. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

8.4 1D saturation of entropy (Law of inertia)

In this section we will need to use the Unruh temperature[29]. As can be reviewed in the citations provided, the Unruh temperature is an exact result obtained from special relativity. The Unruh effect is the prediction that an accelerating observer will observe blackbody

[28] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048

[29] Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. DOI: 10.1103/PhysRevD.7.2850. URL https://link.aps.org/doi/10.1103/PhysRevD.7.2850; P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4): 609, 1975. URL http://stacks.iop.org/0305-4470/8/i=4/a=022; W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14:870–892, Aug 1976. DOI: 10.1103/PhysRevD.14.870. URL https://link.aps.org/doi/10.

radiation (at the Unruh temperature) where an inertial observer would observe none. The Unruh temperature is:

$$T = \frac{\hbar a}{2\pi c k_B} \qquad \text{Unruh temperature} \qquad (8.30)$$

The Unruh temperature connects acceleration to temperature. We will use it here to convert an entropic force expressed in terms of a temperature to an entropic force expressed in terms of acceleration.

First, let us derive a relation between dS and dN. Here N represents the number of bits.

Theorem 8.31. $dS = (\ln 2) k_B dN$

Proof.

$$\begin{aligned} S &= N k_B \ln(2) && \text{binary entropy} && (8.32)\\ \implies dS &= (\ln 2) k_B dN && && (8.33) \end{aligned}$$

□

Second, let us look at the implications of the first term, Fdx in the $S \propto L$ regime.

Theorem 8.34. *The $S \propto L$ scale implies the law of inertia, $F = ma$.*

Proof. First, consider the equation for an entropic force $F = T\Delta S/\Delta x$ such as a the case of a polymer or of osmosis. In the case of a binary entropy, the entropic force takes the form;

$$\begin{aligned} Fdx &= (\ln 2)^{-1} T dS && \text{regime 5.62} && (8.35)\\ F &= (\ln 2)^{-1} T \frac{dS}{dx} && \text{divide } dx && (8.36)\\ F &= (\ln 2)^{-1} T \frac{(\ln 2) k_B dN}{dx} && \text{binary entropy} && (8.37)\\ F &= T k_B \frac{dN}{dx} && \text{entropic force} && (8.38) \end{aligned}$$

An accelerated object implies the Unruh temperature. Here, we start from the other side. We have T and we replace it with the Unruh temperature.

$$\begin{aligned} F &= \left(\frac{\hbar a}{2\pi c k_B}\right) k_B \frac{dN}{dx} && \text{Unruh temperature} && (8.39)\\ F &= \left(\frac{1}{2\pi}\frac{\hbar}{c}\frac{dN}{dx}\right) a && \text{clean up} && (8.40) \end{aligned}$$

Finally, the equation $F = ma$ can be recovered under the hypothesis that the ratio dx/dN is the reduced Compton wavelength multiplied by 2π.

$$\implies 2\pi \frac{dx}{dN} = \frac{\hbar}{mc} \tag{8.41}$$

From this derivation, the reduced Compton wavelength can now be understood as the ratio between inertial mass and entropy.

□

8.5 *Note on the Schwarzschild radius*

As we have seen, the inertial mass is associated with the linear entropy. We have also seen the existence of a saturation principle applicable to linear entropy. As a result, we would expect that the mass in the universe is bounded linearly. Is that the case?

Consider the Schwarzschild radius,

$$R = \frac{2GM}{c^2} \tag{8.42}$$

As we can see, the radius grows linearly with the mass M. Hence, the linear entropy does saturate.

9 *Universal Brownian motion*

As we have seen in section 5.3, thermal spacetime experiences fluctuations along the x and t axis. We recall the fluctuation relations;

	average	fluctuation	
t (time)	$\bar{t} = \frac{-\partial \ln Z}{\partial P}$	$\overline{(\Delta t)^2} = \frac{\partial^2 \ln Z}{\partial P^2}$	(9.1)
x (space)	$\bar{x} = \frac{-\partial \ln Z}{\partial F}$	$\overline{(\Delta x)^2} = \frac{\partial^2 \ln Z}{\partial F^2}$	(9.2)

9.1 *Schrödinger equation*

In section 8.4, we have used the program-size to entropy relation $TdS = Fdx$ to recover $F = ma$. In this section we use the same relation but we extend it with the fluctuations effects of the thermal UTM. Doing so will allow us to recover the Schrödinger equation.

We recall that a thermal UTM encodes position via the dx conjugate associated with program lengths. As a result, the UTM can only express a position if the program with the corresponding size is part of its partition function (e.i. it halts). In this section, we will argue that the missing non-halting programs are responsible for a universal Brownian motion in space applicable to the dx variable. This will be enough to recover the Schrödinger's equation.

Theorem 9.3. *A position described with missing program-sizes will evolve in time according to Schrödinger's equation.*

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(x,t)$$

The proof is slightly more involved than the preceding theorems. First, here is a sketch of the proof.

1. We will show that non-halting programs leave holes in space such that a position cannot be expressed.

2. We will show that these holes are causing a Brownian motion of the encoded position.

3. We will derive its diffusion coefficient to be $\hbar/(2m)$.

4. We will consider that the presence of any external field is experienced as acceleration via $F = ma$.

5. Using the well known Brownian motion equations of Langevin, we show that the above reproduces Schrödinger's equation exactly.

Lemma 9.4. *A spacial encoding based on programs will leave holes in space corresponding to non-halting programs.*

Proof. We use regime 5.63 applicable to the inertial law. We have also seen that the conjugate x denotes program lengths. However, not all programs halt hence some lengths are missing from the sum. These missing programs are holes in space the position of which cannot be expressed by the UTM's positional algorithm. Since Ω is a normal number, we can expect the position of these holes to be algorithmically random. □

Lemma 9.5. *A particle in space will experience Brownian motion due to the holes.*

Proof. We will calculate the average displacement $\overline{\Delta x}$ of a particle subjected to entropic positioning and space holes. Since Z is a normal number, we conclude that half of the program's lengths are available to describe position and half are not. Therefore, to describe a particle at position x, there is a 50% chance there is a halting program available to express it. And in the case where there is no program at exactly x, then there is a 50% chance that there will be one at position

$x+1$, and so on. In other words, a particle at x has 50% chance of being at x, 25% chance of being at $x+1$, 12.5% chance of being at $x+2$, etc. Expressed as a sum, we obtain

$$\overline{\Delta x} = \frac{1}{2}0 + \frac{1}{4}1 + \frac{1}{8}2 + \frac{1}{16}3 + ... \tag{9.6}$$

$$= \sum_{i=0}^{\infty} \frac{i}{2^{i+1}} \tag{9.7}$$

$$= 1 \tag{9.8}$$

On average, as it moves through space, a position will shift by $\overline{\Delta x} = 1$ at each iteration of the Brownian motion. □

Lemma 9.9. *The diffusion coefficient of the described Brownian motion is*

$$D = \frac{\hbar}{2m}$$

Proof. From Einstein paper the diffusion coefficient of Brownian motion is given by

$$D = \frac{l^2}{2\tau} \tag{9.10}$$

where l is the length of the random step and τ is the frequency of the occurrence of the steps. As we have previously connected the reduced Compton wavelength to $F = ma$ taking the role of the system's characteristic length associated with positional encoding for a mass of bits, it makes sense to use it here as well. We get a scaling factor of

$$\lambda = \frac{\hbar}{mc} \tag{9.11}$$

Since entropic positioning can only express position as multiples of λ, we take it as the Brownian step of length l. The diffusion coefficient becomes

$$D = \left(\frac{\hbar}{mc}\right)^2 \frac{1}{2\tau} \tag{9.12}$$

This leaves of us with the need to define τ. For τ, we take the characteristic frequency of the wave $E = \hbar\omega$. This is related to proof-step frequency. Solving for $\tau = 1/\omega$, we obtain

$$\omega = \frac{E}{\hbar} \tag{9.13}$$

$$\omega^{-1} = \frac{\hbar}{E} = \tau \tag{9.14}$$

Replacing τ in the equation for D, we obtain

$$D = \frac{\hbar^2}{m^2c^2}\left(\frac{E}{2\hbar}\right) \tag{9.15}$$

Using $E = mc^2$, and reducing the constants, we obtain our final expression of D,

$$D = \frac{\hbar^2}{m^2c^2}\frac{(mc^2)}{2\hbar} \tag{9.16}$$

$$= \frac{\hbar}{2m} \tag{9.17}$$

□

Lemma 9.18. *The Langevin equations for Brownian motion with a diffusion coefficient of $\hbar/(2m)$ and an external inertial field experienced as $F = ma$ reproduces Schrödinger's equation.*

Proof. We recall the Langevin equation,

$$d\left[x(t)\right] = v(t)dt \tag{9.19}$$

$$d\left[v(t)\right] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \tag{9.20}$$

where $W(t)$ is a random force and a stochastic variable giving the effect of a background noise to the motion of the particle.

From $F = ma$ and replacing the acceleration $d[v(t)]/dt$ with F/m, Edward Nelson [30] is able to show that the Langevin equation becomes,

[30] Edward Nelson. Derivation of the schrodinger equation from newtonian mechanics. Phys. Rev., 150:1079–1085, Oct 1966. DOI: 10.1103/PhysRev.150.1079. URL https://link.aps.org/doi/10.1103/PhysRev.150.1079

$$\nabla\left(\frac{1}{2}\vec{u}^2 + D\nabla\cdot\vec{u}\right) = \frac{1}{m}\nabla V \tag{9.21}$$

where D is the diffusion coefficient of $\hbar/(2m)$ obtained in lemma 9.9, where $\vec{F} = -\nabla V$, where $\vec{u} = v\nabla\ln\rho$ and ρ is the probability density of $x(t)$. For brevity, the proof of 9.21 is omitted here but can be reviewed in Nelson's paper. Eliminating the gradients on each side and simplifying the constants, we obtain

$$\frac{m}{2}\vec{u}^2 + \frac{\hbar}{2}\nabla\cdot\vec{u} = V - E \tag{9.22}$$

where E is the arbitrary integration constant. This equation in non-linear because of the term $\vec{u}^2$ but it can be made linear by a change of dependant variable. To make it linear, let us pose

$$\vec{u} = \frac{\hbar}{m}\frac{1}{\psi}\nabla\psi \tag{9.23}$$

and replace it into equation 9.22 , we obtain

$$
\begin{aligned}
V - E &= \frac{m}{2}\left(\frac{\hbar}{m}\frac{1}{\psi}\nabla\psi\right)^2 + \frac{\hbar}{2}\nabla\cdot\left(\frac{\hbar}{m}\frac{1}{\psi}\nabla\psi\right) && (9.24)\\
&= \frac{\hbar^2}{2m}\frac{1}{\psi^2}(\nabla\psi\cdot\nabla\psi) + \frac{\hbar^2}{2m}\left[\nabla\cdot\left(\frac{1}{\psi}\nabla\psi\right)\right] && (9.25)\\
&= \frac{\hbar^2}{2m}\frac{1}{\psi^2}(\nabla\psi\cdot\nabla\psi) + \frac{\hbar^2}{2m}\left[\frac{\psi\nabla\cdot\nabla\psi - \nabla\psi\cdot\nabla\psi}{\psi^2}\right] && \text{(Identity)}\\
&= \frac{\hbar^2}{2m}\frac{1}{\psi^2}(\nabla\psi\cdot\nabla\psi) + \frac{\hbar^2}{2m}\left[\frac{1}{\psi}\nabla\cdot\nabla\psi - \frac{1}{\psi^2}(\nabla\psi\cdot\nabla\psi)\right] && (9.26)
\end{aligned}
$$

The first and the last terms cancel each other.

$$\frac{\hbar^2}{2m}\frac{1}{\psi}\nabla^2\psi = V - E \tag{9.27}$$

Finally, it simplifies to

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V - E\right]\psi = 0 \tag{9.28}$$

which is the time independent Schrödinger's equation.

□

We are now ready to derive the time dependent Schrödinger equation and prove theorem 9.3.

Proof. We use the same proof used by Edward Nelson in the same paper. Starting from the time dependent Schrödinger equation, we show that a replacement of $\psi = e^{R+iS}$ leads to the Langevin equation of Brownian motion.

$$\frac{\partial\psi}{\partial t} = i\frac{\hbar}{2m}\nabla^2\psi - i\frac{1}{\hbar}V\psi \tag{9.29}$$

Replacing ψ with e^{R+iS}, we obtain

$$\frac{\partial\left(e^{R+iS}\right)}{\partial t} = i\frac{\hbar}{2m}\nabla^2\left(e^{R+iS}\right) - i\frac{1}{\hbar}V\left(e^{R+iS}\right) \tag{9.30}$$

Taking the derivatives and the gradients, we obtain

$$\left[\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t}\right]\left(e^{R+iS}\right) = \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla(R+iS))^2\right]\left(e^{R+iS}\right) - i\frac{1}{\hbar}V\left(e^{R+iS}\right) \tag{9.31}$$

Eliminating e^{R+iS} from each side and simplifying, we obtain

$$
\begin{aligned}
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla(R+iS))^2\right] - i\frac{1}{\hbar}V && \text{(eliminating } e^{R+iS}\text{)}\\
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{i\hbar}{2m}\left[\nabla^2 R + i\nabla^2 S + (\nabla R)^2 + 2i\nabla R\cdot\nabla S - (\nabla S)^2\right] - i\frac{1}{\hbar}V && \text{(taking the product)}\\
\frac{\partial R}{\partial t} + i\frac{\partial S}{\partial t} &= \frac{\hbar}{2m}\left[i\nabla^2 R - \nabla^2 S + i(\nabla R)^2 - 2\nabla R\cdot\nabla S - i(\nabla S)^2\right] - i\frac{1}{\hbar}V && \text{(distributing the } i\text{)}
\end{aligned}
$$

We obtain two equations by separating the real and the imaginary parts

$$\frac{\partial R}{\partial t} = \frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R \cdot \nabla S\right] \tag{9.32}$$
$$\frac{\partial S}{\partial t} = \frac{\hbar}{2m}\left[\nabla^2 R + (\nabla R)^2 - (\nabla S)^2\right] - \frac{1}{\hbar}V \tag{9.33}$$

This is equivalent to the Langevin equations with some replacements

$$\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m}\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{v} \cdot \vec{u}) \tag{9.34}$$
$$\frac{\partial \vec{v}}{\partial t} = \frac{\hbar}{2m}\nabla(\nabla \cdot \vec{u}) + \frac{1}{2}\nabla(\vec{u}^2) - \frac{1}{2}\nabla(\vec{v}^2) - \frac{1}{m}\nabla V \tag{9.35}$$

Lemma 9.36. *Equation 9.32 with the replacements $\nabla R = (m/\hbar)\vec{u}$ and $\nabla S = (m/\hbar)\vec{v}$ produces 9.34*

Proof.

$$\frac{\partial R}{\partial t} = \frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R \cdot \nabla S\right] \qquad \text{(equation 9.32)}$$
$$\nabla\frac{\partial R}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[-\nabla^2 S - 2\nabla R \cdot \nabla S\right]\right) \qquad \text{(taking the gradient)}$$
$$\frac{\partial \nabla R}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[-\nabla \cdot \nabla S - 2\nabla R \cdot \nabla S\right]\right) \tag{9.37}$$
$$\frac{m}{\hbar}\frac{\partial \vec{u}}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[-\nabla \cdot \left(\frac{m}{\hbar}\vec{v}\right) - 2\left(\frac{m}{\hbar}\vec{u}\right) \cdot \left(\frac{m}{\hbar}\vec{v}\right)\right]\right) \qquad \text{(replacing } \nabla R \text{ and } \nabla S\text{)}$$
$$\frac{\partial \vec{u}}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[-\nabla \cdot \vec{v} - 2\frac{m}{\hbar}\vec{u} \cdot \vec{v}\right]\right) \qquad \text{(eliminating } m/\hbar\text{)}$$
$$\frac{\partial \vec{u}}{\partial t} = -\frac{\hbar}{2m}\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u} \cdot \vec{v}) \qquad \text{(equation 9.34)}$$

□

Lemma 9.38. *Equation 9.33 with the replacements $\nabla R = (m/\hbar)\vec{u}$ and $\nabla S = (m/\hbar)\vec{v}$ produces 9.35*

Proof.

$$\frac{\partial S}{\partial t} = \frac{\hbar}{2m}\left[\nabla^2 R + (\nabla R)^2 - (\nabla S)^2\right] - \frac{1}{\hbar}V \qquad \text{(equation 9.33)}$$
$$\nabla\frac{\partial S}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[\nabla \cdot \nabla R + (\nabla R)^2 - (\nabla S)^2\right]\right) - \frac{1}{\hbar}\nabla V \qquad \text{(taking the gradient)}$$
$$\frac{m}{\hbar}\frac{\partial \vec{v}}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[\nabla \cdot \left(\frac{m}{\hbar}\vec{u}\right) + \left(\frac{m}{\hbar}\vec{u}\right)^2 - \left(\frac{m}{\hbar}\vec{v}\right)^2\right]\right) - \frac{1}{\hbar}\nabla V \qquad \text{(replacing } \nabla R \text{ and } \nabla S\text{)}$$
$$\frac{\partial \vec{v}}{\partial t} = \nabla\left(\frac{\hbar}{2m}\left[\nabla \vec{u} + \frac{m}{\hbar}\vec{u}^2 - \frac{m}{\hbar}\vec{v}^2\right]\right) - \frac{1}{m}\nabla V \qquad \text{(eliminating } m/\hbar\text{)}$$
$$\frac{\partial \vec{v}}{\partial t} = \frac{\hbar}{2m}\nabla(\nabla \cdot \vec{u}) + \frac{1}{2}\nabla(u^2) - \frac{1}{2}\nabla(v^2) - \frac{1}{m}\nabla V \qquad \text{(equation 9.35)}$$

□

This completes the proof of theorem 9.3. □

9.2 *Dirac equation*

In a previous section, we have used $TdS = Fdx$ to recover $F = ma$. In section 7, we have used $TdS = Pdt + Fdx$ to recover special relativity. We have then used a random walk on dx to recover the Schrödinger equation which is the quantum analogue to $F = ma$. Of course, the natural question to ask is, will using $TdS = Pdt + Fdx$ and applying a random walk to both dt and dx be enough to recover the Dirac equation, the quantum analogue to special relativity? The answer is yes!

In this section, we will see that applying a random walk to both the dt and the dx variables is enough to recover the Dirac equation for relativistic quantum mechanics. Let us begin by answering why would there be a random walk on dt.

First we consider that, as is the case with program length, program runtime varies from one UTM to the next. Programs that are difficult to solve on one UTM are likely to be difficult to solve on other UTMs. For example the travelling salesman problem is hard to solve on every UTM. The runtime of these programs will be randomly distributed and centred around a mean runtime.

Second, we consider an analogous argument to the one used to justify a random walk on dx, but applied to dt. On some UTM a program of size x might have halted and on others it might not have. Therefore a particle can be defined to be at a time t only if a program halting at time t is in the partition function. If there is no such available halting program at time t, then the particle will be a time $t \pm \Delta t$, the runtime of the next available halting program. Since the halting problem is algorithmically random and non-computable, we consider this behaviour as a random walk in time.

A connection between a random walk in time and space and the telegraphic equation has been linked to the Dirac equation before[31]. D. G. C. McKeon and G. N. Ord proposes a random walk model in space and in time. Starting from the equation for a random walk in space, we have

[31] D Mckeon and G N. Ord. Time reversal in stochastic processes and the dirac equation. Physical review letters, 69:3–4, 08 1992; and D. G. C. McKeon and G. N. Ord. On how the (1+1)-dimensional dirac equation arises in classical physics. Foundations of Physics Letters, 9 (5):447–456, Oct 1996. ISSN 1572-9524. DOI: 10.1007/BF02190048. URL https://doi.org/10.1007/BF02190048

$$P_{\pm}(x, t + \Delta t) = (1 - a\Delta t)P_{\pm}(x \mp \Delta x, t) + a\Delta t P_{\mp}(x \pm \Delta x, t) \tag{9.39}$$

then, D. G. C. McKeon and G. N. Ord extend this equation with a random walk in time. They obtain

$$F_{\pm}(x,t) = (1 - a_L\Delta t - a_R\Delta t)F_{\pm}(x \mp \Delta x, t - \Delta t) + a_{L,R}\Delta t B_{\pm}(x \mp \Delta x, t + \Delta t) + a_{R,L}\Delta t F_{\mp}(x \pm \Delta x, t - \Delta t) \tag{9.40}$$

where $F_{\pm}(x,t)$ is the probability distribution to go forward in time and $B_{\pm}(x,t)$, backward in time. They then introduce a causality condition such that $F_{\pm}(x,t)$ and $B_{\pm}(x,t)$ only depends on probabilities from the past.

$$F_{\pm}(x,t) = B_{\mp}(x \pm \Delta x, t + \Delta t) \tag{9.41}$$

From equation 9.2 and 9.41, they get

$$B_{\pm}(x,t) = (1 - a_L\Delta t - a_R\Delta t)B_{\pm}(x \mp \Delta x, t + \Delta t) + a_{L,R}\Delta t B_{\mp}(x \pm \Delta x, t + \Delta t) + a_{R,L}\Delta t F_{\pm}(x \mp \Delta x, t - \Delta t) \tag{9.42}$$

In the limit $\Delta x, \Delta t \to 0$ and with $\Delta x = v\Delta t$, they get,

$$\pm v\frac{\partial F_{\pm}}{\partial x} + \frac{\partial F_{\pm}}{\partial t} = a_{L,R}(-F_{\pm} + B_{\pm}) + a_{R,L}(-F_{\pm} + F_{\mp}) \tag{9.43}$$

$$\pm v\frac{\partial B_{\mp}}{\partial x} + \frac{\partial B_{\mp}}{\partial t} = a_{L,R}(-B_{\mp} + F_{\mp}) + a_{R,L}(-B_{\mp} + B_{\pm}) \tag{9.44}$$

Posing these changes of variables,

$$A_{\pm} = (F_{\pm} - B_{\mp})\exp[(a_L + a_R)t] \tag{9.45}$$

$$\lambda := -a_L + a_R \tag{9.46}$$

then 9.44 becomes

$$v\frac{\partial A_{\pm}}{\partial x} \pm \frac{\partial A_{\pm}}{\partial t} = \lambda A_{\mp} \tag{9.47}$$

Finally, they pose $v = c$, $\lambda = mc^2/\hbar$ and $\psi = F(A_+, A_-)$, they get

$$i\hbar\frac{\partial \psi}{\partial t} = mc^2\sigma_y\psi - ic\hbar\sigma_z\frac{\partial \psi}{\partial x} \tag{9.48}$$

which is the Dirac equation in 1+1 spacetime.

9.3 Discussion

The mathematical derivation of the Schrödinger and the Dirac equation are borrowed from the field of stochastic mechanics. The field has different goals and provides a different interpretation that what is suggested in this paper. The goal of stochastic mechanics is to recover and explain the laws of quantum mechanics as emergent from microscopic stochastic processes such as Brownian motion. They admit punctual particle type of interpretation in which the particle is imagined to undergo a random walk creating a diffusion in the probability of its position. The position is recovered when the point-like particle is measured again.

In this paper[32], Edward Nelson reviews the pros and cons of the stochastic mechanics approach. In in, he lists the following successes and failures of the approach.

[32] Edward Nelson. Review of stochastic mechanics. In Journal of Physics: Conference Series, volume 361, page 012011. IOP Publishing, 2012

"4. Successes of stochastic mechanics

Here is a list of the main successes of stochastic mechanics.

1. A classical derivation of the Schrödinger equation, by Guerra and Morato[33].
2. The probability density ρ of the Markov process agrees with $|\Psi|^2$ at all times.
3. A stochastic explanation of the relation between momentum and the Fourier transform of the wave function, by David Shucker[34].
4. A proof of the existence of the Markov process under the physically natural assumption of finite action, by Eric Carlen[35]. This is perhaps the most technically demanding work in the entire subject.
5. A stochastic explanation of why identical particles satisfy either Bose-Einstein or Fermi-Dirac statistics if $d \geq 3$, with para-statistics possible if $d = 2$. This is not contained in §20 of [this reference][36], but it follows from the discussion there.
6. A stochastic explanation of spin and why it is integral or half-integral, work of Thaddeus Dankel[37], Timothy Wallstrom[38], and of Daniela Dohrn and Francesco Guerra jointly[39].
7. If the force is time-independent, the expected stochastic energy $E_t(\frac{1}{2}u^i u_i + \frac{1}{2}v^i v_i + \varphi)$ is conserved; see §14 of [this reference][40].
8. A stochastic picture of the two-slit experiment, explaining how particles have trajectories going through just one slit or the other, but nevertheless produce a probability density as for interfering waves; see §17 of [this reference][41]."

- - Edward Nelson

[33] Francesco Guerra and Laura M Morato. Quantization of dynamical systems and stochastic control theory. Physical review D, 27(8):1774, 1983

[34] David S Shucker. Stochastic mechanics of systems with zero potential. Journal of Functional Analysis, 38(2): 146–155, 1980

[35] Eric A Carlen. Existence and sample path properties of the diffusions in nelson's stochastic mechanics. In Stochastic ProcessesâĂŤMathematics and Physics, pages 25–51. Springer, 1986

[36] Edward Nelson. Quantum fluctuations. Princeton University Press, 1985

[37] Thad Dankel Jr. Higher spin states in the stochastic mechanics of the bopp–haag spin model. Journal of Mathematical Physics, 18(2):253–255, 1977

[38] Timothy C Wallstrom. On the derivation of the schrödinger equation from stochastic mechanics. Foundations of Physics Letters, 2(2):113–126, 1989

[39] Daniela Dohrn and Francesco Guerra. Nelson's stochastic mechanics on riemannian manifolds. Lettere al Nuovo Cimento (1971-1985), 22(4):121–127, 1978

[40] Nelson, 1985

[41] Nelson, 1985

Edward Nelson, in the same paper also suggests the following failure of the theory.

"5. Failures of stochastic mechanics

[...]

Since $\rho = |\Psi|^2$ at all times, stochastic mechanics gives the same prediction as quantum mechanics for a measurement performed at a single time. But it can give wrong predictions for measurements performed at two different times; see Chapter 10 of [this reference][42]. Consider two entangled but dynamically uncoupled harmonic oscillators. Let $X_i(t)$ be the Heisenberg position operator of oscillator i at time t. Each is periodic in t, so the correlation of $(X_1(t_1), X_2(t_2))$ does not decay as $t_2 \to \infty$. Let $x_i(t)$ be the position of oscillator i at time t according to stochastic mechanics. Then $x_i(t)$ has the same probability distribution as $X_i(t)$ for each i and each t, but $(x_1(t_1), x_2(t_2))$ does not have the same probability distribution as $(X_1(t_1), X_2(t_2))$. In fact, the correlation of $(x_1(t_1), x_2(t_2))$ decays to 0 as $t_2 \to \infty$. The oscillators are uncoupled, so $X_1(t_1)$ and $X_2(t_2)$ commute, and according to quantum mechanics, the probability distribution is that of $(X_1(t_1), X_2(t_2))$. If $(x_1(t_1), x_2(t_2))$ represented the real physical situation, theirs would be the probability distribution. Thus stochastic mechanics and quantum mechanics give different predictions for the result. Why do I not suggest that the experiment be done? Because if a record of the observation of the first oscillator at time t_1 is made by some physical means, and similarly for the second oscillator, and the two records are compared at a common later time t_3, this is an observation at a single time, for which quantum mechanics and stochastic mechanics agree. The non-locality of stochastic mechanics conspires to bring the records into agreement.

How can a theory to be so right and yet so wrong? The most natural explanation is that stochastic mechanics is an approximation to a correct theory of quantum mechanics as emergent. But what is the correct theory?"

- - Edward Nelson

[42] William G Faris. Diffusion, Quantum Theory, and Radically Elementary Mathematics.(MN-47). Princeton University Press, 2014

This failure only occurs because stochastic mechanic attempt to explain quantum effects (such as entanglement) by replacing them with a statistical and classical approach. However, this failure does not apply to us because we are not trying to eliminate quantum entanglement. As a result, we are fine accepting entangled states onto which the Schrödinger equation is applied as per the standard quantum mechanics theory.

10 Characteristic units

Our goal in this section is to show how the definition of the Planck units naturally follows from the state equation (5.57). To do so, we must first obtain definitions for G, c and $\hbar$ by deriving from it known laws of physics that contain them.

10.1 Gravitation constant

We start by obtaining the gravitational constant G from Newton's law of gravitation.

Theorem 10.1. *The gravitational constant G is defined as $c^3L^2/\hbar$.*

Proof. A derivation of Newton's law of gravitation from the entropic perspective has been done before by Erik Verlinde[43]. Here to obtain the law of gravitation, we work in regime 5.64. This regime contains the 2D-holographic principle and, as a result, the entropy of the system grows via x^2, an area law. We further consider that the entropy of this area law is given by bits exclusively occupying a small area L^2 on the surface. In this case, the total number of bits on the surface is given by

[43] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

$$N = \frac{4\pi x^2}{L^2} \tag{10.2}$$

The equipartition theorem applies to energy terms of the partition function, which are quadratic. The term $kxdx$ is $\frac{1}{2}kx^2$ in the partition function. As a result its average energy is $E = \frac{1}{2}Nk_BT$ as per the equipartition theorem.

$$E = \frac{1}{2}\left(\frac{4\pi x^2}{L^2}\right)k_BT \tag{10.3}$$

$$\implies T = \frac{L^2}{2\pi k_B}\frac{E}{x^2} \tag{10.4}$$

We obtain a constant temperature throughout the system indicating that it is at thermodynamic equilibrium. As our goal is to recover the gravitational constant, we inject this temperature in the entropic force relation.

$$F = Tk_B\frac{dN}{dx} \qquad \text{entropic force (8.38)} \tag{10.5}$$

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)k_B\frac{dN}{dx} \qquad \text{derived temperature} \tag{10.6}$$

We then replace the ratio dx/dN by the reduced Compton wavelength.

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)k_B\left(2\pi\frac{mc}{\hbar}\right) \tag{10.7}$$

$$F = \left(\frac{L^2c}{\hbar}\right)\frac{Em}{x^2} \qquad \text{clean up} \tag{10.8}$$

We then convert E to its rest mass via $E = mc^2$.

$$F = \left(\frac{L^2c^3}{\hbar}\right)\frac{Mm}{x^2} \tag{10.9}$$

We obtain the Newton's law of gravitation along with a definition for G.

$$F = G\frac{Mm}{x^2} \tag{10.10}$$

$$\implies G = \frac{L^2c^3}{\hbar} \tag{10.11}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{(Planck's length)}$$

□

10.2 *Speed of light*

Theorem 10.12. *The speed of light c is defined by P/F.*

Proof. We refer to the proof for theorem 7.1 where P/F is a characteristic speed associated with an inversion in the direction of the second law of thermodynamics. Then, under the principle that the second law is irreversible, the speed P/F is a boundary and defines c. □

10.3 *Planck's constant*

Theorem 10.13. *The action $\mathcal{S}$ is defined by $\hbar$.*

Proof.

$$\frac{1}{\ln 2}TdS = 2\pi\mathcal{S}df \qquad \text{regime 5.62} \tag{10.14}$$

$$dE = \frac{1}{\ln 2}TdS = 2\pi\mathcal{S}df \qquad \text{units of energy} \tag{10.15}$$

$$dE = 2\pi\mathcal{S}df \qquad \text{posing } dS = 0 \tag{10.16}$$

Switching to the angular frequency,

$$dE = \mathcal{S}d\omega \qquad df = d\omega/(2\pi) \tag{10.17}$$

$$\int dE = \int \mathcal{S}d\omega \tag{10.18}$$

$$E = \mathcal{S}\omega + C \tag{10.19}$$

Posing $C = 0$, this is the photon angular-frequency to energy relation $E = \hbar\omega \implies \mathcal{S} = \hbar$. □

10.4 *Planck's units*

We have now obtained a definition for three of the fundamental constants.

$$\hbar = \mathcal{S} \qquad c = \frac{P}{F} \qquad G = \frac{L^2c^3}{\hbar} \tag{10.20}$$

We can now definite characteristic units applicable to the thermal UTM,

$$G = \frac{L^2 c^3}{\hbar} \implies L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{(Planck's length)}$$

$$t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} \qquad \text{(Planck's time)}$$

$$E = \mathcal{S}/t \implies E = \sqrt{\frac{\hbar c^5}{G}} \qquad \text{(Planck's energy)}$$

$$P = t^{-2}\mathcal{S} = \frac{c^5}{G} \qquad \text{(Planck's power)}$$

$$\frac{P}{F} = c \implies F = \frac{c^4}{G} \qquad \text{(Planck's force)}$$

which agrees with the physical Planck units.

11 *Discussion*

A convincing scientific theory is one that survives falsification. Meaning, the theory should make predictions that can be either verified or falsified via physical observations. The concept of falsifiability, in principle, serves as an ideal. In practice however, there is an additional informal criteria whose mention is often neglected but one that is nonetheless also important - the prediction must be *remarkable*. And indeed, looking into the history of science we find that the more remarkable the prediction is, the more convinced we are of the validity of the theory predicting it. Being remarkable is an aesthetic; it is connected to the uniqueness of the explanation as well as to the impact of the prediction on the current state of the art.

For example, a theory which predicts a slight correction of less than one thousand of a percentage point on some measured quantity (while everything else remains equal) will not be considered a remarkable prediction. The prediction might be absolutely correct, but it would very unlikely come to replace the existing textbook theory within a reasonable timeframe. First, the cognitive burden of learning a new approach hardly justifies the near-negligible improvement. Second, many alternative theories would be presumed to be able to account for such a small variation and its uniqueness will be questioned. And indeed, in the literature, we find this is quite often the case.

However, the situation is different when the prediction is remarkable. For example, before Einstein's theory of relativity, time was assumed to be absolute and constant. Hence, the prediction that it was not was remarkable. Once experimental evidence was found to confirm this unexpected prediction, then the adoption of the new theory was favoured.

11.1 *Reductio ad 400*

Our goal here will be to produce a prediction that is both remarkable and falsifiable. One that, ideally, no other theory has predicted before. One that is of significant impact not only to physics but also to philosophy and one that if confirmed, must alter our deepest conception of reality. This prediction, if it is observed to be true, will lend tremendous credibility to this theory.

Our prediction will be a numerical estimation of the minimum number of bits required to encode the full complexity of the ensemble Z describing the universe. In other words, its compressibility ratio. Evidence of this prediction should be abundant in the universe as it affects all things. At $\bar{t}$ the numerical value of Z converges towards Ω up to an error rate. Up until the error rate, the first n bits of Z are the first n bits of Ω. We will calculate the numerical value of n as predicted by this theory for the current size and age of the universe.

To calculate it, we will consider the case where the entropy saturation principles competes with each other. In this case, we can take any saturation scale and suppose that it holds all the entropy.

Let us recall the ensemble Z.

$$Z = \sum_{i=1}^{\infty} 2^{-Af_i - |p_i|} \tag{11.1}$$

From this equation, we can calculate n as follows; Under the holographic principle, the entropy of the universe is restricted by the number of bits occupying the Planck area that can fit on the surface of a sphere enclosing the universe with radius equal to the cosmic event horizon. This is approximately 10^{122} bits of entropy. Hence, as the first n bits of Ω can decide the first 2^n theorems, we calculate n as follows:

$$n = \log_2 \left(10^{122}\right) \approx 400 \text{ bits} \tag{11.2}$$

Four hundred is the number of leading bits of Ω required to decide 10^{122} theorems. This calculation suggests that the entire informational description of the universe can be compressed to a mere 400 bits of data, enough to fit into the memory of a pocket calculator. These 400 bits are the leading part of Ω which itself is algorithmically random and cannot be compressed any further by any possible algorithm. The value of the bits of Ω as they are not given by an algorithm, cannot be deduced from pure reason. Consequently, we will argue that these bits are better interpreted as the axioms of the theory of everything of the universe. Hence, the theory of everything which describes the universe at its current size and age must have approximately 400 algorithmically random bits of axioms.

An observer knowing these 400 bits could calculate the entire informational description of the universe from first principles. To the knowledge of the author, no other theory has suggested such a strong compressibility applicable to the universe.

How credible is 400 bits? Well, we will grant that it is mind bogglingly low. But, for purposes of falsification this is a good thing. If we had obtained a compressibility of say 10^{100} bits instead of 400 bits then it would have been a much less remarkable prediction. The fact that it is so low is precisely why it is so remarkable. As far as to its credibility, consider the axioms of vanilla non-relativistic quantum mechanics. Copy-pasting the text of the axioms in notepad taken from wikipedia [44] and applying a compression algorithm, I obtain 1235 byte of data as a zip file. The file is very small, yet it can explain a large percentage of the universe. The Dirac equation takes only a handful of compressed bytes to express yet it explains an even larger part of the universe. The point is that axioms contain tremendous amount of information in a small amount of bits. Nonetheless, the compressibility of the whole universe to 400 bits of data should still surprise us. Evidence of such a low bound on complexity should be plentiful in the universe.

[44] Wikipedia - Mathematical formulation of quantum mechanics https://en.wikipedia.org/wiki/Mathematical_formulation_of_quantum_mechanics

It is worth mentioning that a similar number was obtained by Paul Davies[45] in the context of the maximal number of qubits usable by a general quantum computer. Here, we suggest that the bound of ≈ 400 as described by Davies in the context of quantum computers and qubits is essentially the same bound described here but in terms of Ω bits. The bound should serve as the primary falsifiable prediction of an informational theory of the universe. It predicts an ultimate compressibility of the universe to 400 bits of data. We can consider that the data is so compressed that its decompression algorithm operates over billions of years - the amount of time it takes to produce approximately 10^{122} uncompressed facts from 400 Ω bits using thermal dovetailing.

[45] Paul CW Davies. The implications of a cosmological information bound for complexity, quantum information and the nature of physical law. Fluctuation and Noise Letters, 7(04):C37–C50, 2007

11.2 An axiom-free theory

If the bits are the theory of everything, and we have not explicitly specified any of these bits, why is it that we were able to obtain physical laws? Are the physical laws not supposed to be encoded within the bits (Therefore, if the bits are unknown then the laws should also be unknown)? And if not, then what exactly do the bits represent?

Before we answer the question, let us imagine a computer program which constructs a virtual world out of a seed. The seed is a short sequence of random or pseudo-random numbers. As a result, the seed can be shared between two users and the world building program, as it is deterministic, will always rebuild the same world from the same seed. Hence, the world building program is the same for all worlds. Changing the seed changes the world, but it does not change the rules used to build the world.

We essentially suggest a similar interpretation. The theory of everything is the seed and the laws of physics is the program that builds the world from the seed. This is why the axiom-free methodology of removing all formal axioms and rules of inference from Miniversal logic was so critical to deriving the laws of physics. The theory of everything only contains the seed. As surprising as this might sound, the laws of physics are not part of the theory of everything - they are independently deducible from pure reason by any and all observers. This turns out to be an absolute necessity. When we reproduced the universal doubt method of Descartes within formal logic by removing rules of inference and formal axioms, we set up the only logical system capable of proving the laws of physics.

11.3 The unreasonable effectiveness of mathematics in the natural sciences

As Wigner[46] once wrote, mathematics is unreasonably effective in the natural sciences. In the present theory, the universe contains 400 bits of mathematically unexplainable information, and 10^{122} of mathematically explainable information. The entropy of 10^{122} is produced by a deterministic algorithm. This explains why mathematic is so effective. The world building program is deterministic and follows mathematical and repeatable patterns for all of its facts.

[46] Eugene P. Wigner. The unreasonable effectiveness of mathematics in the natural sciences. richard courant lecture in mathematical sciences delivered at new york university, may 11, 1959. Communications on Pure and Applied Mathematics, 13(1): 1–14, 1960. ISSN 1097-0312. DOI: 10.1002/cpa.3160130102. URL http://dx.doi.org/10.1002/cpa.3160130102

11.4 Boltzmann brains

The prediction that the universe is describable by only 400 bits lends weight to the Boltzmann brain hypothesis. The hypothesis states that a brain is most likely to be in the simplest possible universe that is capable of producing a brain. At 400 bits of complexity, this might be as simple as it gets.

The argument is actually stronger because the complexity of the universe is grown bit by bit. At the beginning of the universe, 1 bit of Ω was sufficient to produce all of its facts. As time advances, the number of bits required to describe it must also grow. At some point it would have grown to 100 bits, then to 200 bits, and so. Eventually it would become sufficiently large and complex to encode sentient life. The development of sentient life would occur more or less when n is sufficiently large to allow it. Hence, the first sentient life in the universe are invariably Boltzmann brains.

11.5 *Why do all observers agree on the laws of physics?*

All subjective observers who can produce complex thought such as the cogito will be able to deduce the same laws of physics independently of any observations. The occurrence of complex thought defined as being able to in principle verify the proof of any theorem of any assumption, guarantees the laws of physics as we know them.

11.6 *Undecidable future*

The world building program of the universe unpacks the Ω seed starting from leftmost bit and moving to the right. This allows it to extract facts and as they are calculated store them in the large entropy of the universe. Facts stored as such are immediately accessible as part of the entropy of the universe at little algorithmic time cost. The future as it contains more fact, will use more bits of Ω to be produced from. Hence the value of n, currently $\approx$ 400 will grow with time and was smaller in the past. The future as it is connected to a larger n than the present cannot mathematically be decided from the present as the bits of Ω are non-computable.

12 *Conclusion*

We note an affinity between a thermal universal Turing machine and the laws of physics. The affinity occurs when we consider a prefix-free UTM calculating its Ω number in a manner so as to maximize the entropy throughout the calculation. When the entropy is maximized, the halting probability becomes a Gibbs ensemble.

Understanding physics from the perspective of an thermal UTM holds several conceptual advantages. First, the system is at thermo-equilibrium hence it doesn't impose a 'special' case or a 'fine-tuning'. Second, it is a universal Turing machine hence it defines a universal system capable of arbitrary computation which can match the universe's complexity. More specifically, the representation can define a non-computable future with a computable singular past whose halting entropy is 0. This provides us with an arrow of time closely matching human experience. The entropy of the complete system (which includes future possibilities as well as an encoding scheme for the past) does stay constant over time as the change of entropy of one is offset by the other. The second law of thermodynamics, understood as an increase in entropy over time, is perceived in the exfoliation variables while the larger system, made to include future possibilities, has a constant entropy over time. In this system future possibilities are consumed to produce encoding possibilities. The second law of thermodynamics is therefore corrected to a law of conservation of entropy for the larger system comprised both thermal time and thermal space.

The decomposition of the program encoding scheme used by the thermal UTM via a Taylor expansion produces terms which can be linked to a various saturation scales. For the first Taylor expansion term, we recover special relativity (speed of light (7.1), light-cones (figure 1) and the Lorentz's factor (figure 2)) and the law of inertia (8.34). For the second term, we recover general relativity (8.12) and the holographic principle (8.2). Finally, the third term is related to an entropic origin of dark energy (8.20). Quantum mechanics is recovered as a result of the random walk produced on dx and dt and associated with fluctuating thermodynamic variables. The Lagrange multipliers of the partition function are the Planck units.

The derivation of the representation can be achieved from pure reason. It does not require an appeal to experimental evidence. It contains a metaphysical proof that the solution is unique hence it provides an explanation for why the universe is the way it is, and not an alternative. Finally, in the last part of the paper, we have calculated to compressibility of the universe under the holographic assumption to be approximately 400 bits. Those bits are the theory of everything for the current size and age of the universe and can be loosely interpreted as a random seed. This can serve as a remarkable prediction which opens the theory to the possibility of falsification. As the 400 bits are the leading bits of Ω the formulation, as it is non-compressible, is necessarily the simplest theory.

As a reference, I presented many of these ideas in an earlier publication [47].

[47] Alexandre Harvey-Tremblay. A derivation of the laws of physics from pure information. http://vixra.org/abs/1705.0274, 2017

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