Shapiro-like step patterns due to femtometer-scale charge motions: evidence from experimental data for the baryons.

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Keywords: Aharonov-Bohm effect, quantized flux, Shapiro steps.

Abstract

Aharonov and Bohm predicted (and Chambers measured) interference patterns related to phase differences in the wavefunctions of two coherent electron beams traveling around a concentrated magnetic field source. The phase difference is proportional to the magnetic flux linked between the beams, and should be an integer number $n$ of flux quanta $\frac{hc}{e}$ in the case the wave functions are single-valued around a closed path of integration. This latter condition would occur in the case of a closed ring of moving charge instead of two independent beams, a situation that should be inaccessible experimentally in ordinary conditions. If such experiment could actually be undertaken magnetic flux should form quantized, Shapiro-like steps in a plot of confined flux against some variable. The objective of this paper is to display evidence for such flux quantization from the examination of rest masses and magnetic moments data for baryon octet particles, which would play the role of rings of current. Our main result is a Shapiro-like step plot of flux against the magnetic moments of baryons.
1. Introduction.

Aharonov and Bohm (A-B) [1] devised a set of ingenuous experiments to directly investigate the role of gauge invariance and the associated influence of field potentials in quantum mechanics. Although such issues had already been raised in the early days of quantum physics by Schrodinger [2] and London [3], Aharonov and Bohm’s investigations took place at a time in which an actual experiment could be set up to confirm their predictions. Following the directions in [1], Chambers [4] was able to actually measure the interference fringes of two split coherent beams of electrons separated by a concentrated magnetic fiber. The phase difference was proportional to the magnetic flux linked by the pair of beams and the fringe pattern could be shifted by changing the amount of flux, in agreement with A-B. A particular detail (pointed out in a footnote in the A-B paper) could not be checked by such experiment, namely that if the wave functions remained single valued throughout, the linked flux would be quantized in units $n$ of $\phi_0 = \hbar c/e$. An experiment comprising a single particle’s wave-function might produce such result.

The objective of the present paper is to point out that a series of data for a long time available in the literature might be interpreted in terms of the foregoing proposal in the A-B paper.

2. Theory.

Isolated current-loops containing a single quantum of flux of value $\phi_0/2$ are well known from type-II superconductivity [5]. The formation of superconductor current loops is a many-body effect, though. We are going to investigate if there might exist single-particle systems confining flux in a similar manner, since this would in principle result in the limit case
considered by A-B. It is essential that such proposal be quantitatively supported by experimental data. Let’s consider the actual case of particles of the baryon octet. In spite of their rather short mean-lives all the eight particles have well-established rest masses and magnetic moments. E.J. Post [6] considered how to associate these latter two variables in a tentative model for the electron. Post showed that the magnetic moment for the electron could be obtained up to the first-order correction (from QED) with the equation:

\[ mc^2 = \phi i / c + eV \]  

(1)

Here the left side is the rest energy of the electron, which from the right side is considered as fully describable by electromagnetic terms. The first one on the right side is the magnetic energy of an equivalent (hypothetic) current ring of value \( i \) linking an amount of flux \( \phi \), that should occur in a number \( n \) of flux quanta \( \phi_0 \). The second (electrostatic energy) term is much smaller than the first (will be neglected hereafter) and accounts for the radiation-reaction correction for the magnetic moment which is proportional to the fine structure constant \( \alpha \), as is well known [6]. Post associates the current with the magnetic moment \( \mu \) and the size \( R \) of the ring with the equation:

\[ \mu = \pi R^2 i / c \]  

(2)

It must be pointed out that albeit useful, the parameters in these equations cannot be strictly interpreted as their macroscopic counterparts do. For instance, Barut developed a full theory for the leptons [7-9] in which he shows that radiation-reaction terms from QED can be translated in the simple picture of a conventional interaction between the self magnetic field of a lepton and an anomalous magnetic moment, characterized by a \( g \)-factor which is introduced to make the bridge between the two pictures.
In the present case we are interested in assessing a sufficiently large group of particles in order that flux quantization can be properly demonstrated, as predicted by A-B, and the baryons form such a group. The parameter $R$ must be determined for substitution into (2). In the case of nucleons, experimental determinations of the radius of a proton have been undertaken since the 1950s, and the most recent value is of about 0.85 fm\(^{10}\). Miller [11] has carried out detailed theoretical calculations of the charge distribution around nucleons. His plot of charge distribution [11] points towards an averaged radius of 0.6 fm, which will be adopted as discussed below.

The model by Post was devised to fit a single fundamental particle, the electron. There is however consistent evidence that the constituents inside baryons behave like a highly-correlated “quasiparticle”, resembling a Cooper pair in superconductors, and thus the individual character of quarks and gluons will be entirely neglected. Taking the model of Post in the case of baryons implies accepting such kind of analogy with a fundamental single particle. One then inserts equation (2) into equation (1) (without the electrostatic small term) and thus eliminates the current. The parameter $R$ has been calculated/measured for the nucleons only, but it remains part of the final expression for all baryons obtained after the combination of (1) and (2). We may conveniently eliminate $R$ from this treatment by adopting for all baryons an expression which is valid for the leptons (for $R = \lambda$, the Compton wavelength), namely:

$$\mu = eR/2$$

which is certainly valid for the proton for $R = 0.6$ fm from [11](cf. Table 1). The combination of equations (1)-(3) with $\phi = n(hc/e)$ can therefore be cast in the final form (inserting $\alpha = e^2/hc$):

$$\mu = eR/2$$
\[ n = \left( 2e^2 \alpha/e^3 \right) \mu m. \] (4)

3. Analysis.

Equation (4) is the main result of this work. It has been derived from the assumption that a femtometer-scale particle (baryon) can be represented by a loop of current, based upon an analogy from superconductivity and from Post’s model for the electron. The continuity of the wavefunction around the loop requires \( n \) to be an integer, which would constitute the limiting case of the A-B effect (as pointed out by Post[6]). All the parameters on the right side are known for the eight baryons of the octet, and are listed in Table 1. Figure 1 shows the plot of the calculated \( n \) against the magnetic moment for each particle. There is indeed a tendency to form Shapiro-like steps at integer numbers of flux quanta in agreement with the A-B conditions (Shapiro steps are obtained in plots of voltage against current across irradiated Josephson junctions between superconductors, as a consequence of gauge invariance and thus flux quantization[5]). As stressed earlier, it was shown by Barut that radiation terms from QED can be treated in the rather simple classical picture of the interaction between the anomalous magnetic moment with the particles self-field. A classical electromagnetic calculation carried out in ref. [12] considers the cyclotron extra rotations produced by the effect of the magnetic field due to an electron’s spin magnetic moment. The calculation is used to predict that one quantum of flux across the area covered by the particle is associated with each (Bohr- or) nuclear-magnetron of magnetic moment. As shown in Table 1, some values of \( n \) calculated to simultaneously fit mass and magnetic moment follow such classical result quite closely. However, the plot in Figure 1 displays clear deviations from the classical result towards the horizontal steps characteristic of compliance with the flux quantization condition, the limit case considered by the Aharonov-Bohm effect. There is
a competition between these effects, so that the steps are not sharply defined. Such deviations are probably related also to the fact that current and confined magnetic field superpose each other and this imposes an uncertainty in the amount of enclosed flux (something carefully avoided by Chambers[4]).


In conclusion, we have analyzed well-known tabulated data for the masses and magnetic moments of the baryon octet particles. The relatively simple model adopted here has no fitting parameters. The model associates magnetic energy with the rest energy, and gauge invariance implies flux quantization within the area covered by the particles charge motion. Figure 1 displays such a tendency for flux to adopt quantized values, similar to Shapiro steps[5], with deviations associated with self-magnetic field interactions. We argue that this serves as a femtometer-scale realization of the Aharonov-Bohm effect for a single-valued wave function describing each baryon’s closed paths.
References

Table 1: Data for the baryon octet (moments $\mu$ from ref. [13]). According to equation (4) in gaussian units, $n = 1.16 \times 10^{47} \mu m$. The plot of $n$ against $\mu$ (n.m.) is shown in Figure 1.

<table>
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<tr>
<th></th>
<th>abs $\mu$ (n.m.)</th>
<th>$\mu$ (erg/G) $\times 10^{23}$</th>
<th>$m$(Mev/c$^2$) $\times 10^{24}$</th>
<th>$m$(g) $\times 10^{24}$</th>
<th>$n$ from eq.(4)</th>
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Figure 1: Plot of $n$ against the magnetic moment following eq (4) and Table 1. The diagonal line is the classical prediction of one flux quantum per nuclear magneton (n.m.)[12]. Horizontal steps at integer values of $n$ are shown. The data display a tendency to follow the steps, like in the Shapiro experiments with Josephson junctions between superconductors, which are associated with flux quantization.