How scale factors determine starting time step from minimum non zero value, using superfluid universe model, with an initial Hubble parameter H = 0 , with later Hubble H ~ T(temperature), after formation of causal structure to obtain graviton production, and investigation of the Penrose Weyl tensor conjecture, and its possible breakdown

Andrew Walcott Beckwith

Physics Department, Chongqing University, College of Physics, Chongqing University Huxi Campus, No. 44 Daxuechen Nanlu, Shapinba District, Chongqing 401331, People's Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract

We start where we use an inflaton value due to use of a scale factor $a \sim a_{\min} t^{\gamma}$ .Also we use $\delta g_{tt} \sim a_{\min}^2 \cdot \phi_{initial}$ as the variation of the time component of the metric tensor g_{tt} in Pre-Planckian Space-time. In doing so, what we lead up to using the Huang Superfluid universe model, which is by the modified superfluind cosmology model leading to examining $a^2 = Curvature / energy - density$ with in the Pre Planckian regime, Curvature, small but non zero, and energy density $\frac{\dot{\phi}^2}{2} + V(\phi)$. The Potential energy is given by what it would be if $a \sim a_{\min} t^{\gamma}$ leading to a relationship of $a_{initial} \propto initial - time$, where we will isolate conditions for the initial time and compare them against a root finder procedure given in another paper written by the author. Then, afterwards, assuming a modified Hubble parameter, with an initial Hubble parameter after the Causal surface with , right after a quantum bounce, determined by $H_{causal-structure-quantum-bounce} = 0$, is then $H_{initial} \sim 1/\Delta t \sim 1.66\sqrt{g_*} \cdot T^2 / m_{Planck}$. and g_* is an initial degrees of freedom value of about 110. Then, the graviton production rate is a function of time leading to a temperature T dependence, with M here is a chosen Mass scale, M of about 30 TeV, with d greater than or equal to zero, representing the Kaluza Klein dimensions assumed with the number of gravitons produced after the onset of Causal structure given by $n(T) \sim T^2 \cdot m_{Planck} \cdot (T/M)^{d+2}$. This $n(T)$ by Infinite quantum statistics is proportional to entropy. We close with a caveat as far as the implications of all this to the Penrose Conjecture about the vanishing of the Weyl tensor, in the neighborhood of a cosmological initial singularity. And what we think should be put in place instead of the Penrose Weyl tensor hypothesis near a 'cosmological' singularity. And we close with a comment about the Weyl curvature tensor, in Pre Planckian to Planckian physics, and also a final appendix on the Mach's principle as written by Sciama, $G\rho\tau^2 \sim 1$ in defining the initial space-time non singular 'bubble.

Key words Inflaton physics, Causal structure Entropy, temperature dependent initial Graviton production, Kaluza Klein dimensions, Penrose Weyl tensor conjecture

1. Referral to the Huang Superfluid Universe model

We look at [1] by Huang, as to a critical density affecting scale factor 'size of the universe' as given by

$$\begin{aligned}
&H^2 = \frac{-k(curvature)}{a^2} + \frac{2}{3}\cdot\rho_c \\
&\& \\
&\rho_c = \frac{\dot{\phi}^2}{2} + V\left(\phi\right) \\
&\& \\
&H^2(Quantum - bounce) = 0 \\
&\Leftrightarrow a^2 = \frac{3k(curvature)}{2\rho_c} \\
&\Leftrightarrow a_{bounce} = \sqrt{\frac{3k(curvature)}{\dot{\phi}^2 + 2V\left(\phi\right)}}
\end{aligned} \qquad (1)$$

This curvature, in the vicinity of Pre-Planckian space-time is of minimal value. Whereas Huang delineates the evolution of the scale factor as [1]

$$\ddot{\phi}_n = -3H\dot{\phi}_n - \frac{\partial V\left(\phi_n\right)}{\partial \phi_n} \qquad (2)$$

The scalar field which Huang accesses is ϕ_n , with this being due to setting V as dependent upon the Kummel function, as written up in page 58 of [1] with, here, n going from 1 to N, in terms of scalar fields ,and

$$\begin{aligned}
&V = \tilde{\Lambda}^4 U_b\left(z\right) \\
&\& \\
&z = 8\pi^2\phi^2 / \tilde{\Lambda}^2 \\
&\& \\
&\phi^2 = \sum_{n=1}^{N}\phi_n^2 \\
&\& \\
&\tilde{\Lambda} = Momentum - cutoff \\
&U_b\left(z\right) = c_0\tilde{\Lambda}^b \cdot \left[M(-2 + b/2, N/2, z) - 1\right] \\
&M(p,q,z) = 1 + \frac{p}{q}z + \frac{p(p+1)}{q(q+1)}\frac{z^2}{2!} + ..
\end{aligned} \qquad (3)$$

As given by [1], this potential system is from one loop Feynman diagrams as given in [2] .Our approximation is to set N as equal to 1, in the Pre Planckian regime, with the Causal structure creation zone, at the 'bubble' of space-time leading to a bifurcation of additional structure and additional space-time scalar fields, as delineated by ϕ_n. However before this happens to delineate the initial scalar field , with N=1 as within the bubble of space-time . What we are doing is to review what was put in [3] and contrast it to a (single field?) version of Eq. (3) above. In doing so we are using the Padmanbhan treatment of the linkage between scale factor, inflaton, and what was done in [3] while assuming that the Eq. (4) and Eq.(5) is for the regime of the quantum bubble, possibly of radii Plank length, and then match it to EqI3) above. i.e. probably of Planck dimensions, as having[3] .

We will remark upon utilization of the following two scalar potentials and the potential system in the following manner. In Eq.(3) we explicitly refer to a multi scalar inflaton field, which we can all as ϕ_n with values from 1 to N. But in the pre Planckian regime, we are looking at a single inflaton field version of the dynamics, which is given in Eq. (5) below.

In this case, the dyna mics of our problem will be laid out as follows

$$\phi(Before-Planckian)\xrightarrow[Causal-boundary]{}\phi_{n=1}\xrightarrow[Past-Causal-bundary]{}\phi_{n=1,...N}\,(Planckian) \qquad (4)$$

The first stage of this evolution, is given by Eq. (4) below. The Second stage has the scalar field as given in $\phi_{n=1}$ as stated for Eq. (4) below, but then mapped as the first admitted scalar field as given in Eq.(3), and then the final stage, has scalar fields which can be ranging from 1 to N in labels, which would be a physical transformation of the problem from a single field regime, to a multi scalar field regime, with similarities to super fluid helium.

In appendix A, we argue that this is similar to a particle in a quantum state, in a box, when the box is then suddenly opened up. I.e. in that quantum experiment which is in Appendix A, we have a ground state probability of P(1)=.41 that a ground state wave function would be n=1 and stay there if the length of the box were changed from L/2 to L, and we argue that we have an analogous situation here, for the linkage given for Eq.; (3), Eq.(4) and Eq. (5) given here. Having said that let us look at the Pre Planckian inflaton field, which motivates the start of our analysis

In short a single inflaton field will dominate the interior of an inflaton bubble, and then be considered as bridged to a single field version of Eq. (3) above initially. I.e. the single field inflaton, will obey the relations which were cited as given in [3] which we reproduce below as

$$\begin{aligned}
&a \approx a_{\min} t^{\gamma} \\
&\Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right\} \\
&\Leftrightarrow V \approx V_0 \cdot \exp\left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}
\end{aligned} \qquad (5)$$

To employ this Eq. (5) we are using, as was done in ‘[3], the following boundary condition of the bubble of Space-time as was given in [3] which we put in as being the boundary of a purported quantum bounce. This is also substantially using [4] which using the material so cited.

In doing this, we also can state that there is a commensurate internal wave function, within this bubble. We will allude to this later. See the conclusion.

Note that this all has profound linkage to the Penrose suggestion that the Weyl tensor vanishes at an initially assumed singularities of space-time. As given in [5]/ i.e. the Penrose suggestion in [5] is that an “effective Weyl Curvature of a given frequency and amplitude allegedly adds an effective “gravitational energy” contribution to the Ricci Tensor of magnitude of the square of the amplitude of the Weyl tensor, times 1 over the frequency of the alleged Weyl tensor oscillatory frequency, squared.

Penrose suggestion leads to the suggestion that if the amplitude of the Weyl tensor is zero, then there would be no "gravitational energy"

We suggest here, that the Weyl tensor would NOT vanish, if our formulas Eq.(3) to Eq.(5) hold, and that instead there would be gravitational energy. In **Appendix A**, we examine some quantum mechanical arguments as to our problem, at the boundary of a nonsingular initial bubble of space-time, and in **Appendix B**, we will examine and amplify what we mean as to the consequences of Gravitational potential energy. I.e. in doing so, we cite a different interpretation of [5] as given, as a way confirming the existence of initial non zero entropy at the start of cosmological expansion.

Note that we have in our document access to looking at the interior of the presumed initial space-time bubble of Pre Causal space time. This will be in lieu of [6,7, and 8] which yields us Eq. (6) below

$$
\begin{aligned}
& g_{tt} \sim \delta g_{tt} \approx a_{\min}^{2} \phi_{initial} << 1 \\
& \xrightarrow[\mathrm{Pr}e-Planck\to Planck]{} \delta g_{tt} \approx a_{\min}^{2} \phi_{Planck} \sim 1 \\
& \Leftrightarrow \left(\frac{R_c\big|_{initial} \sim c\cdot\Delta t}{l_{Planck}} \right) \sim \vartheta(1)\Bigg|_{Planck}
\end{aligned}
\tag{6}
$$

The n=N version would have ONE component of the potential largely dominated by the Eq.(4) write up, and the rest of the structure would be additional add ons according to the Kummel potential write up as given in Eq. (1) given above,

From now on, we will be examining the physics implications of finding and using Δt

2. Examining Δt from the vantage point of a minimum scale factor calculation.

To do this, we have that interpretation of Eq. (1) will lead to the following linkage of scale factor of the Universe, minimum, and the time derivative of the inflaton field, as given in Eq. (5) for the Pre Planckian regime, about the Causal structure as given in Eq. (6) above, mainly, then

$$
a_{bounce} = \sqrt{\frac{3k(curvature)}{\dot{\phi}^{2} + 2V(\phi)}} \sim \sqrt{\frac{12\pi G\cdot k(curvature)}{\gamma}}\cdot t\cdot\sqrt{1+2V_{0}\cdot\gamma^{2}\cdot\frac{(3\gamma-1)}{32\pi}}
\tag{7}
$$

This is for a minimum time step, t, which in our re write is, then

$$
a_{bounce} \sim \Delta t\cdot\sqrt{\frac{12\pi G\cdot k(curvature)}{\gamma}}\cdot\sqrt{1+2V_{0}\cdot\gamma^{2}\cdot\frac{(3\gamma-1)}{32\pi}}
\tag{8}
$$

What we are doing is to contrast different ways of obtaining a time step Δt and then employing the tools used in [3] and [4]

Then making use of [9] while using the tools given in reference [8] with g_{*} is an initial degrees of freedom value of about 110 [10] , and T in Eq.(8) as a temperature, right after the formation of Causal structure, and with M here is a chosen Mass scale, M of about 30 TeV [11] we find that Eq. (9) below as given then will lead to via use of the ideas of [9] used again and again .

$$
H_{Early-Universe} \sim 1.66\cdot\sqrt{g^{*}}\cdot\frac{T_{Early-Universe}}{M_{mass-scale}}
\tag{9}
$$

Note that we will in due course, also amplify this results linkage to Appendix B, in our conclusion.

Implying for a value right at the causal boundary of space time, i.e. the bounce radii of emergent

$$\Delta t \sim 1/\left(1.66\cdot\sqrt{g^{*}}\cdot\frac{T_{Early-Universe}}{M_{mass-scale}}\right) \qquad (10)$$

This will, if we utilize [8] tie in with a graviton production expression we give as, if d is the extra dimensions of assumed Kaluza – Klein space-time

$$n(T) \sim T^{2}\cdot m_{Planck}\cdot\left(T/M\right)^{d+2} \qquad (11)$$

As stated before, this assumes, that Eq. (10) is by Ng. Infinite quantum statistics [12], an entropy count, with at the Causal boundary, a nonzero value, in line with [13]. And the non zero value of the scale factor is largely in tune with the ideas of quantum bounces as given in Loop quantum gravity [14] and also the non linear electrodynamic suggestions given by Camera, et. al. [15].

Having said that, we will then cite a result as given in [16] which involves a non linear equation for the Δt values used in Eq. (7) and Eq.(9) which in turn affects Eq. (10) which by infinite quantum statistics [12] implies that at a causal surface boundary, that we do not have non zero entropy.

3. Examination of the minimum time step, in Pre-Planckian Space-time as a Root of a Polynomial Equation

We initiate our work, citing [16] to the effect that we have a polynomial equation for the formation of a root finding procedure for Δt , namely if

$$\Delta t\cdot\left|\left(\sqrt{\frac{8\pi GV_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)-\frac{\left(\sqrt{\frac{8\pi GV_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)^{2}}{2}+\frac{\left(\sqrt{\frac{8\pi GV_{0}}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)^{3}}{3}-...\right| \qquad (12)$$

$$\approx\left(\sqrt{\frac{\gamma}{\pi G}}\right)^{-1}\frac{48\pi\hbar}{a_{\min}^{2}\cdot\Lambda}$$

From here, we then cited, in [15], using [12]a criteria as to formation of entropy, i.e. If Λ is an invariant cosmological ‘constant’ and if Eq. (12) holds, we can use the existence of nonzero initial entropy as the formation point of an arrow of time. given in Eq. (1) with a counting algorithm of created gravitons giving a nonzero entropy which can also be cited as similar to the Entropy given below Note that this is the boundary between the single inflaton treatment given in Eq. (5) and the more general equation

$$S_{\Lambda}\Big|_{Arrow-of-time}=\pi\cdot\left(\frac{R_{c}\big|_{initial}\sim c\cdot\Delta t}{l_{Planck}}\right)^{2}\neq 0 \qquad (13)$$

This should be compared with Eq. (11) as a nonzero value for initial entropy at a causal surface/ boundary.

Note that the most likely result of a solution for Eq. (12) would be in the case that

$$\left(\sqrt{\frac{8\pi G V_0}{\gamma\cdot(3\gamma-1)}}\cdot\Delta t-1\right)\sim\varepsilon^{+}\sim tiny \\ \Leftrightarrow \Delta t\sim Planck-time \tag{14}$$

What Eq. (13) gives us then is an estimate as to a truncated value of time step which is tied into the arrow of time consideration as to the later part of this document. This is also linked to the causal barrier idea also alluded to in this document.

All this leads to a conclusion which is to the inter connectivity of initial conditions and nonzero entropy.

4. Conclusion. Inter connection between minimum scale factor, Δt, and Eq. (10). Much more to explore

That there may be a linkage between a minimum scale factor, a minimum time step and initial graviton production is nothing other than stunning. Also, this can be linked to possible falsification of a prior suggestion brought up in [16] which we cite below. Can we also, in all of this, examine if there is an invariant cosmological constant, or if it varies with an initial electromagnetic field, as is suggested next.

One way to look at it would be to suggest that as done by H. Kadlecova [17] in the 12 Marcel Grossman meeting that the typical energy stress tensor, using, instead, Gyratons, with an electro-magnetic energy density addition to effective Electromagnetic cosmological value as given by

$$\rho_{E\&M-contribution}\sim 8\pi G\cdot\left(E^2+B^2\right) \tag{15}$$

I.e. that there be, due to effective E and M fields a boost from an initially low vacuum energy to a higher ones, as given by Kadlecova [17.18]

$$\Lambda_{+}=\Lambda+\rho_{E\&M-contribution} \tag{16}$$

If true, this may affect Eq. (12) as given in the text. Ere also should keep in mind the issues brought up by Abbot et.al. and Corda, as far as foundational gravity as cited in [19,20, and 21] as well. I.e. parsing correctly would entail understanding the foundations of experimental gravity.

Finally, and not least, this construction of a single field inflaton field, as given up to the Causal structure boundary is, if it is done correctly, probably linked to one of the many post causal inflaton fields, as referenced in [1], and Eq. (1) of this presentation. The transition from one to possibly many inflaton fields, and a super fluid model of the universe be a way, as the author visualizes, of initiating turbulence at the start of the formation of a causal structure, with an analogy to superfluid induced turbulence as alluded to in [1]. A topic the author will explore later. And also if we can observe the following generated GW, as given with defined Frequency

$$frequency\big|_{initial}\sim 1/\left(R_c\big|_{initial}\sim c\cdot\Delta t\right)\xrightarrow[c\equiv 1]{}1/\Delta t \tag{17}$$

This frequency is in part due to the following argument as given by [21] as far as the article by Halliwell, as far as quantum cosmology,[22] as far as the intenior wave function for the wavefunction in the interior of the buibble of space time, closely matched to the causal surface of the one Plancklength radii of initial space-time. We then get an interior no boundary wavefunctional of the form

$$\begin{aligned}\psi_{No-boundary} &\sim \left[\exp(1/3V(\phi))\right]\cdot\cos\left((a^2V\left(\phi\right)-1)^{3/2}-\pi/4\right)\\ &\sim\left[\exp(8\pi t^2/3\gamma\left(3\gamma-1\right))\right]\cdot\cos\left((a^2(8\pi t^2/\gamma\left(3\gamma-1\right))^{-1}-1)^{3/2}-\pi/4\right)\end{aligned}\tag{18}$$

This is an interior no boundary condition for an interior wavefunctional as given by [22] and the important question to ask is how to match this WKB argument with the physics as represented by Eq. (17) above, as in sync with [22]

If so, is this idea in sync with Cyclic conformal cosmology? [23]?

One of the open questions this also leads to , is, if [23], in terms of the Cyclic conformal cosmology of Penrose is admissible, with this construction or is ruled out.

What we are also considering is, although not explicitly stated, a similar mechanism as is given in the Higgs formation of mass as is written up in pages 480 to page 483 of [24] , and also a way to a possible linkage to [25] in terms of gravitons, and Higgs theory. In particular,

quote:

Higgs mechanism at the graviton level as a consequence of the Vainshtein mechanism ,

end of quote,

from [25] may be developed in a future update of this document. Another alternative, to consider, in this temperature dependent regime, is also given by [26].

One final consideration. In [26] Oda has a rendering of the Cosmological Constant as given by the paragraph right after equation (42) of [26]

Quote

where the cosmological constant takes the form Λ = (2−(5 /4)times D) times m^2, which is negative for D > 1. We conjecture that in this class of potentials, the cosmological constant might be always negative since the ’t Hooft model belongs to this class.

End of quote

The radical suggestion the author has, that in the Pre Planckian regime, in the regime right next, or included within the bubble, that the effective spatial dimension, D, would be 1, i.e. a dramatic reduction of effective ‘dimensionality’ with the effect that in the Pre-Planckian space-time, that one has, due to this, an effective POSITIVE cosmological constant. I.e. that the Oda conjecture applied literally should be with respect to the nucleated bubble of Pre Planckian space-time.

The author welcomes disagreements with this conjecture, and also wishes constructive engagement as to this point from interested readers.

We also wish to point to a recent paper, by Canate, Jime, and Salgado [27] as to the question if Geometric hair, in black hole theory is supported . by analytical and geometric models. The authors refer to several modified gravity models which impact the expansion of the universe. Minding that the Corda suggestion [20] as to how early universe models as to Tensor-Scalar models influence what is known about early universe experimental gravity data sets which could be expected, the additional benefit of our analysis, may be in helping to delineate what modified gravity models are admissible as far as the early universe, which in turn will directly impact the characterization of if or not black holes, indeed have geometic hair. If we go in addition to this, a review of [28] , where the author did a thought experiment as to what a causal discontinuity did as to the available fluctuations, and [29] on an inquiry as to if extra dimensions are necessary at all, and [30] as to how certain black hole results may be replicated, as far as the question of entanglement entropy in the early universe, we find that the model so given above, may have some very unexpected inter relationships with black hole physics, but also with the early universe at the same time.

Finally in a reminder as to purported bridges between the pre Planckian bubble, as would be for the physics, of linkage between Eq. (3) and Eq. (4) the author wishes to reiterate the following points

Eq,.(4) in Pre Planckian physics up to a causal barrier, would be for a single field inflaton. The author is stating that the INITIAL inflaton field, if the causal structure structure is linked to the forming of Eq. (3) by assuming that the Eq.(4) construction would go to N=1 of Eq. (3). This would be equivalent , with the other inflaton fields, N= 2 to N= N, being filled out at the same time the physics of [28] was fulfilled.

The details of this would be in some respects also similar to a 2nd order phase transition. [30] which is a point which will require additional modeling,. That is the transition from N=1 initial scalar field potential, to many scalar field potentials. We state unequivocally though that the details would have some overlap with the ideas outlined in [29] as to the quark gluon plasma and electroweak, but would not have the convenient simple phase diagrams as outlined in [31] / Amd then using Eq. (4) and Eq. (A2) of **Appendix A**, below, we argue we then will have a probability of the suddenly liberated from just n = 1 ground state, of what we were looking at the causal barrier to be, that instead we will have a probability of P(1)~ .41, as given by approximation in Appendix A, that the single field inflaton would be held to, in main value, with a 59- 60 per cent probability that other inflaton states would be evolved to, as implied by [32]. The exact particulars of this would be in refinement of an argument as qualitatively alluded to in [32] below, with major refiments.

We close with our arguments for further investigation of the results of **Appendix B**, which suggests that if there is not a singularity, that there exists contributions to "gravitational energy" as cited on page 615 of [5]. If we conflate gravitional energy, with the production of gravitons, and we use the Ng hypothesis [12] of infinite quantum statistics, to conflate a production of say, a number of relic gravitons with a count of entropy, what we are suggesting is that our non singular resuls for starting expansion are in tandem, due to [5] with nonzero initial entropy. Which would have profound observational data consequences.

Our final goal in this document, is to eventually come up with a detailed pre-Planckian physics analysis of a precursor to redoing the presumed Weyl cosmological tensor, and to in part modify [5] in terms of conclusions, relic gravitational energy at the start of cosmological expansion, and if [12] holds as far as gravitons, come up with a detailed analysis as to why the initial expansion of the universe starts off with nonzero entropy.

That will be the conclusion which we hope to reach in a future document. And this will by necessity, be reviewing Eq. (4), Eq. (5) and Eq. (18) of our document as well as a re do of the assumed conclusions given in [5] as written up by Penrose, in 1978-1979.

It also requires a further elaboration of Eq. (14) as well, which we intend to do in a future document which will also relate the discussion to the future projects alluded to in **Appendix B.**

We also will consider, in Appendix C, what the Weyl tensor, for at least 4 dimensions concludes as far as the Friedman-Walker-Lemaintre 'perfect fluid' cosmology pertains to , with a comment in it as far as what the Pre Planckian to Planckian transition would say as far as the Penrose conjecture

The tale away in Appendix C is that Eq. (C1) has its simplified form, right after the Causal boundary, but that we would have to consider the transformations needed from Pre Planckian space time to Planckian, in order to come up with full analytical development as far as fi the Penrose Weyl tensor would indeed lead to a vanishing behavior at near singular condtions.

The development of this would be tied into fuller development of the point raised in Appendix B in a future publication. In addition, we also will make reference, to **Appendix D**, i.e. where we have Mach's principle as , $G\rho\tau^2 \sim 1$ in defining the initial space-time non singular 'bubble. If , here, τ is a time unit, which is interpreted slightly differently than being the Hubble time, but instead is the recounting is given as

$$\tau \xrightarrow[\Pr e-Planckian \to Planckian]{} \Delta t \tag{19}$$

Whereas we will be interpreting

$$\rho \xrightarrow[\Pr e-Planckian \to Planckian]{} N\left(number-of-gravitons\right) \times \frac{m_g\left(mass-of-graviton\right)}{Volume} \quad (20)$$

The discussion of the applications of Eq. (19) and Eq. (20) are linked with suitable references, and also are tied, into an interpretation of Eq. (4) in a way which introduces the idea of quantum mechanics being introduced, near the causal boundary surface as referenced in Eq. (14) in a way which makes our interpretation of space-time gravitational wave signals being produced, also, in effect, linked to when $G\rho\tau^2 \sim 1$ holds, in effect transforming it to a data set we will call

$$\begin{aligned}
&G\rho\tau^2 \sim 1 \xrightarrow[\Pr e-Planckian \to Planckian]{} \\
&G \cdot N\left(number-of-gravitons\right) \times \frac{m_g\left(mass-of-graviton\right)}{Volume} \cdot \Delta t^2 \sim 1 \\
&\xrightarrow[\Pr e-Planckian-Volume \to Volume-scaled-to-1]{} \\
&G \cdot N\left(number-of-gravitons\right) \times m_g\left(mass-of-graviton\right) \cdot \Delta t^2 \sim 1
\end{aligned} \quad (21)$$

This is our final future works project which we will attempt to confirm or to analyze via future data sets, and we will do it while asserting that Δt is tied into a solution to Eq. (12)

All these suppositions, plus the idea of when we go from Quantum to quasi-classical will be extended from **Appendix D**, and will be hopefully made congruence with respect to Appendix B, as far as the Weyl conjecture by Penrose, as well as also giving more explicit content to Appendix C, as far as the transition from one inflaton field, to perhaps many inflaton fields. With the initial inflaton field being approximately 41% of being one of the many past the causal boundary multiple inflaton fields. I.e. this transformation, as alluded to in Eq. (4) will be in its end product the graviton / gravitational wave generation of our model, and deserves further future elaboration.

In doing so, the author wishes to add another experimental bench mark to review, namely that one has a mass of the graviton , at near light speed being 5×10^9 greater than the presumed rest mass of the "massive" graviton, which would be a staggering increase in the effective mass of gravitons, traveling near the speed of light, right after the H= 0 causal boundary surface.

The further explanation of this business is in **Appendix D**, and would be important in itself as far as to ascertain the fidelity of GW data sets, with the predictions of [19,20,21].

In itself this would be lending toward trying to ascertain experimental data set confirmation if this is viable and a reasonable datum to consider in this situation. As a datum which may explain the black hole situation where the mass of a graviton, should it exist, have a Compton wavelength 5×10^9 greater than the GW wave fronts ascertained in the LIGO measurements, as well as other issues, in [19,20, 21]

In doing all of this, we urge the readers to keep in mind earlier work done by the author as to a Modified Heinsenberg Uncertainty principle, which is summarized in **Appendix E.** All of what we have here should be summarized and compared to a result which is to be held in sync with the physics of Pre Planckian to Planckian physics as outlined below. It goes without saying that a major task of our future work should be comparing the results of Eq. (1) to Eq. (4) of our main document with the modified Heisenberg Uncertainty principle, in Appendix E. Also, and not least will be in doing further computational matching of our presumed Causal boundary, as given of about Planck Length in radii, as the pre Planckian to Planckian physics, boundary with the requirements of the Mach's principle, as given in **Appendix C**, and **Appendix D**..

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APPENDIX A

Summary of material from [32] as to quantum mechanical probability for particle to stay in ground state. For a box, with a wave functional as described below.

Assume a normalized quantum mechanical wave functional, ψ, as given by

$$\psi = \sqrt{\frac{2}{L}}; if \quad 0 \le x \le L/2 \qquad \text{(A1)}$$

$$\psi = 0; \quad if \quad L/2 \le x \le L$$

If so then, the probability that one has a wave functional value with n=1 in the situation defined by Eq. (A1) is given as

$$\begin{aligned} \psi(x) &= \sum_{n=1}^{\infty} A(n) \sqrt{\frac{2}{L}} \sin\left[\frac{n\pi x}{L}\right] \\ A(n) &= \int_{0}^{L/2} \sqrt{\frac{2}{L}} \sin\left[\frac{n\pi x}{L}\right] \sqrt{\frac{2}{L}} dx = \frac{4}{n\pi} \sin^2\left[\frac{n\pi}{4}\right] \\ P(n) &= \left(\frac{4}{n\pi} \sin^2\left[\frac{n\pi}{4}\right]\right)^2 \\ \therefore P(1) &= \frac{4}{\pi^2} \cong .41 \end{aligned} \qquad \text{(A2)}$$

Appendix B;

Making sense of the Penrose reference, as to nonzero initial entropy, and other cosmological issues.

In [5] Penrose, makes the following claim, and we will be examining its implications. He claims that

"An oscillatory Weyl curvature of frequency ϖ and complex amplitude Ψ supplies an effective "gravitational-energy" contribution to the Ricci tensor [33] of magnitude

$$Gravitational - energy - contribution, Ricci - tensor \sim \left|\Psi\right|^2 \varpi^{-2} \qquad \text{(B1)}$$

Before approach to a singularity

The assumption, is that at the singularity, that the complex amplitude, Ψ , is set equal to zero. And so there is no gravitational energy, at a singularity.

Our suggestion is that Eq. (B1) never goes to zero, due to Eq. (3) to Eq. (5) of our text, and that due to this, we will be having, instead, that the nonzero value of Eq. (B1) is a condition for initial graviton production. Hence, then, using [12] and infinite quantum statistics, we are having, then that graviton production, then will be linked to entropy production, at the start of a causal boundary, of space-time.

This should be compared with [34] and with [35] which according to Penrose gives a far more detailed proof, and also we can connect it with [36] in terms of an eventual calculation which will be linked to some of the Pre-Planckian space-time results of [4] and also [28].

The long and short of it is also that if we understand what the consequences of a causal discontinuity are, we will be able to perform a detailed calculation of the Weyl curvature tensor in the neighborhood of a near singular starting point of space-time. Doing that is equivalent to the following

a. Detailing a relic initial graviton rate, for the start of expansion from a causal discontinuous bubble of space-time
b. Detailing a physical mechanism for the production of nonzero entropy at the start of cosmological expansion
c. Re do of Eq.(4), Eq. (5) and Eq. (18) of our document, due to a re interpretation of [5], with a nod to [34,35,36] of our document
d. Detailed calculations of the Weyl tensor in the neighborhood of the Causal boundary.
e. A review of the physics, of presumed singularity theorems as given in [37], plus [38]

Appendix C.

The Weyl curvature tensor, in the Friedman-Lemaitre-Robertson-Walker (FLRW) metric and what it says about Pre Planckian-Planckian transformations

We initiate this section by stating the n=4 (three spatial dimensions and one time dimension) Weyl Tensor, in the case of a fluid cosmology , a.k.a. the Friedman-Lamaitre-Roberson-Walker metric

We write for the Weyl curvature Tensor, a formulation given by [39], which we rewrite as

$$\begin{aligned} C_{abcd} &= \frac{3}{a^2}\cdot\left(a\cdot\ddot{a}+\dot{a}^2+k(Curvature)\right)\cdot\left(g_{ac}g_{bd}-g_{ad}g_{bc}\right) \\ &+\frac{1}{6}\cdot\left(g_{ac}g_{db}-g_{ad}g_{cb}\right)-\frac{1}{2}\cdot\left(g_{ac}R_{db}+g_{bd}R_{ca}-g_{ad}R_{cb}-g_{bc}R_{da}\right) \end{aligned} \tag{C1}$$

The entries into the above, assuming c=1 (speed of light) in the Friedman-Lemaitre-Roberson-Metric would be right after the Causal boundary, in the Neighborhood of Planckian Physics, given as [40], namely looking at the so called entries into the following expressions, namely if we go by [1]

$$
\begin{aligned}
& g^{00} = -1 \\
& g^{11} = \frac{a^2}{1 - k(Curvature) \cdot r^2} \\
& g^{22} = a^2 \cdot r^2 \\
& g^{33} = a^2 \cdot r^2 \sin^2 \theta \\
& R_{00} = -3 \frac{\ddot{a}}{a} \\
& R_{ij} = \frac{3}{a^2} \cdot \left(a \cdot \ddot{a} + 2\dot{a}^2 + k(Curvature) \right) \cdot g_{ij}
\end{aligned}
\tag{C2}
$$

In the Pre Planckian space-time,. We will have that about the Causal boundary, were H=0 we will have

$$
\left| \delta g^{00} \right| \sim a^2 \left| \phi(\inf) \right| << 1 \tag{C3}
$$

It so happens, that for very small time steps, with the inflaton, as given by Eq.(5) in the main text would be negative, i.e.

$$
\operatorname{sgn}(\phi(\inf)) = -1 \tag{C4}
$$

Our task would be to fill in the details of the evolution of the metric tensor, as far as Pre Plankian space-time and to find a way analytically to obtain an expression, which would in some sense have an analytical linkage, prior to the space-time given in (C1) above.

This will be the task of our future analytical work, and its possible connection to the Penrose Weyl Hypothesis, and singularities, as given in [5]

Appendix D, Mach's principle as written by Sciama, $G\rho\tau^2 \sim 1$ in defining the initial space-time non singular 'bubble.

For the sake of completeness we reproduce Eq. (19) to Eq. (21) of the text, but with the commensurate references, and include in their references, with suitable explanations included.

Our starting equation is given by Sciama, as given in [41], which is rendered as

$$
G\rho\tau^2 \sim 1 \tag{D1}
$$

Reference [42] gives further insights, into how Sciama worked with this insight, but for the Pre-Planckian to Planckian space-time physics, we will stick for the moment with looking at the intricacies of the Eq.(D1) in defining the initial space-time non singular 'bubble. If , here, τ is a time unit, which is interpreted slightly differently than being the Hubble time, but instead is the recounting is given as

$$
\tau \xrightarrow[\Pr e-Planckian \to Planckian]{} \Delta t \tag{D2}
$$

Whereas we will be interpreting

$$\rho \xrightarrow[\Pr e-Planckian \to Planckian]{} N\left(number-of-gravitons\right) \times \frac{m_g\left(mass-of-graviton\right)}{Volume} \quad \text{(D3)}$$

The discussion of the applications of Eq. (D2) and Eq. (D3) are linked with suitable references, and also are tied, into an interpretation of Eq. (D4) in a way which introduces the idea of quantum mechanics being introduced, near the causal boundary surface as referenced in Eq. (14) in a way which makes our interpretation of space-time gravitational wave signals being produced, also, in effect, linked to when $G\rho\tau^2 \sim 1$ holds, in effect transforming it to a data set we will call

$$\begin{aligned}
& G\rho\tau^2 \sim 1 \xrightarrow[\Pr e-Planckian \to Planckian]{} \\
& G \cdot N\left(number-of-gravitons\right) \times \frac{m_g\left(mass-of-graviton\right)}{Volume} \cdot \Delta t^2 \sim 1 \qquad \text{(D4)} \\
& \xrightarrow[\Pr e-Planckian-Volume \to Volume-scaled-to-1]{} \\
& G \cdot N\left(number-of-gravitons\right) \times m_g\left(mass-of-graviton\right) \cdot \Delta t^2 \sim 1
\end{aligned}$$

This is our final future works project which we will attempt to confirm or to analyze via future data sets, and we will do it while asserting that Δt is tied into a solution to Eq. (12) of the main text.

Here we refer to these equations as having to be checked against the predictions given in gravitational wave physics problems [43]

We also state for the record that this is assuming massive gravitons. i.e. we work with the following given value

$$m_g < 1.2 \times 10^{-22}\ \text{eV}/c^2. \qquad \text{(D5)}$$

This is, then of course in sync with [44] as well, and should be made consistent with respect to future gravitational wave astronomy data sets. A future works project which we think is essential, where one has to keep in mind that the Compton wavelength of the graviton is not equal to the gravitational wave wavelength. Instead, the lower-bound graviton Compton wavelength is 5 × 10^9 times greater than the gravitational wavelength for the GW150914 event, which was ~ 2000 km.

Clarifying this last point with sufficient data analysis, will entail a close check with [19, 20, 21}. And of course interested readers are invited to look at the theoretical massive gravity theoretical details which are in [45]. Also, in [46] there is a very detailed discussion of a quantum oscillator, assuming a mass , m, distance d, and temperature T, which is a length of the traversing of our formed quantum mechanical states, possibly of gravitons, to emerge as a classically inclined decoherence state, in a time, t, as given by the [46] result, as

$$t > \frac{1}{\gamma} \cdot \left[\frac{h}{d\sqrt{2mKT}} \right]^2$$

&

$$\gamma \equiv \left(Gh / c^5 \right) \cdot \omega_0^3 \qquad \text{(D6)}$$

&

$$\omega_0^3 = cube - of - graviton - frequency$$

Of course, reconciling Eq. (D5) and the discussion between Eq. (D5) and Eq. (D6) should be part of our future works program, as well as all the other issues alluded to by Dr. Corda in [20] which is very relevant. In doing so, this should give more detail as to Eq. (4) in the main text.

One possible end run about the difference in Graviton Compton wavelength, and of Gravitational wave wavelength, this of the fact that lower-bound graviton Compton wavelength is 5 × 10^9 times greater than the gravitational wavelength

Look at the special relativistic proportionality factor of increase in mass is included in, we would be obtaining the very high relativistic

$$\begin{aligned}
&\beta \equiv 1/\sqrt{1-\left[v(velocity-graviton)/c\right]^2} \\
&\Rightarrow 5 \times 10^9 \sim 1/\sqrt{1-\left[v(velocity-graviton)/c\right]^2} \\
&\Leftrightarrow 25 \times 10^{18} \sim 1/\left(1-\left[v(velocity-graviton)/c\right]^2\right) \\
&\Leftrightarrow \frac{1}{25} \times 10^{-18} \sim \left(1-\left[v(velocity-graviton)/c\right]^2\right) \qquad \text{(D7)} \\
&\left[v(velocity-graviton)/c\right]^2 \sim 1-\frac{1}{25} \times 10^{-18} \\
&\Rightarrow v(velocity-graviton) \sim \left(1-\frac{1}{50} \times 10^{-18}\right) \times c
\end{aligned}$$

I.e. this would be in line with the situation where one has a mass of the graviton , at near light speed being

5×10^9 greater than the presumed rest mass of the "massive" graviton, which would be a staggering increase in the effective mass of gravitons, traveling near the speed of light, right after the H= 0 causal boundary surface.

In itself this would be lending toward trying to ascertain experimental data set confirmation if this is viable and a reasonable datum to consider in this situation. As a datum which may explain the black hole situation as outlined above.

APPENDIX E.

Summary of the changes of the Pre Planckian to Planckian Heisenberg Uncertainty principle to keep in mind.

We use the approximation as presented in [6] which we reproduce below as also in [47, 48]

$$
\begin{aligned}
(\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\
(\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A
\end{aligned}
\qquad \text{(E1)}
$$

If we use the following, from the Roberson-Walker metric [47, 48, 49]

$$
\begin{aligned}
g_{tt} &= 1 \\
g_{rr} &= \frac{-a^2(t)}{1-k\cdot r^2} \\
g_{\theta\theta} &= -a^2(t)\cdot r^2 \\
g_{\phi\phi} &= -a^2(t)\cdot \sin^2\theta \cdot d\phi^2
\end{aligned}
\qquad \text{(E2)}
$$

Following Unruh [47, 48] , write then, an uncertainty of metric tensor as, with the following inputs

$$
a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters \qquad \text{(E3)}
$$

Then, if $\Delta T_{tt} \sim \Delta\rho$ [6, 47, 48]

$$
\begin{aligned}
& V^{(4)} = \delta t \cdot \Delta A \cdot r \\
& \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\
& \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}
\end{aligned}
\qquad \text{(E4)}
$$

This Eq. (E4) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [49]

$$
T_{ii} = diag(\rho, -p, -p, -p) \qquad \text{(E5)}
$$

Then by [6]

$$
\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}} \qquad \text{(E6)}
$$

Then, by [6]

$$
\begin{aligned}
& \delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \\
& Unless \quad \delta g_{tt} \sim O(1)
\end{aligned}
\qquad \text{(E7)}
$$

This leads to us estimating of the Δg_{tt} term in Modified HUP , as a summary of what we obtain here, is if we use something similar to the Chapygin gas model [50]

$$\begin{aligned}
&\rho \sim \frac{3}{\tilde{\tilde{\alpha}}} \cdot (1 \pm A) \cdot \Lambda + H.O.T \sim \frac{\Delta E}{l_p^3} \\
&\& A = 1/3 \ \ (radiation) \\
&\Leftrightarrow \Delta g_{tt} \sim \frac{\hbar \tilde{\tilde{\alpha}}}{(t_{\min} \sim Planck - time)} \cdot l_p^3 \cdot (1 \pm A) \cdot \Lambda_{Today's-value}
\end{aligned} \tag{E8}$$

For our purposes, this corresponds to having $\tilde{\tilde{\alpha}}$ fairly large but not infinite,

$$\begin{aligned}
&\delta t \Delta E \geq \left. \frac{\hbar}{\delta g_{tt}} \right|_{\Pr e-Octonionic} \xrightarrow{change\ in\ phase,\ given\ byp\ \ phase\ \ \delta_0} \delta t \Delta E \geq \hbar \big|_{Octonionic} \\
&with \ \ \delta t \geq \frac{\hbar}{\delta g_{tt} \Delta E} FIXED
\end{aligned} \tag{E9}$$

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