The Recursive Future And Past Equation Based On The Ananda-Damayanthi Normalized Similarity Measure Considered To Exhaustion

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Abstract

In this research investigation, the author has presented a Recursive Past Equation and a Recursive Future Equation based on the Ananda-Damayanthi Normalized Similarity Measure considered to Exhaustion [1].

The Recursive Future Equation

Given a Time Series $Y = \{y_1, y_2, y_3, \ldots\ldots, y_{n-1}, y_n\}$

we can find y_{n+1} using the following Recursive Future Equation

$$y_{n+1} = \underset{p\to\infty}{Limit} \frac{\left\{\sum_{k=1}^{n} y_k \left\{\left\{\frac{S_k}{L_k}\right\} + \left\{\frac{S_{k+1}}{L_{k+1}}\right\} + \left\{\frac{S_{k+2}}{L_{k+2}}\right\} + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\} + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}\right\}\right\}}{\sqrt{\sum_{k=1}^{n} \left\{\left\{\frac{S_k}{L_k}\right\}^2 + \left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2 + \left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2 + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2 + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}}$$

where

$S_k = Smaller\ of\ (y_{n+1}, y_k)$ and $L_k = Larger\ of\ (y_{n+1}, y_k)$

$S_{k+1} = Smaller\ of\ ((L_k - S_k), y_k)$ and $L_{k+1} = Larger\ of\ ((L_k - S_k), y_k)$

$S_{k+2} = Smaller\ of\ ((L_{k+1} - S_{k+1}), y_k)$ and $L_{k+2} = Larger\ of\ ((L_{k+1} - S_{k+1}), y_k)$

$S_{k+p-1} = Smaller\ of\ ((L_{k+p-2} - S_{k+p-2}), y_k)$ and $L_{k+p-1} = Larger\ of\ ((L_{k+p-2} - S_{k+p-2}), y_k)$

$S_{k+p} = Smaller\ of\ ((L_{k+p-1} - S_{k+p-1}), y_k)$ and $L_{k+p} = Larger\ of\ ((L_{k+p-1} - S_{k+p-1}), y_k)$

where p is a Number which makes the aforementioned Difference Residual $\left(L_{k+p-1}-S_{k+p-1}\right)$ tend to Zero.

From the above Recursive Equation, we can solve for y_{n+1}.

Proof:

We consider y_1 and find the Ananda-Damayanthi Similarity [1] between y_1 and y_{n+1} which turns out to be $\left\{\frac{S_1}{L_1}\right\}$. We now consider the lack of similarity part, i.e., $\left(L_1-S_1\right)$ and again find the Similarity between y_1 and $\left(L_1-S_1\right)$ which turns out to be $\left\{\frac{S_{1+1}}{L_{1+1}}\right\}=\left\{\frac{S_2}{L_2}\right\}$. And similarly, we find $\left\{\frac{S_{1+2}}{L_{1+2}}\right\}=\left\{\frac{S_3}{L_3}\right\}$, $\left\{\frac{S_{1+3}}{L_{1+3}}\right\}=\left\{\frac{S_4}{L_4}\right\}$, ..., $\left\{\frac{S_{1+p-1}}{L_{1+p-1}}\right\}=\left\{\frac{S_p}{L_p}\right\}$, $\left\{\frac{S_{1+p}}{L_{1+p}}\right\}$. We now add them all. Similarly, we consider y_2, y_3,....., upto y_{n-1} and y_n and compute such aforementioned quantities and add them all. We now Normalize, i.e., divide each of this value by the quantity $\sqrt{\sum_{k=1}^{n}\left\{\left\{\frac{S_k}{L_k}\right\}^2+\left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2+\left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2+\ldots\ldots\ldots+\left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2+\left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}$. We equate this value to y_{n+1} as the RHS is the Total Normalized Similarity contribution from each element of the Time Series Set $Y=\left\{y_1,y_2,y_3,\ldots\ldots\ldots,y_{n-1},y_n\right\}$ with respect to y_{n+1}.

General Form

We can note that the above equation

$$y_{n+1} = \underset{p\to\infty}{Limit} \frac{\left\{\sum_{k=1}^{n} y_k \left\{\left\{\frac{S_k}{L_k}\right\} + \left\{\frac{S_{k+1}}{L_{k+1}}\right\} + \left\{\frac{S_{k+2}}{L_{k+2}}\right\} + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\} + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}\right\}\right\}}{\sqrt{\sum_{k=1}^{n} \left\{\left\{\frac{S_k}{L_k}\right\}^2 + \left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2 + \left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2 + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2 + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}}$$

is in general of the form

$$y_{n+1} = \underset{p\to\infty}{Limit} \frac{\left\{a_1 y_{n+1} + \frac{a_2}{y_{n+1}}\right\}}{\sqrt{\left\{(a_3 y_{n+1})^2 + \left(\frac{a_4}{y_{n+1}}\right)^2\right\}}}$$

where, a_1, a_2, a_3 and a_4 are some positive integers.

We can further write the above equation as

$$(y_{n+1})^2\left\{(a_3y_{n+1})^2+\left(\frac{a_4}{y_{n+1}}\right)^2\right\}=\left\{a_1y_{n+1}+\frac{a_2}{y_{n+1}}\right\}^2$$

$$(a_3)^2(y_{n+1})^6-(a_1)^2(y_{n+1})^4+\{(a_4)^2-(2a_1a_2)\}(y_{n+1})^2-(a_2)^2=0$$ Equation A

Defining Error

We define Error in the following fashion:

For the Recursive Future Equation:

Method 1

Given a Time Series $Y=\{y_1,y_2,y_3,.......,y_{n-1},y_n\}$ we consider only $Y=\{y_1,y_2,y_3,.......,y_{n-1}\}$ and use the aforementioned Recursive Future Equation to find the n^{th} term. Say this is ${}^p y_n$ where the p stands for the 'predicted' or 'forecasted' value. Then, the Error is defined by

$$\varepsilon_F=\left(\frac{y_n-{}^p y_n}{y_n}\right)$$

Method 2

Given a Time Series $Y=\{y_1,y_2,y_3,.......,y_{n-1},y_n\}$ we consider it and use the aforementioned Recursive Future Equation to find the $(n+1)^{th}$ term. Say this is ${}^p y_{n+1}$ where the p stands for the 'predicted' or 'forecasted' value. We now consider the Time Series Set $Y=\{y_2,y_3,.......,y_{n-1},y_n,y_{n+1}\}$ and use the aforementioned Recursive Past Equation to generate the term previous to y_2, i.e., ${}^{p}y_1$. Then, the Error is defined by

$$\varepsilon_F = \left(\frac{y_1 - {}^{p}y_1}{y_1} \right)$$

Therefore, simple Error can be given by

$$\varepsilon_F = \left(y_1 - {}^{p}y_1\right) = \left\{ y_1 - Desired\ \ Root\ Of \left\{ \begin{matrix} (c_3)^2 (y_{n+1})^6 - (c_1)^2 (y_{n+1})^4 + \\ \{(c_4)^2 - (2c_1c_2)\}(y_{n+1})^2 - (c_2)^2 = 0 \end{matrix} \right\} \right\}$$

where the Equation $(c_3)^2 (y_{n+1})^6 - (c_1)^2 (y_{n+1})^4 + \{(c_4)^2 - (2c_1c_2)\}(y_{n+1})^2 - (b_2)^2 = 0$ is analogously developed as equation B using the Time Series Set $Y = \{y_2, y_3, \ldots\ldots., y_{n-1}, y_n, y_{n+1}\}$ to find y_1, where where, c_1, c_2, c_3 and c_4 are some positive integers.

The Functional Form Equation For Making Future Forecast

We consider the equation shown below

$$\varepsilon_F = \left(y_1 - {}^{p}y_1\right) = \left\{ y_1 - Desired\ \ Root\ Of \left\{ \begin{matrix} (c_3)^2 (y_{n+1})^6 - (c_1)^2 (y_{n+1})^4 + \\ \{(c_4)^2 - (2c_1c_2)\}(y_{n+1})^2 - (c_2)^2 = 0 \end{matrix} \right\} \right\}$$

and minimize the Error w.r.t y_{n+1}, i.e.,

$\frac{d\varepsilon_F}{dy_{n+1}} = 0$ with $\frac{d^2\varepsilon_F}{dy_{n+1}^2} > 0$ at the value of $y_{n+1}\big|_{\varepsilon_F \min}$ where is ε_F minimum. The Equation at which this error is Minimum i.e., $\frac{d\varepsilon_F}{dy_{n+1}} = 0\Big|_{\varepsilon_F\ Min}$ can be used to re-calculate the a_1, a_2, a_3 and a_4 and say these are a_{1new}, a_{2new}, a_{3new} and a_{4new}, The Functional Form Equation For Making Future Forecast becomes

$$(a_{3new})^2(y_{n+1})^6 - (a_{1new})^2(y_{n+1})^4 + \{(a_{4new})^2 - (2a_{1new}a_{2new})\}(y_{n+1})^2 - (a_{2new})^2 = 0$$

The Recursive Past Equation

Given a Time Series $Y = \{y_1, y_2, y_3, \ldots\ldots, y_{n-1}, y_n\}$

we can find y_0 using the following Recursive Past Equation

$$y_n = \underset{p\to\infty}{Limit} \frac{\left\{\sum_{k=0}^{n-1} y_k \left\{\left\{\frac{S_k}{L_k}\right\} + \left\{\frac{S_{k+1}}{L_{k+1}}\right\} + \left\{\frac{S_{k+2}}{L_{k+2}}\right\} + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\} + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}\right\}\right\}}{\sqrt{\sum_{k=0}^{n-1}\left\{\left\{\frac{S_k}{L_k}\right\}^2 + \left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2 + \left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2 + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2 + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}}$$

where

$S_k = Smaller\ of\ (y_n, y_k)$ and $L_k = Larger\ of\ (y_n, y_k)$

$S_{k+1} = Smaller\ of\ ((L_k - S_k), y_k)$ and $L_{k+1} = Larger\ of\ ((L_k - S_k), y_k)$

$$S_{k+2} = Smaller\ of\ \left(\left(L_{k+1} - S_{k+1}\right), y_k\right) \text{ and } L_{k+2} = Larger\ of\ \left(\left(L_{k+1} - S_{k+1}\right), y_k\right)$$

$$S_{k+p-1} = Smaller\ of\ \left(\left(L_{k+p-2} - S_{k+p-2}\right), y_k\right) \text{ and } L_{k+p-1} = Larger\ of\ \left(\left(L_{k+p-2} - S_{k+p-2}\right), y_k\right)$$

$$S_{k+p} = Smaller\ of\ \left(\left(L_{k+p-1} - S_{k+p-1}\right), y_k\right) \text{ and } L_{k+p} = Larger\ of\ \left(\left(L_{k+p-1} - S_{k+p-1}\right), y_k\right)$$

where p is a Number which makes the aforementioned Difference Residual $\left(L_{k+p-1} - S_{k+p-1}\right)$ tend to Zero.

From the above Recursive Equation, we can solve for y_0.

Proof:

We consider y_0 and slate the Ananda-Damayanthi Similarity [1] between y_0 and y_n which turns out to be $\left\{\frac{S_0}{L_0}\right\}$. We now consider the lack of similarity part, i.e., $\left(L_0 - S_0\right)$ and again find the Similarity between y_0 and $\left(L_0 - S_0\right)$ which turns out to be $\left\{\frac{S_{0+1}}{L_{0+1}}\right\} = \left\{\frac{S_1}{L_1}\right\}$. And similarly, we find $\left\{\frac{S_{0+2}}{L_{0+2}}\right\} = \left\{\frac{S_2}{L_2}\right\}$, $\left\{\frac{S_{0+3}}{L_{0+3}}\right\} = \left\{\frac{S_3}{L_3}\right\}$, ………., $\left\{\frac{S_{0+p-1}}{L_{0+p-1}}\right\} = \left\{\frac{S_{p-1}}{L_{p-1}}\right\}$, $\left\{\frac{S_{0+p}}{L_{0+p}}\right\} = \left\{\frac{S_p}{L_p}\right\}$. We now add them all. Similarly, we consider y_2, y_3,……, upto y_{n-1} and compute such aforementioned quantities and add them all. We now Normalize, i.e., divide each of this value by the quantity

$$\sqrt{\sum_{k=0}^{n-1}\left\{\left\{\frac{S_k}{L_k}\right\}^2 + \left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2 + \left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2 + \ldots\ldots\ldots + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2 + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}$$

. We equate this value to y_n

as the RHS is the Total Normalized Similarity contribution from each element of the Time Series Set $Y = \{y_0, y_1, y_2, y_3, ..., y_{n-1}\}$ with respect to y_n.

General Form

We can note that the above equation

$$y_n = \underset{p\to\infty}{Limit} \frac{\left\{\sum_{k=0}^{n-1} y_k \left\{\left\{\frac{S_k}{L_k}\right\} + \left\{\frac{S_{k+1}}{L_{k+1}}\right\} + \left\{\frac{S_{k+2}}{L_{k+2}}\right\} + ... + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\} + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}\right\}\right\}}{\sqrt{\sum_{k=0}^{n-1}\left\{\left\{\frac{S_k}{L_k}\right\}^2 + \left\{\frac{S_{k+1}}{L_{k+1}}\right\}^2 + \left\{\frac{S_{k+2}}{L_{k+2}}\right\}^2 + ... + \left\{\frac{S_{k+p-1}}{L_{k+p-1}}\right\}^2 + \left\{\frac{S_{k+p}}{L_{k+p}}\right\}^2\right\}}}$$

is in general of the form

$$y_n = \frac{\left\{b_1 y_n + \frac{b_2}{y_n}\right\}}{\sqrt{\left\{(b_3 y_n)^2 + \left(\frac{b_4}{y_n}\right)^2\right\}}}$$

where, b_1, b_2, b_3 and b_4 are some positive integers.

We can further write the above equation as

$$(y_n)^2\left\{(b_3 y_n)^2+\left(\frac{b_4}{y_n}\right)^2\right\}=\left\{b_1 y_n+\frac{b_2}{y_n}\right\}^2$$

$$(b_3)^2(y_n)^6-(b_1)^2(y_n)^4+\{(b_4)^2-(2b_1b_2)\}(y_n)^2-(b_2)^2=0$$ Equation B

where, b_1, b_2, b_3 and b_4 are some positive integers.

Defining Error

We define Error in the following fashion:

For the Recursive Past Equation:

Method 1

Given a Time Series $Y=\{y_1, y_2, y_3, \ldots\ldots., y_{n-1}, y_n\}$ we consider only $Y=\{y_2, y_3, \ldots\ldots., y_{n-1}, y_n\}$ and use the aforementioned Recursive Future Past to find the 1^{st} term. Say this is ${}^p y_1$ where the p stands for the 'predicted' or 'forecasted' value. Then, the Error is defined by

$$\varepsilon_P=\left(\frac{y_1 - {}^p y_1}{y_1}\right)$$

Method 2

Given a Time Series $Y=\{y_1, y_2, y_3, \ldots\ldots., y_{n-1}, y_n\}$ we consider it and use the aforementioned Recursive Future Equation to find the term previous to y_1. Say this is ${}^p y_0$ where the p stands for the 'predicted' or 'forecasted' value. We now consider the Time Series Set $Y = \{y_0, y_1, y_2, y_3, \ldots\ldots, y_{n-1}\}$ and use the aforementioned Recursive Future Equation to generate the term next to y_{n-1}, i.e., ${}^p y_n$. Then, the Error is defined by

$$\varepsilon_F = \left(\frac{y_n - {}^p y_n}{y_n} \right)$$

Functional Form Equation For Making Past Forecast

A Seasoned reader of author Literature, especially the section on '*Functional Form Equation For Making Future Forecast'* can infer the procedure for the Past Forecast which is very much similar to the Future Forecast.

Computation Complexity

For the World's fastest Chineese Super-Computer which can compute 33,860 Trillion Computations per second

we can use the equation

$2^{(m+n)} = 33860 \times 10^{12}$ to calculate the Maximum Number of Terms of the Time Series n for which we wish to predict the $(n+1)^{th}$ term and m is the Number Of Difference Residual Terms we wish to consider for each term, to find the n for a given m so that the $(n+1)^{th}$ term is computed in one second.

Furthermore, if we take $m = 8\, or\, 10$ (beyond which the value of the Difference Residuals is near vanishing) and for different amounts of times we can spare for getting the computed answer, the Number of Terms of the Time Series n that we can consider is given below:

Serial Number	*Duration Of Computation*	*Number of Terms n To Consider*	
		$m=8$	$m=10$
1	1 Second	46.91043	44.91043
2	1 Hour	58.72421	56.72421
3	1 Day	63.30917	61.30918
4	1 Week	66.11653	64.11653
5	1 Month (31 Days)	68.26337	66.26337
6	1 Year	71.82093	69.82093

That is, if the Time Series Set were to contain n number of terms (as shown in the table for varying values of m, namely 8 and 10, then the Duration of Computation is tabulated above.

For Forecasting Future Element

We have $2^{(m+n)}$ number of 6th Order Polynomial Equations of the kind as shown in equation A to solve as these account for all the cases of the Time Series Set Elements being greater or lesser than the future $(n+1)^{th}$ element to be computed, as these equations are being represented by the aforementioned Recursive Future Equation. Only one among them is the correct equation and this can be found by using this thusly computed $(n+1)^{th}$ value and omitting the first element y_1, using the Time Series Set $Y=\{y_2,y_3,.......,y_{n-1},y_n,y_{n+1}\}$ we predict the element y_1 using the aforementioned Recursive Past Equation. And one of the $2^{(m+n)}$ number of 6th Order Polynomial Equations of the kind as shown in equation A which gives the best true value of y_1 can be considered as the correct equation and its future element forecast of y_{n+1} as the correct forecast.

For Forecasting Future Element

We have $2^{(m+n)}$ number of 6th Order Polynomial Equations of the kind as shown in equation A to solve as these account for all the cases of the Time Series Set Elements being greater or lesser than the past element y_0 to be computed, as these equations are being represented by the aforementioned Recursive Past Equation. Only one among them is the correct equation and this can be found by using this thusly computed y_0 value and omitting the latest element y_n, using the Time Series Set $Y=\{y_0,y_1,\ y_2,y_3,.......,y_{n-1}\}$ we predict the element y_n using the aforementioned Recursive Future Equation. And one of the $2^{(m+n)}$ number of 6th Order Polynomial Equations of the kind as shown in equation A which gives the best true value of y_n can be considered as the correct equation and its past element forecast of y_0 as the correct forecast.

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