

**PROPOSED HYPOTHESIS OF A**  
**PROCESS LINKED TO GRAVITY**  
**AFFECTING MASS-ENERGY**

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## ABSTRACT

**The proposal assumes that the distortion of space-time due to relative velocity (Special Relativity), and the distortion of space-time produced by gravitational fields (General Relativity) are linked to changes of state that affect to mass-energy.**

The hypothesis proposes the existence of a process linked to gravity, this phenomenon would affect mass-energy. It would be required to add an additional condition (being a more restrictive scenario) keeping the field equations that define space-time curvature, but by adding the condition linked to the proposed phenomenon, the trajectory that would follow mass-energy in that curved space-time, changes with respect to the established by the officially accepted model. The effect is negligible if the distortion of space-time caused by a gravitational field does not have a significant value. The proposed hypothesis allows to mathematically calculate the discrepancy that would exist with respect to the current model. In case of being correct, the proposal would have important implications in diverse areas of science and its effect would be determinant in the study of black holes or questions related to Cosmology.

## **BACKGROUND, PROBLEMS JUSTIFYING NEW CONTRIBUTIONS**

The mathematical model of General Relativity has allowed to carry out accurate predictions and calculations, however there are certain issues about gravity that have not been resolved satisfactorily and lead to the conclusion that there is something wrong or that is not being interpreted properly, or there is something else that is not being taken into account. Below are briefly described some of the problems concerning gravity:

- Theoretically the mathematical model of relativity predicts or gives rise to singularities at certain circumstances. The rules established by quantum mechanics require an increasing energy in order to increase the degree of confinement of a particle. However, the model defined by relativity, there is not such an impediment to that circumstance, quite the contrary, what the theory seems to indicate is that under certain conditions bodies would inexorably follow a path to singularity. Other forces such as electromagnetism where initially there were divergences at certain conditions, have been renormalized thus avoiding such divergences, which has not yet been possible with gravity. These have been some of the reasons for defining alternative models such as String Theory.

- Stephen Hawking's contributions to black-holes radiation that lead to the paradox of information loss for a body that crosses the horizon of events was a problem without a clear resolution, until the middle of the 1990s, when the Holographic Principle was proposed, which currently has the consensus and majority support of the scientific community.

- At 2012 arose a new conflict presented by Ahmed Almheiri, Donald Marolf, Joseph Polchinski and James Sully. Taking into account the officially accepted model, including the Holographic Principle, a particle would have at the same time two quantum entanglements, while being entangled with a particle that crosses the event horizon and at the same time with the duplicate information linked to that event horizon. The fact of a double quantum entanglement contravenes the quantum rules, this has generated a new conflict that has in some way divided the scientific community and still does not have a clear resolution. The proposal or solution presented by Ahmed Almheiri, Donald Marolf, Joseph Polchinski and James Sully. It included the existence of a Firewall at the event horizon of a black hole, whereby an observer on reaching the event horizon or in the vicinity of it would encounter quantum energy that would prevent the passage through the event horizon. However that proposal is yet a controversial one, critics arguing that energy firewall seems an "Ad Hoc" solution and that firewall seems to come from nothing because only would appear in the vicinity of the black hole.

If the proposed hypothesis at this paper is a correct one, it would have important implications and should be taken into account in relation to the phenomena described above.

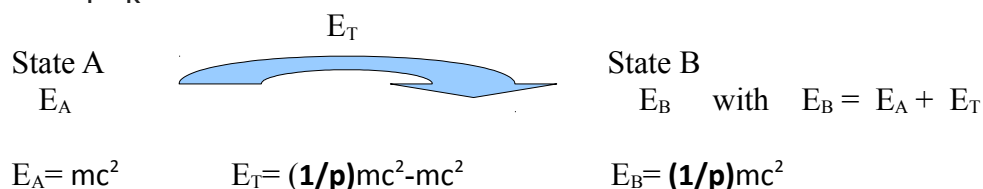
## **DISCUSSION**

The proposal assumes that the distortion of space-time due to relative velocity (Special Relativity), and the distortion of space-time that is produced by gravitational fields (General Relativity) are linked to changes of state that affect to mass-energy. Mass-energy would be affected, changing its state, generically denoted "State A" and "State B". The present paper establishes as reference "State A" while "State B" is taken as referenced to A (This will be the criterion followed at the paper if it is not said otherwise). Concerning relative velocity, these states correspond to relative velocities between bodies. Taking as reference "State A", if a body moves at velocity  $v$  relative to another body, we would say that the reference body is in "State A", while the other body that moves at velocity  $v$  relative to A, is in "State B". Considering General Relativity, the distortion of spacetime generated by the gravitational field will also be associated with different states, if we take as reference the state for which the gravitational field effect is zero, then "State A" would be associated with time  $dt$ , while the "State B" would be in a generic way, characterized by the proper time  $d\tau$ , which would depend on the space-time distortion. That is to say, concerning relative velocity, "State B" would be generically characterized by  $v$ , meanwhile concerning gravitational fields "State B" would be generically characterized by proper time  $\tau$ .

Note: Concerning General Relativity the proposed hypothesis adds an additional condition to the system of ten non-linear partial differential equations corresponding to the Standard Model of Relativity. As example (at this paper) of General Relativity scenario with the presence of a gravitational field, will be considered the Schwarzschild metric for the vacuum solution of the field outside the homogeneous sphere, uncharged, non rotating.

**Quantum mechanics have linked processes where particles interact, passing from an "α state" to a "β state", with a probability associated to that process. The proposal introduces a process linked to gravity, where mass-energy would be affected changing its state from A to B.**

The hypothesis proposed at this paper assumes that there is a contribution of energy between both states. That would be already taken into account by the Standard Model of Relativity for Special Relativity ( $E_T = E_K$ ), but not for General Relativity with the presence of a gravitational field.



Where  $p$  is a factor that relates the referenced state (State B) to the reference state (State A).

- Concerning Special Relativity:  $p = 1/\gamma$  **Being  $\gamma$  The Lorentz factor**

Then  $E_T = \gamma mc^2 - mc^2$  kinetic energy.

$E_B = \gamma mc^2$  This value includes the factor  $p = 1/\gamma$ , So that a relation between the reference (State A) and the state referenced to it (State B) is established, so that the value  $E_B = \gamma mc^2$  implies that "State A" is the reference, if we take as reference "State B" and we consider it as reference to itself (for example an observer situated at "State B" observing something which is at "State B" as well) then velocity 0,  $\gamma = 1$  and the factor  $p = 1$ , so  $E_B = mc^2$  because the observer is at reference (State B) and the object observed is at "State B" as well and  $E_T$  between them has a null value.

This case (Special Relativity) from the practical point of view, there would be no changes with respect to the current official model. The energy required to pass from one state to another would be the kinetic energy, and mass-energy at "State B" would have a value increased by the factor  $\gamma$  with respect to "State A" and we would say that "State B" has reference at A or is relative to A. Thus this approach would be compatible with the model established by Special Relativity.

- Concerning General Relativity: "State A" is linked to dt and "State B" linked to dτ, hypothetically with no relative velocity between both states, the relation between times is given by:  $p=dt/dt$  denoting  $p=1/\phi$ ; **Being**  $\phi=dt/d\tau$

If the reference is "State B" and the referenced state is also "State B":  $p=dt/d\tau=1$

If the reference is "State B", and "State A" is referenced to it, then  $p=dt/d\tau$

(Both observers, one of them situated at "State A" the other at "State B" would agree on time linked to A is t, while time linked to the "State B" would be τ. Concerning relativistic velocity, both observers would take as value p for the other state as  $p=1/\gamma$ )

If the reference is a state associated with  $d\tau_1$ ; and another state, with  $d\tau_2$  is referenced to it, then  $p=dt/d\tau_1$

Considering "State A" as reference and B with reference at A:

$$E_A=mc^2 \quad E_T=(dt/d\tau)mc^2-mc^2 \quad E_B=(dt/d\tau)mc^2$$

**Considering an observer in "State A", and an object in State A as well, with associated energy  $E_A=mc^2$ . If that object passes to "State B", while the observer is sit in "State A", the value of the energy associated to that object relative to the observer fixed in "State A", changes to  $E_B=(dt/d\tau)mc^2$ , and the value of the energy required for that process to take place is  $E_T=(dt/d\tau)mc^2-mc^2$**

The value of the energy  $E_B$  is the value at B with reference the "State A", indicating that energy at B is with respect to A, already implies that relation, although for this concept to be explicitly represented would be required a notation of the type:

$$E^A_A=mc^2 \quad E^A_B=(dt/d\tau)mc^2$$

Upper index A indicates that the reference is "State A", so the value of the energy at A with reference A has value  $mc^2$  while the value at B with reference A would have value:  $(dt/d\tau)mc^2$

If we consider the value at B with reference B then  $E^B_B=mc^2$

and the value at A with reference B would be  $E^B_A=(d\tau/dt)mc^2$

The energy between two states B and C taking as reference A (with associated time t. at A):

$$E^A_C - E^A_B = \int_{dt/d\tau_B}^{dt/d\tau_C} mc^2 d\phi \quad \text{Energy linked to the proposed process}$$

$\tau_B$  proper time at B;  $\tau_C$  proper time at C

When the states B and C correspond to A and B respectively, and denoting generically  $\tau_B=\tau$ , then:  $E^A_B - E^A_A = (dt/d\tau)mc^2 - mc^2$

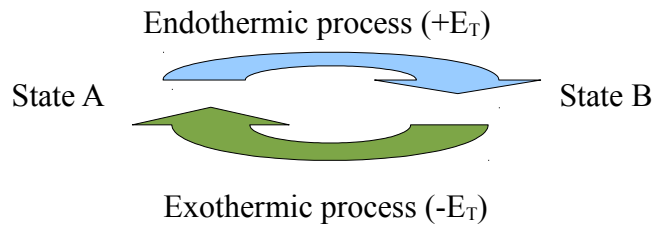
The value at B for an observer at State A  $E^A_B = (mc^2) + ((dt/d\tau)mc^2 - mc^2) = dt/d\tau mc^2$

The value at A for an observer at State B  $E^B_A = (mc^2) + ((d\tau/dt)mc^2 - mc^2) = d\tau/dt mc^2$

Because concerning General Relativity, both observers do agree on the values of dt and dτ, so that the parameters dt and dτ have inverse position for an observer at B.

If the observer is fixed at "State B", then the object at "State A" has associated energy  $E^B_A = (d\tau/dt)mc^2$  (meanwhile, for the observer fixed at "State A" the associated energy of the object is  $E^A_A = mc^2$ ). now for the observer fixed at "State B", when the object changes from "State A" to "State B", its associated energy would change to  $E^B_B = mc^2$  being  $E^B_T = mc^2 - (dt/d\tau)mc^2$  the energy required (taking as reference B) for that process to take place. Considering the reverse process, if the object with associated energy  $E^B_B = mc^2$  changes to "State A" ( $E^B_A = (d\tau/dt)mc^2$ ), then instead of requiring energy, would be an exothermic process, having  $((d\tau/dt)mc^2 - mc^2)$  a negative value.

Concerning the process linked to General Relativity:



If the object changes its state from “State A” to “State B”, the process is an endothermic one, requiring energy, the reverse process, changing from “State B” to “State A” is an exothermic one, releasing energy

The proposed process, as defined, implies an additional effect to the currently accepted model, caused by the gravitational waves. The endothermic process from A to B would be at the expense of velocity corresponding to the Kinetic Energy, while the exothermic process from B to A would increase velocities of bodies for an expansive scenario.

**Elements involved in the proposed process:**

- Gravitational waves, which will interact with mass-energy.
- Mass-energy with a starting reference value  $mc^2$ . (value of mass-energy at “State A” with reference the state linked to dt,  $E_A^A = mc^2$  ).
- Potential Energy, part of it will be absorbed by the process and the rest would be transformed into the velocity term of the kinetic energy. Or equivalently, without the potential energy concept, the process from “State A” to “State B” requires energy (it is an endothermic process), that energy will be at the expense of the velocity.

**Result (state B):**

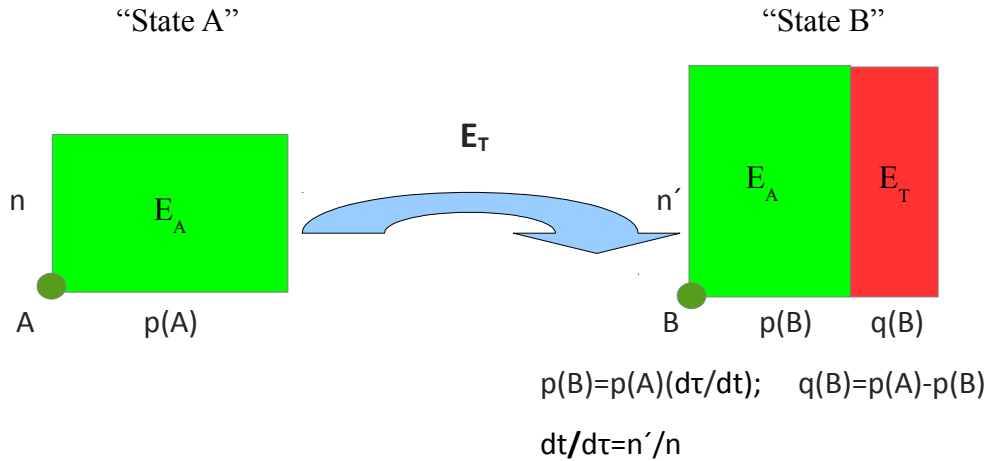
- Space-time distortion at the space-time position linked to the “State B” (as defined by Einstein field equations, plus the additional effect of the energy required by the process).
- Mass-energy with value  $E_B^A = (dt/d\tau)mc^2$
- Kinetic energy that will have the altered mass-energy in the new state. Noticing that mass-energy of the body will depend on the reference taken, and the velocity term of that kinetic energy is modified, because the process will take place at the expense of the velocity. (value linked to  $\Upsilon_{mod}$  ( $\Upsilon_{mod} = 1/\sqrt{(1- v_{mod}^2/c^2)}$ ), while  $\Upsilon = 1/\sqrt{(1- v^2/c^2)}$  would be the value corresponding to the theoretical Kinetic energy without taking into account the proposed process.

In order to calculate  $\Upsilon_{mod}$ , it is necessary to take into account the combination of the energy required by the process and the gravitational field as defined by the E.F.E.

It is noteworthy that the process as described is similar to the phenomenon corresponding to the photoelectric effect (each with its own characteristics):

Photons interact with electrons, part of the energy is absorbed by the process and the rest goes to kinetic energy.

**Hypothesis:** Gravitational waves would interact with mass-energy, part of the energy is absorbed by the process, the rest goes to the velocity term of the kinetic energy.



$p(A)=1$ , then  $p(B)= dt/dt$  and  $q(B)= 1- dt/dt$   
 $E_T = q(B) E_B^A = (1- dt/dt)(dt/dt)mc^2 = (dt/dt)mc^2 - mc^2$   
 $E_A^A = p(B)E_B^A = (dt/dt) (dt/dt)mc^2 = mc^2$   
 $E_B^A = E_T + E_A^A = (dt/dt)mc^2$

Taking as reference the “State A”:

$$E_A^A = mc^2 \quad E_B^A = (dt/dt)mc^2 \quad E_T^A = (dt/dt)mc^2 - mc^2$$

Taking as reference the “State B”:

$$E_A^B = (dt/dt)mc^2 \quad E_B^B = mc^2 \quad E_T^B = mc^2 - (dt/dt)mc^2$$

$E_T$  is the energy required by the process to pass from “State A” to “State B”, if  $dt/dt=10$  (considering quite an extreme distorted spacetime scenario) then  $E_T^A=9 mc^2$  it would be 9 times the reference A,  $E_T^B = (9/10) mc^2$  it would be (9/10) times the reference B, because the value of energy at B is 10 times the value at A.

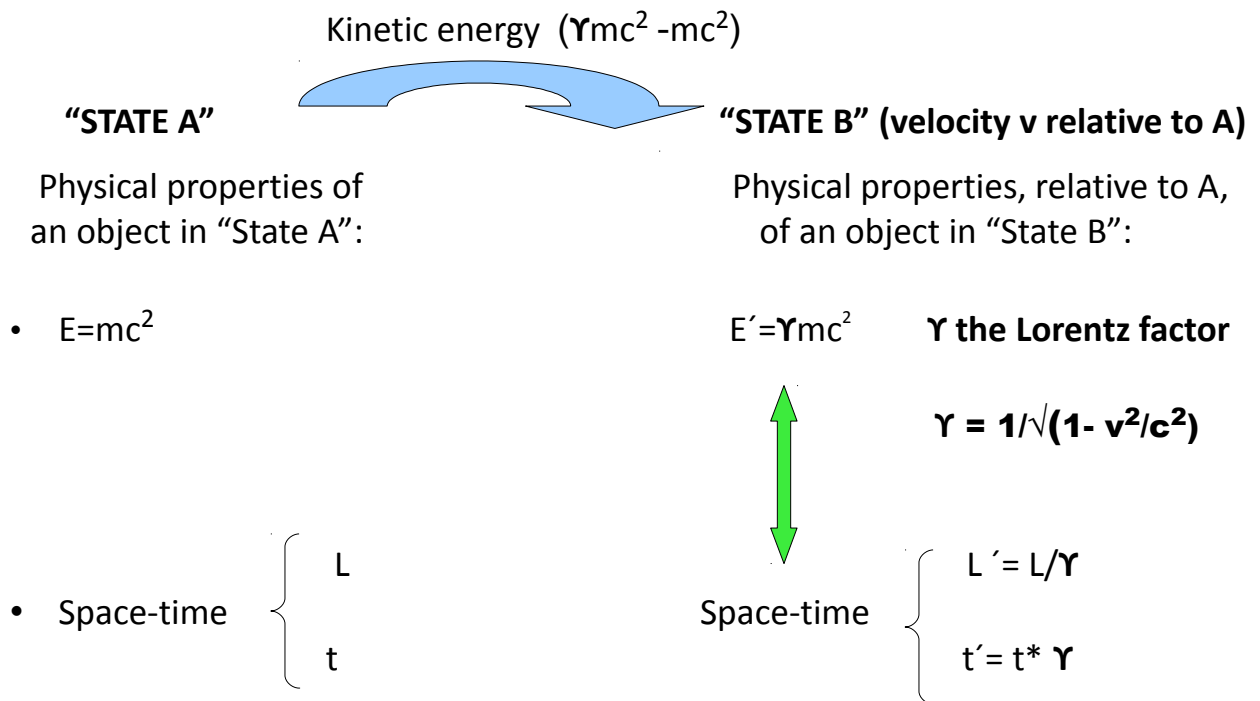
The proposal implies that not all the potential energy would transformed into the velocity term of the kinetic energy. Considering a free fall body, if  $v$  is the velocity when all the potential energy transforms into kinetic energy,  $v_{mod}$  would be the velocity taking into account the process as defined. Discrepancy would be an extremely small one, insofar the space-time curvature has not a significant value.

Noticing that  $v$  depends on the reference as well (as it is well Known the Kinetic Energy is not invariant, as it includes velocity), so that a strong gravitational field, for example a black hole,  $v$  would show different values depending on the reference, for example an observer fixed at position far away from the black hole,  $v=(1-r_s/r)c\sqrt{r_s/r}$  being  $r_s$  the Schwarzschild radius, meanwhile an observer in free fall, would observe the free fall velocity.

## Special Relativity scenario

Considering a hypothetical pure Special Relativity scenario, if a body has no velocity relative to the reference, we might say that the body is in "State A", if it is applied kinetic energy ( $\gamma mc^2 - mc^2$ ) to the body, then the state of the body changes (physical properties mass-energy and space-time do change) the body is now in "State B" with mass-energy  $\gamma mc^2$  relative to the "State A" (the previous reference) and with space-time distortion relative to the "State A".

Minkowski space  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$



"Considering a hypothetical pure Special Relativity scenario (with negligible gravitational fields), now taking into account the proposed hypothesis, the result would be the same: we would say that mass-energy ( $mc^2$ ) of the body would be affected by the Kinetic energy ( $\gamma mc^2 - mc^2$ ), as result would be obtained: Mass-Energy ( $\alpha$ -state)(quantity  $mc^2$ ) plus Mass-Energy ( $\beta$ -state) (quantity  $\gamma mc^2 - mc^2$ ) with total quantity of mass-energy ( $\gamma mc^2$ ) and space-time distortion as defined by relativity."

The expression "space-time distortion as defined by relativity." means that  $L' = L/\gamma$  and  $t' = t * \gamma$



There is an interrelation between the physical properties: mass-energy and space-time

$$(1) \quad E' = \gamma mc^2 \longleftrightarrow \text{Space-time}$$

Mass-energy is affected, being altered in accordance with what is modified the space-time, or inversely we could say as well that space-time distortion is in accordance with the alteration of mass-energy.

Knowing that  $L' = L/\gamma$  and using the defined concepts:

$$(L-L') * (\text{total energy}) = (L-L') * (\text{mass-energy}(\alpha\text{-state}) + \text{mass-energy}(\beta\text{-state})) = (L-L/\gamma) * (\gamma mc^2) = L(\gamma mc^2 - mc^2) = L * (\text{mass-energy}(\beta\text{-state}))$$

**then the proportion:  $(L-L')/L = (\text{mass-energy}(\beta\text{-state})) / (\text{total energy})$**

While  $(L') * (\text{total energy}) = L(mc^2) = L(\text{mass-energy}(\alpha\text{-state}))$

**then**  $\gamma = L / L' = (\text{total energy}) / (\text{mass-energy}(\alpha\text{-state}))$

That is  $E' = \gamma mc^2 = (L / L') * (\text{mass-energy}(\alpha\text{-state}))$

From a practical point of view, considering a hypothetical pure Special Relativity scenario, the result is the same whether assuming the proposed process or the model corresponding to the Standard Model of Relativity. But when the gravitational fields effects are not negligible, then the proposed process establishes discrepancies with respect to the Standard Model of Relativity, that discrepancy is negligible if the graviational field is a weak one, but the discrepancy increases the stronger the graviational field.

**General Relativity scenario**

Standard Model of Relativity , without taking into account the proposed process:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\Pi G}{c^4}T_{\mu\nu}$$

Einstein Field Equations corresponding to the Standard Model of Relativity.

Considering the Schwarzschild metric for the vacuum solution of a homogeneous sphere, uncharged, non rotating.

$$ds^2 = c^2 d\tau^2 = (1 - r_s/r) c^2 dt^2 - (1 - r_s/r)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

**$\tau$ : proper time**

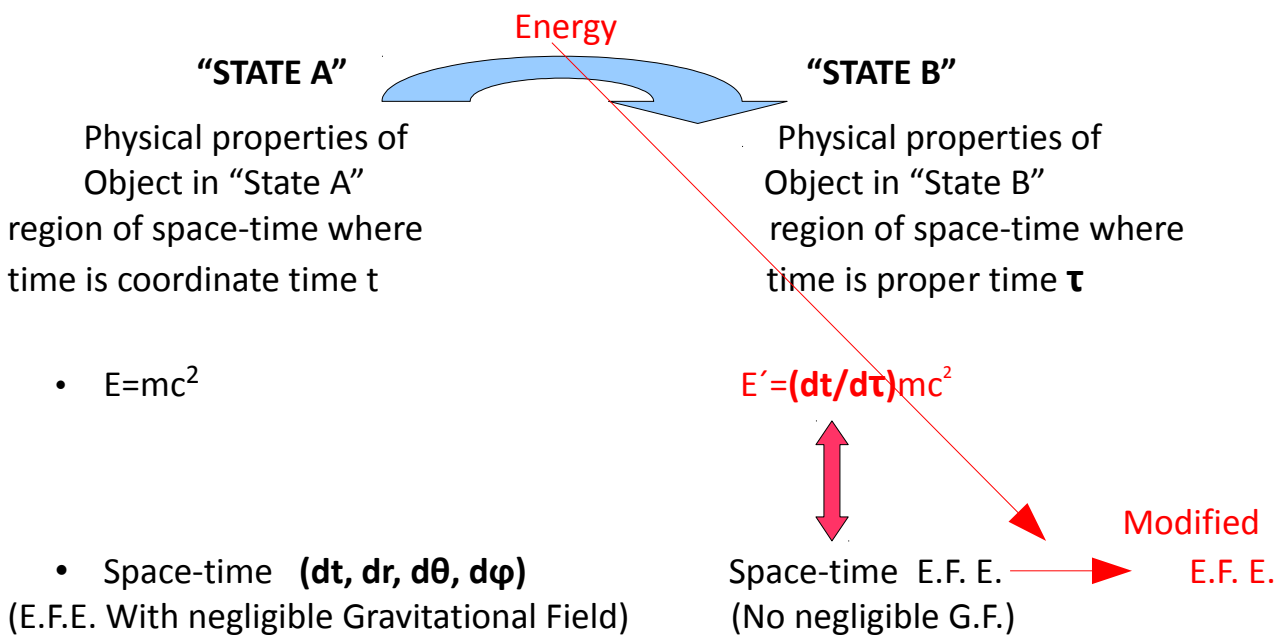
**$t$ : time coordinate**

**$r$ : Schwarzschild radial coordinate**

**$\theta$ : colatitude**

**$\phi$ : longitude**

$$r_s = 2GM/c^2$$



Previous Scheme, without including the terms in red (**Energy;  $E'=(dt/d\tau)mc^2$  ; Modified E.F. E.** ) would correspond to the Standard Model of General Relativity.

Modified Einstein Field Equations means that the Stress Energy Tensor of the E.F.E. Should include the effect of the energy required by the process to take place, so it might be denoted as well modified Stress Energy tensor of the E.F.E.

The hypothesis proposes that the physical properties of an object will change according to the region of the gravitational field that it occupies, and there would be an interrelation between mass-energy and the curvature of space-time defined by the E.F.E. (that is how it is calculated the expression  $(dt/d\tau)mc^2 - mc^2$ ), in a similar manner as it was defined when considering the Special Relativity scenario with

$$(1) E' = \Upsilon mc^2 \longleftrightarrow \text{Space-time.}$$

Considering a gravitational field scenario, mass-energy of the object would depend on the E.F.E. Being  $E'=(dt/d\tau)mc^2$  That process requires **Energy** to take place and the counterpart effect of that energy will be the corresponding of energy opposed to the free fall of the object, which has to be included in the E.F.E. So that instead of the E.F.E. Corresponding to the Standard Model of Relativity it is defined the **Modified E.F. E.** which are the same E.F.E. But taking into account that effect.

Gravitational waves will interact with mass-energy, that process requires energy (it is an endothermic process), will be at the expense of reducing the velocity term so that instead of  $v$  will be  $v_{\text{mod}}$ . That effect is the corresponding to energy opposed to the free fall so that the factors  $(1 - r_s/r)c^2$  and  $(1 - r_s/r)^{-1}$  linked to  $dt^2$  and  $dr^2$  will be modified giving as result  $v_{\text{mod}}$  instead of  $v$ .

**Mass-Energy  $mc^2$  ( $\alpha$ -state) interaction with G.W. + Energy  $\rightarrow$**

**Mass-Energy ( $\alpha$ -state)(quantity  $mc^2$  ) + Mass-Energy ( $\beta$ -state) (quantity  $(dt/d\tau)mc^2 - mc^2$ ) = total mass-energy (quantity  $(dt/d\tau)mc^2$ ). Taking as reference "State A"**

**Energy  $((dt/d\tau)mc^2 - mc^2)$**

**The energy required implies that the process is an endothermic one, it takes place at the expense of taking energy from the physical system. We have:**

**+  $((dt/d\tau)mc^2 - mc^2)$  energy required by the process and**

**-  $((dt/d\tau)mc^2 - mc^2)$  the counterpart of the energy required by the process**

The counterpart of the energy required by the process  $- ((dt/d\tau)mc^2 - mc^2)$  has a negative value and produces the effect corresponding to energy opposed to the free fall, so that instead of  $dr$  and  $d\tau$  (values corresponding to the Standard Model of Relativity) will be  $dr_{mod}$  and  $d\tau_{mod}$  values due to the effect of the energy required by the process.

Note:  $dr_{mod}$  and  $d\tau_{mod}$  do represent that the factors  $(1 - r_s/r)c^2$  and  $(1 - r_s/r)^{-1}$  linked to  $dt^2$  and  $dr^2$  are affected by the energy linked to the proposed process, giving as result  $v_{mod}$  instead of  $v$ .

If the proposed process has an effect such that there is conservation of energy, then:  
 $(\Upsilon_{mod}(dt/d\tau)mc^2 - (dt/d\tau)mc^2) = (\Upsilon mc^2 - mc^2)$

obtaining  $\Upsilon_{mod} = 1 + \Upsilon * d\tau/dt - dt/d\tau$

being  $\Upsilon_{mod} = 1/\sqrt{(1 - v_{mod}^2/c^2)}$

**Considering the Special Relativity scenario, There is a contribution of energy (kinetic energy) between the states, an interrelation between space-time distortion and the alteration of mass-energy, and the value of mass-energy depends on the reference taken.**

**Considering the presence of gravitational fields, it is proposed a process that takes place at the expense of the energy from the physical system, there is as well an interrelation between space-time distortion and the alteration of mass-energy, and the value of mass-energy depends on the reference taken.**

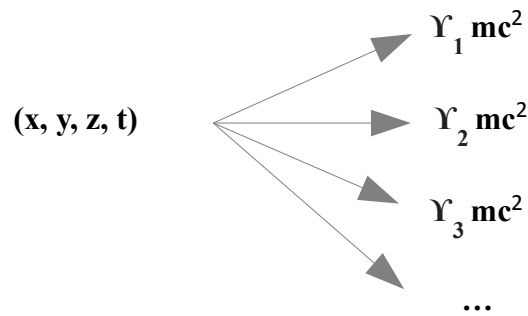
$$E_C^A - E_B^A = \int_{dt/d\tau_B}^{dt/d\tau_C} mc^2 d\phi \quad \text{Energy linked to the proposed process}$$

$p = 1/\phi$  Being  $\phi = dt/d\tau$  (t coordinate time;  $\tau$  proper time)

$\tau_B$  proper time at B;  $\tau_C$  proper time at C

When the states B and C correspond to A and B respectively, and denoting generically  $\tau_B = \tau$ , then:  $E_B^A - E_A^A = (dt/d\tau)mc^2 - mc^2$

Concerning Relativity, it is not enough the assignation of four space-time coordinates to define the properties of the physical system. For instance, considering the Special Relativity scenario.



Mass-energy of a body located in  $(x, y, z, t)$  depends on the reference taken.  $\Upsilon$  relates the “State B” (referenced state) of the body situated at  $(x, y, z, t)$  with the “State A” (the reference).

In order to establish the physical properties of a body, it is required to define  $x, y, z, t$  and  $\Upsilon$ .

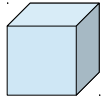
The present paper suggests that there is not just three space-like dimensions and one time-like dimension, but four spatial dimensions and one time dimension  $x, y, z, w, t$  where the space-time framework fulfills:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = dw^2 - dx^2 - dy^2 - dz^2$$

- There is a dependent relation between them as established by the expression  $c^2 d\tau^2 = dw^2 - dx^2 - dy^2 - dz^2$  linking the time-like coordinate to the space-like coordinates.
- The distortion of the space-like coordinate  $w$  is the same than the distortion of the time-like coordinate  $t$ . That is what is representing  $c^2 dt^2 - dx^2 - dy^2 - dz^2 = dw^2 - dx^2 - dy^2 - dz^2$
- There is an interrelation between mass-energy and space-time.

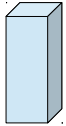
In order to explain those concepts it is used the following simplified schemes:

A) Supposing a three dimension euclidean space.



Isotropic properties, with a uniform distribution of mass-energy within a region of that euclidean 3d space.

B) Now supposing another scenario where there is distortion of space.



Contraction of one of the coordinates at the expense of dilation of another coordinate.

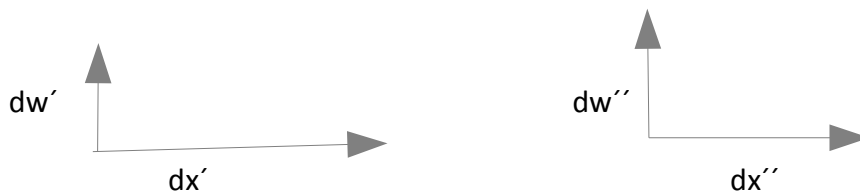
The value of the mass-energy distributed on scenario A) and B) is the same, but changes the configuration, it is further distributed configuring the dilated dimension at the expense of the contracted dimension.

Now considering the Minkowski space-time  $ds^2=c^2d\tau^2=c^2dt^2-dx^2-dy^2-dz^2$  for the Special Relativity scenario and the Riemannian geometry corresponding to a curved space-time framework due to the presence of mass-energy, for instance the Schwarzschild metric for the vacuum solution of a homogeneous sphere, uncharged, non rotating.

The proposal implies that the distortion of the time coordinate is the same than the distortion of w coordinate.

Representing just the space-like distorted coordinate and the coordinate w:

Special Relativity scenario, Minkowski space-time, velocity on x direction  $dw^2-dx^2-dy^2-dz^2$   
 (1) "State A" (2) "State B"



The scheme represents that there is space distortion producing contraction on the space-like distorted coordinate and dilation of the w coordinate. Considering those four space-like dimensions (without taking into account the time coordinate) the value of mass-energy is the same at (1) and (2), just changes the distribution of mass-energy.

The value of mass-energy (considering those four space-like dimensions) is  $mc^2$  in both cases (1) and (2), but changes de distribution of that energy, increasing the quantity of energy linked to the w coordinate in case (2), that situation would correspond with the quantity of mass-energy ( $\beta$ -state) in "State B".

The proposal implies that there is an interrelation between the quantity of mass-energy that changes to mass-energy ( $\beta$ -state) and the distortion of space-time.

Time has to be taken into account, with distortion of time coordinate the same than distortion of  $w$ .



Taking into account now the effect due to the distortion of the time coordinate, the amount of mass-energy will be:

- $mc^2$  value of mass-energy corresponding to "State A"
- $\Upsilon mc^2$  value of mass-energy corresponding to "State B" relative to "State A"

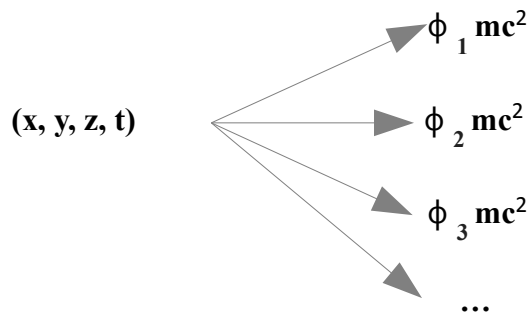
Similarly to the Special Relativity Scenario, will be the General Relativity Scenario with the presence of gravitational fields.

- $mc^2$  value of mass-energy corresponding to "State A" being space-time region with negligible gravitational field effect where time is coordinate time  $t$ .
- $(dt/d\tau)mc^2$  value of mass-energy corresponding to "State B" being  $\tau$  proper time in "State B"

The energy required to change from one physical state to another is  $(dt/d\tau)mc^2 - mc^2$

Denoting  $\phi = (dt/d\tau)$

The hypothesis proposed implies that considering gravitational fields scenarios, it is not enough the coordinates  $(x, y, z, t)$  to define the properties of the physical system.



The value of mass-energy will depend on the reference taken.

It is needed to define  $(x, y, z, t)$  plus  $\phi$ , required to characterized a value with the coordinates  $(w, x, y, z, t)$ .

## Probabilistic approach

It is defined the following Physical System: Special Relativity Scenario.

A radioactive element that has a hypothetical uniform distribution of mass,  $dm/(dx dy dz)$  homogeneous density through a region of space-time between  $(0, 0, 0)$  and  $(x, y, z)$ .

The decay of that radioactive element over a period of time  $t$  (0 starting point of the experiment and  $t$  moment of the measurement, with  $dt$  time element for the time coordinate), follows the Poisson's distribution:  $P(\lambda, x)$  where  $\lambda$  represents the frequency of occurrence of a given event (this case, the decay of the radioactive element) and  $x$  would represent the amount or number of events in an interval.

The Poisson distribution is related to another discrete probability distribution, the binomial distribution. Considering  $n$  Bernoulli statistical trials, each of them with probability  $p \cdot P$  that a certain event takes place, fulfilling the following conditions:

$0 < p \cdot P \ll 1$  very small probability of success.

$n \uparrow \uparrow$  very high number of statistical trials.

$n \cdot p \cdot P = \lambda$  The product of the number of statistical events multiplied by the probability associated with each of the trials is equal to the frequency of occurrence  $\lambda$

If these three conditions are met, both distributions give very similar values, at the limit when  $n \rightarrow \infty$  are equivalent ones.

This leads to the proposal that at the Physical System is taking place the occurrence of statistical events each one with probability  $p \cdot P$  of being successful. That is to say, during the period of time  $dt$ , it takes place  $n$  statistical events,  $P$  representing the probability of an event taking place at a particular trial, this case corresponds to the decay of the radioactive element, meanwhile  $p$  is a factor (which depends on distortion of space-time) that modifies the value of  $P$ . The value of  $p$  will be linked to the alteration of mass-energy, if all the mass-energy corresponds to mass-energy ( **$\alpha$ -state**) then the value of  $p$  is 1 (that is the case for a reference frame with no velocity), when there is velocity, the kinetic energy do alter the state of mass-energy. The higher the quantity of mass-energy that is affected changing to ( **$\beta$ -state**), the lower the value of  $p$ , the value of mass-energy corresponding to mass-energy ( **$\alpha$ -state**) is  $p \cdot P$ , while the corresponding to mass-energy ( **$\beta$ -state**) is  $q \cdot P = (1-p) \cdot P$ . Each of the  $n$  statistical events will have associated a value for mass-energy ( **$\alpha$ -state**) and mass-energy ( **$\beta$ -state**), the quantity of mass-energy ( **$\beta$ -state**) is linked to the dilation of the  $w$  coordinate and the contraction of the coordinate corresponding to the direction of velocity.

$p$  and  $n$  are dimensionless factors that depend on distortion of space-time, the product of both is a constant value. "State A" ( $1, n$ ) "State B" ( $p, n'$ ) with  $1 \cdot n = p \cdot n'$  then  $p = n/n' = t/t'$

The value of  $p$  for the "State A" (no velocity) is 1, the value of  $p$  for the "State B" is linked to the quantity of mass-energy ( **$\alpha$ -state**), but increasing the number of statistical events to  $n'$  instead of  $n$ , the physical system reaches the same total amount of mass-energy ( **$\alpha$ -state**)

where  $n \cdot 1 \cdot P = \lambda = n' \cdot p \cdot P$

**"State A"                      "State B"**

The kinetic energy alters the amount of mass-energy ( **$\alpha$ -state**), and that value is linked to the distortion of space-time. Considering scenarios with presence of gravitational fields, it is proposed that the curvature of space-time is linked to the alteration of mass-energy, so knowing the E.F.E. might be calculated the alteration of mass-energy and in turn the energy required for that alteration to take place, which is responsible of an additional distortion of space-time.

This way it is linked the curvature of space-time with the probability of events taking place.



## The weak equivalence principle

□The trajectory of a point mass in a gravitational field depends only on its initial position and velocity, and is independent of its composition and *structure*□

### Space-time position A

body with mass-energy  $m_1$   
body with mass-energy  $m_2$

### Space-time position B

body with mass-energy  $m_1$  and velocity  $v$   
body with mass-energy  $m_2$  and velocity  $v$

In order to keep that equality, as the value of mass-energy at space-time position B has changed,  $v$  has to change as well. So taking into account the process:

### “State A”

body with mass-energy  $m$

### “State B”

body with mass-energy  $(dt/d\tau)m$  and velocity  $v_{mod}$   
Relative to “State A”

Energy conservation would allow us to know the value of  $v_{mod}$

Below is calculated the value of  $v_{mod}$  considering different references, using conservation of energy and obtaining a result which would be a function of  $v$ . For all the references the expression is the same  $\gamma_{mod} = 1 + \gamma dt/d\tau - dt/dt$ , taking into account just the proposed effect, the expression would be invariant, but the value of  $v$  and consequently  $v_{mod}$  would depend on the reference. In fact what the expression tries to formulate is that in order to preserve the Noether Theorem, the velocity has to be modified. The kinetic energy available for the velocity term would be reduced depending on the factor  $(dt/d\tau)$ :

$(\gamma_{mod} mc^2 - mc^2) = (1/(dt/d\tau))(\gamma mc^2 - mc^2)$  that would be the same for all the references, because they agree on those values, but the value of  $v$  and consequently  $v_{mod}$  would depend on the reference.

The trajectory of a free fall object would be modified considering  $v_{mod}$  instead of  $v$ , that modified trajectory can be defined knowing  $v_{mod}$  at each space-time position, that would be the effect corresponding to the proposed process, added to that we must take into account that the trajectory whether modified or not, would depend as well on the reference of the observer, so the modified trajectory should be transformed to the reference of the observer.

Taking as reference “State A”

Mass-Energy at States A and B with reference A would be

$$E_A^A = mc^2 \quad E_B^A = (dt/d\tau)mc^2$$

The kinetic Energy of the body, now with energy  $E_B^A = (dt/d\tau)mc^2$  when it reaches the “State B” has to be the same than the kinetic energy of the body if all the Potential Energy would transform into Kinetic Energy corresponding to velocity  $v$ .

$$\begin{aligned} \gamma_{mod} (dt/d\tau)mc^2 - (dt/d\tau)mc^2 &= \gamma mc^2 - mc^2 \\ (\gamma_{mod} - 1) (dt/d\tau)mc^2 &= (\gamma - 1) mc^2 \end{aligned}$$

$$\gamma_{mod} = 1 + \gamma dt/d\tau - dt/dt$$

This expression does not depend on mass, so bodies with different mass will still have the same velocity, but  $v_{mod}$  instead of  $v$

Taking as reference “State B”

Energies at States A and B with reference B would be

$$E_{A}^B = (d\tau/dt)mc^2 \quad E_B^B = mc^2$$

Now for the observer fixed at “State B” the value of the mass-energy linked to the body when it was at the “State A” is  $E_{A}^B = (d\tau/dt)mc^2$  so if all the Potential Energy transforms into Kinetic Energy, it would obtain  $\Upsilon(d\tau/dt)mc^2 - (d\tau/dt)mc^2$

But the value of the energy at “State B” would be  $E_B^B = mc^2$  which is higher than the corresponding to  $E_{A}^B = (d\tau/dt)mc^2$  as consequence of the process  $\Upsilon_{mod} mc^2 - mc^2$

The equality between them:

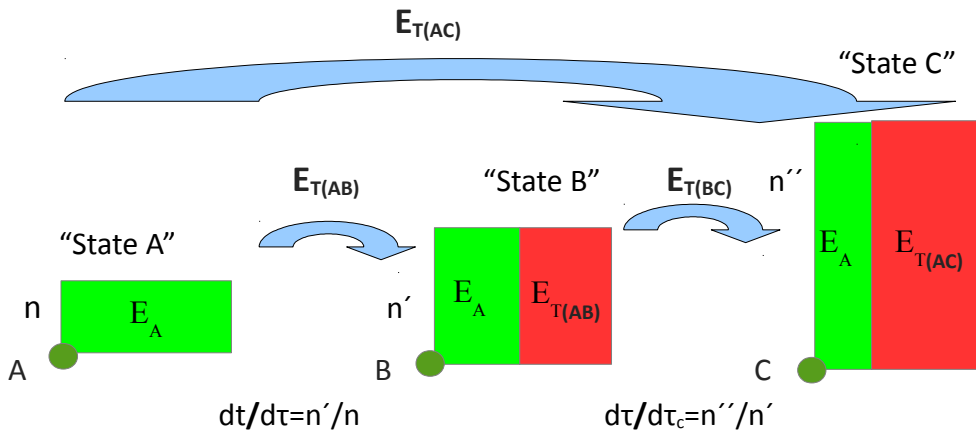
$$\Upsilon_{mod} mc^2 - mc^2 = (\Upsilon(d\tau/dt)mc^2 - (d\tau/dt)mc^2)$$

Changing the reference, changes  $(d\tau/dt)$  to  $(dt/d\tau)$  at the second term (for the observer at B), so the result is the same:  $\Upsilon_{mod} = 1 + \Upsilon dt/dt - d\tau/dt$

Discrepancy between  $\Upsilon_{mod}$  and  $\Upsilon$  is negligible insofar distortion of time does not change significantly. **It should be notice, that this discrepancy is the one accumulated during the whole of the trajectory from A to B**, it would lose kinetic energy with respect to the expected value at each space-time position, the accumulated of the whole trajectory results in the discrepancy corresponding to  $\Upsilon_{mod}$ .

**The reverse process (from B to A) has a negative value for  $E_T$ ; mass-energy value decreases at “State A” relative to the “State B” that effect is offset by increasing velocity.**

Similarly, it might be calculated the value of  $\Upsilon_{mod}$  from any other point of reference



Considering the relation between A and C:

Taking as reference the “State A”:

$$E_A^A = mc^2 \quad E_C^A = (dt/dt_c)mc^2 \quad E_{T(AC)}^A = (dt/dt_c)mc^2 - mc^2$$

$E_{T(AC)}^A$  is the value of  $E_T$  between A and C with reference A.  $E_{T(AC)}^A = -E_{T(CA)}^A$

Taking as reference the “State C”:

$$E_A^C = (d\tau_c/dt)mc^2 \quad E_C^C = mc^2 \quad E_{T(AC)}^C = mc^2 - (d\tau_c/dt)mc^2$$

Considering the relation between B and C:

proper time at B:  $dt$                       proper time at C:  $dt_c$

Taking as reference the “State B”:

$$E_B^B = mc^2 \quad E_C^B = (d\tau/d\tau_c)mc^2 \quad E_{T(BC)}^B = (d\tau/d\tau_c)mc^2 - mc^2$$

Taking as reference the “State C”:

$$E_B^C = (d\tau_c/dt)mc^2 \quad E_C^C = mc^2 \quad E_{T(BC)}^C = mc^2 - (d\tau_c/dt)mc^2$$

To Calculate the value  $\Upsilon_{\text{mod}}$  (corresponding to “State B” corresponding to the trajectory from A to B), with reference C:

$$\Upsilon_{\text{mod}} (d\tau_c/dt)mc^2 - (d\tau_c/dt)mc^2 = (\Upsilon(d\tau_c/dt)mc^2 - (d\tau_c/dt)mc^2)$$

$$\text{Obtaining the same result: } \Upsilon_{\text{mod}} = 1 + \Upsilon d\tau/dt - d\tau/dt$$

That would be the value at B, for a free fall body, passing from “State A” linked to dt, to “State B” linked to dt  $\Upsilon_{\text{mod}}(\text{AB})$  that value would be the same whatever the reference we take. The value corresponding to  $\Upsilon_{\text{mod}}(\text{AC})$  between A and C:  $\Upsilon_{\text{mod}}(\text{AC}) = 1 + \Upsilon d\tau_c/dt - d\tau_c/dt$

Considering as reference an observer at free fall with the body:

If the observer fixed at A, observed that the free fall body passes to states with lower potential energy (the proposal implies that not all of that potential energy is transformed into Kinetic Energy due to increased mass-energy of the body relative to A, so instead of v we obtain the value  $v_{\text{mod}}$ ). Now the observer linked to the free fall body observes that “State A” passes to higher potential states relative to itself, that increment has now a negative sign, during its transition reaching the negative value  $E_p(\text{B})-E_p(\text{A})$  at the “State B”, (when the observer was fixed at A the variation of potential energies was  $E_p(\text{A})-E_p(\text{B})$ ). Considering the free fall observer, a portion of the value  $E_p(\text{B})-E_p(\text{A})$  would be used to reduce the value of mass-energy from  $E_A^A = mc^2$  to  $E_A^B = (d\tau/dt)mc^2$  to offset that value, the Kinetic Energy available is reduced, so that:

$$\Upsilon_{\text{mod}} mc^2 - mc^2 = (\Upsilon(d\tau/dt)mc^2 - (d\tau/dt)mc^2)$$

- Considering the proposed process for the observer in free fall from A to B

The observer in free fall would have associated energy with reference to itself  $mc^2$  because the reference and the referenced states are the same.

The observer would experience a negative acceleration, would measure a negative acceleration linked or due to the proposed process (although that value would be negligible insofar distortion of time due to the gravitational field does not change significantly).

The energy associated with the previous reference would change, if an object is fixed at A while the observer passes from “State A” to “State B” the energy associated to the object changes from  $mc^2$ , when the observer was at “State A” to  $(d\tau/dt)mc^2$ , when the observer reaches the “State B”.

- Considering the proposed process for the observer passing from B to A

The observer would have associated energy with reference to itself  $mc^2$  because the reference and the referenced states are the same.

The observer would experience a positive acceleration, the observer would measure a positive acceleration linked or due to the proposed process (although that value would be negligible insofar distortion of time does not change significantly).

The energy associated with the previous reference would change, if an object is fixed at B while the observer passes from “State B” to “State A” the energy associated to the object changes from  $mc^2$ , when the observer was at “State B” to  $(dt/d\tau)mc^2$ , when the observer reaches the “State A”.

- Considering an hypothetically “pure Special Relativity scenario” for the observer passing from “State A” to “State B”.

The observer in transition from a “State A” to a “State B” with relative velocity v with respect of A, would have associated energy with reference to itself  $mc^2$  because the reference and the referenced states are the same.

The observer would experience a positive acceleration.

The energy associated with the previous reference would change, if an object is fixed at A while the observer passes from “State A” to “State B” the energy associated to the object changes from  $mc^2$ , when the observer was at “State A” to  $\Upsilon mc^2$ , when the observer reaches the “State B”.

The equations proposed represent an additional condition.

The equation of motion (if there is no external force):

$$m(d^2x^\mu/d\tau^2) = f^\mu - m \Gamma^\mu_{\nu\lambda} (dx^\nu/d\tau)(dx^\lambda/d\tau)$$

That is the equation for the geodesic in the curved space-time

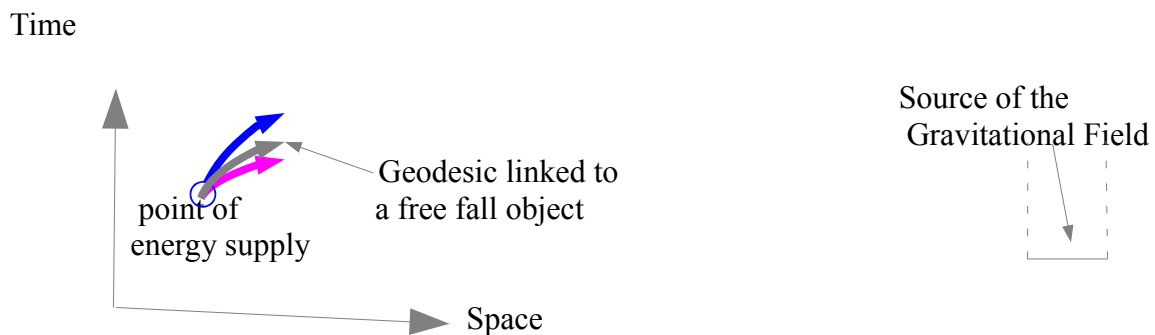
If we add the additional condition proposed, then the system would not correspond to the equation of motion with no external force.

We know the geodesic that the particle or the body would follow if there is no external force, to know the trajectory of the particle adding the additional condition, we have to add the force corresponding to each space-time position of the trajectory (the equations proposed allow us to know that force corresponding to each space-time position) So it has to be applied the Energy term (opposed to the free fall) that takes place between the initial State A and all the space-time positions in its trajectory until reaching the final State B. Knowing the energy required by the proposed process.

The proposal implies that a free fall object would not follow the geodesic that results after apply the Euler-Lagrange equations to the Einstein field equations. The discrepancy would be an extremely small one, insofar the distortion  $dt/d\tau$  does not reach a significant value. The process would require energy and that would be at the expense of Kinetic energy. In other words, what in the officially accepted model corresponds to a scenario of free fall, would not exist as such since that body would have its trajectory in space-time forced by the proposed effect.

### Scenario 1

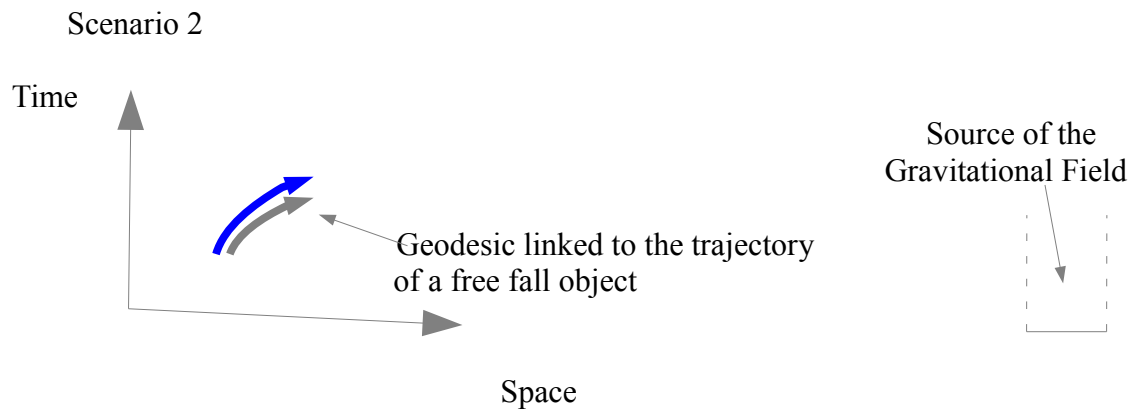
Space-time curvature that affects the three spatial coordinates and the temporal coordinate



Using a simplified scheme (to contrast it against the second scenario), the path that would follow a body affected by a gravitational well is represented schematically.

Gray color represents the trajectory that would follow a body in free fall, that would be linked to the geodesic of the space-time curved by effect of the gravity. The blue and magenta lines correspond to states in which energy has been supplied (in the scheme it has been done at a certain point, from which it diverges from the path of the body in free fall) for or against the gravitational effect (Although according to the scheme does not get to overcome the gravitational effect) what forces the body to leave the initial trajectory of the geodesic linked to the body in free fall.

If the source of the gravitational well corresponds to a sphere not charged electrically and not rotating, with uniform mass distribution, considering the case of a black hole, the body in free fall, when arriving at the event horizon, at that moment all the future trajectories of that body point towards the interior of the black hole, and would inexorably be directed theoretically to the inside of the black hole ending in a singularity.



Considering scenario 2, the evolution of a free-fall body would not be the same as in scenario 1, but would follow the trajectory represented by the blue line rather than the gray line (this gray line represents the path that in theory would have the body in free fall). Since now the body does not behave properly as a body in free fall, as its evolution would be forced by the effect corresponding to the process that is associated with the distortion of space-time linked to gravity. That is to say, the phenomenon that is understood as free fall would be affected by this effect and the body instead of following the gray trajectory, would follow a trajectory like the one corresponding to a case of forced fall.

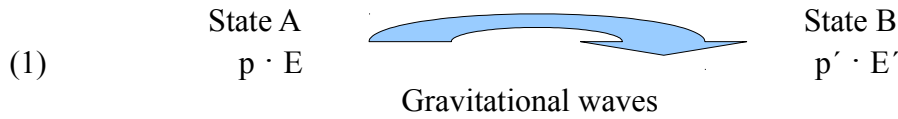
We could know the evolution of the body, if we calculate the effect of the proposed process at each point of the trajectory. We could also make the body follow the path corresponding to the body in free fall (as defined by the officially accepted model, the gray line), if at each point of the path is compensated the energy corresponding to that effect.

Both paths converge at a distance from the source of the gravitational field to which there is no gravitational effect (theoretically at infinite distance from the source). Under usual conditions, weak gravitational fields, the discrepancy between the two is negligible and increases as the distortion produced by the gravitational field increases. For the case of the black hole described in scenario 1, as the body approaches the event horizon, the energy needed to bring the body to the evolution of the geodesic path for the body in Free fall (the corresponding to the gray line) increases until reaching an infinite value at the black hole event horizon.

So, in summary, while scenario 1, the free-fall body evolves following the path marked by the geodesic, at scenario 2, by adding a condition to the equations, the body supposedly in free fall, does not behave as such, as the added effect causes its trajectory to be altered.

The proposed process Schematically:

- General Relativity (Gravitational field)



Notation:  $E'$  energy at State B related to A;  $E' = E_B = E_A + E_T = E^A_B = (dt/d\tau)mc^2$

The change of mass-energy from "State A" to "State B" will be caused by gravitational waves, its effect will depend on the location of mass-energy relative to the source of the gravitational field.

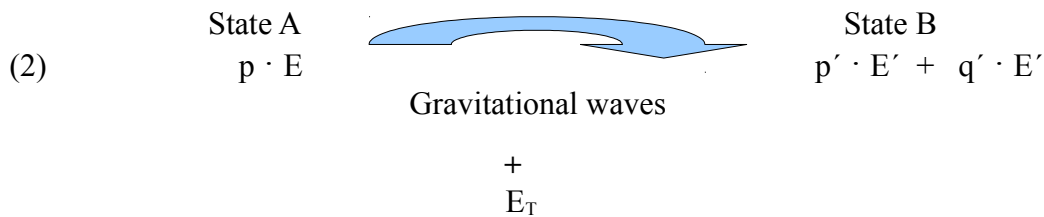
$E=mc^2$  being "State A" the reference, then  $p=1$ , if we take as reference the place where the gravitational field has no effect (theoretically at infinite distance with respect to the source) its temporal coordinate will correspond to  $dt$ . Taking as generic value for state B, the proper time  $d\tau$ , values:

$$p' = d\tau/dt \quad \text{and} \quad E' = (dt/d\tau) mc^2$$

"State A" related to A;  $p \cdot E = mc^2$

"State B" related to A  $p' \cdot E'$  would be  $mc^2$  as well, with the distinction that there would be a different distortion of space-time at B comparing to A, as established by the field equations.

This result would correspond to the one obtained by the officially accepted model. The hypothesis proposed nevertheless leads to the result set out at (2) which is discussed below.



Considering this scenario, changing the state from "A" to "B", in addition to gravitational waves, will require the energy input with value  $E_T$  and at the final state, we would have, besides the initial value of the mass-energy, the value corresponding to the energy added.

$$q' \text{ would be } 1 - d\tau/dt \quad \text{and} \quad E_T = E'_q = q' \cdot E' = (dt/d\tau) mc^2 - mc^2$$

$$E'_p = p' \cdot E' = mc^2$$

$$E' = p' \cdot E' + q' \cdot E' = (dt/d\tau) mc^2 \text{ with reference A (That is to say } E^A_B = (dt/d\tau)mc^2 \text{ )}$$

And with the distortion of space-time as established by the field equations, but needing to add the additional condition corresponding to the proposed process.

It is wellworth noticing that the process as described is similar to the phenomenon corresponding to the photoelectric effect, where we have:

Photons interact with electrons, part of the energy is absorbed by the process and the rest would go to kinetic energy.

Hypothesis: Gravitational waves would interact with mass-energy, part of the energy would be absorbed by the process ( $E_T = E'_q$ ) and the rest would go to kinetic energy.

But there is another analogy or coincidence, since the energies linked to the relative velocity, would follow the same pattern as the one proposed by the hypothesis.

- Special Relativity (relativistic velocity  $v$  at "State B" with refeence A, hypothetically in absence of gravitational fields)

$$(3) \quad \begin{array}{ccc} \text{State A} & \xrightarrow{E_T} & \text{State B} \\ p \cdot E & & p' \cdot E' + q' \cdot E' \\ & & E_T \end{array}$$

This case  $E_T$  would correspond to the Kinetic energy (energy corresponds to the Kinetic energy and is already taken into account in the Relativity model)

$$E = mc^2$$

$$E' = \gamma mc^2$$

$$p = 1 \quad p' = 1/\gamma \quad q' = 1 - 1/\gamma$$

$$p' \cdot E' + q' \cdot E' = \gamma mc^2 \quad \text{with reference A}$$

As it is known, by considering expression (3) if the relativistic velocity  $v$  at "State B" relative to "State A" is equal to the speed of light  $c$ , then an infinite amount of energy is required to pass the mass-energy from "State A" to "State B".

If we now analyze at expression (2), if "State B" corresponds to the event horizon of a black hole, then an infinite amount of energy would be required to pass the mass-energy from "State A" to "State B".

### Special Relativity

$$v = x (l_p/t_p)$$

$$x < 1$$

$x = 1$  that state corresponds to a "saturated Physical System" requires an infinite amount of energy.

### General Relativity

Considering the Schwarzschild metric for the vacuum solution of the field outside the homogeneous sphere, uncharged, non rotating.

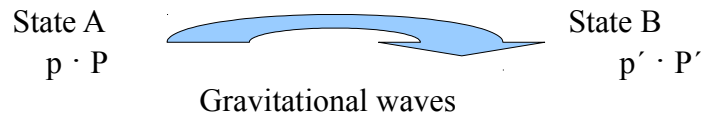
$$2GM/c^2r = (2M/m_p)/(r/l_p)$$

$$\text{with } m_p = \sqrt{hc/2\Pi G}; \quad l_p = \sqrt{hG/2\Pi c^2} \quad \text{then } l_p/m_p = G/c^2$$

$$(2M/m_p) < (r/l_p) \quad \text{or} \quad (2M/r) < (m_p/l_p)$$

$(2M/r) = (m_p/l_p)$  that state corresponds to a "saturated Physical System" requires an infinite amount of energy for a body to reach the event horizon of a black hole event, because the value of the energy required by the proposed process ( $(dt/d\tau) mc^2$ ) to reach that state (corresponding to the event horizon) has an infinite value.

## PROBABILISTIC APPROACH



If  $E$  represents mass-energy, now  $P$  represents the probability that a given event will take place. As example, a particular phenomenon, the decay of a radioactive material, is analyzed. This phenomenon is probabilistically characterized by the Poisson distribution.  $\mathcal{P}(\lambda, x)$  where  $\lambda$  Represents the frequency of occurrence of a given radioactive material decaying and  $x$  would represent the amount of radioactive material that decays. this way  $\mathcal{P}(\lambda, x_1)$  Would give us the probability that a given amount of material  $x_1$  will decay after a stipulated period of time.

The Poisson distribution is related to another discrete probability distribution, the binomial distribution. So if we have  $n$  statistical tests, each of them with a linked probability  $p \cdot P$  that a certain event takes place (taking into account the example, the event would correspond to the decay of the radioactive material), fulfilling the following conditions:

$0 < p \cdot P \ll 1$  very small probability of success.

$n \uparrow \uparrow$  very high number of statistical tests.

$n \cdot p \cdot P = \lambda$  The product of the number of statistical trials multiplied by the probability associated with each of the trials is equal to the frequency of occurrence  $\lambda$

If these three conditions are met, both distributions give very similar values, in fact at the limit when  $n \rightarrow \infty$  are equivalent ones.

This leads to the proposal that this phenomenon might be linked to the occurrence of statistical tests each of them with probability  $p \cdot P$  to be successful.

Analyzing the components of the expression  $n \cdot p \cdot P = \lambda$  (at the end we would have a value or final result set, but would be the result of conjugating various effects).

$P$  would be linked to the probability of this event taking place, if we were able to vary the value of  $P$  to  $P'$  but keeping  $n \cdot p$  as a constant value, then  $\lambda$  value would change to  $\lambda' = n \cdot p \cdot P'$ .

On the other hand,  $p$  and  $n$  would be interrelated, where  $p$  would act as a distribution factor of the probability  $P$  at each trial. Thus if we keep  $P$  constant and what varies is  $p$  to  $p'$  (this value being less than  $p$ ), then the value of  $p' \cdot P$  at each of the trials is smaller, than that we had with  $p \cdot P$  but If we increase the number of trials so that  $n' \cdot p' \cdot P = n \cdot p \cdot P$  then  $\lambda$  would remain a constant value.

The third possible scenario is to change  $P$  and change  $p$  and  $n$  as well (the variation of  $p$  and  $n$  would always go together, what decreases one, increases the other).



All this leads to the proposal that gravitational waves interact with mass-energy by altering the value of the factor  $p$  and consequently the value of  $n$ . The alteration of  $p$  would be linked to the distortion of the space, whereas the variation of  $n$  would be linked to the distortion of the time.

Modern science confers to all the fundamental forces except for gravity a probabilistic approach; gravity causes space-time to be distorted and it is done in such a way that what is contracted space, at the same proportion time expands. The approach presented here, confers a probabilistic approach to the gravitational force, where the distortion of space-time is a direct consequence and there is also an inverse proportion between the distortion of space and the distortion of time.

The factor  $p$ , called here a distribution factor, is the one that would affect the energies, in the section related to energies, it is the factor that relates the reference state (State A) to the referenced state (State B)

As shown for the particular phenomenon of decay of radioactive material, there would be an imbalance between the number of tests  $n$  and the time elapsed

If we consider the same value of  $n$  at A and at B, while  $dt$  is time elapsed at A, then at B with reference A it will have elapsed  $dt$ . The variation of the  $p$  factor would depend on the ratio  $dt/dt$ .

The above analysis corresponds to a particular phenomenon, to extrapolate it to the general behavior of nature, it would be necessary to extend it to all phenomena that take place at Quantum Mechanics.

Should be fulfilled:  $n' \cdot p' \cdot P' = n \cdot p \cdot P'$

Where the variation of  $n \cdot p$  to  $n' \cdot p'$  should be linked to spacetime distortion.

Noticing that the example analyzed is a very particular case where the probability  $P$  in each of the tests remains constant.

On the other hand, the  $p$  factor, which is related to time distortion, should vary with the coordinate axes as established by the field equations while maintaining that the distortion of the space is inversely proportional to the distortion of time. Thus, for the Schwarzschild solution external to the sphere, considering the spherical coordinates,  $p$  would vary with the Schwarzschild radial coordinate and would remain constant for the other two spatial-type coordinates. The value of  $p$ , taking as reference A (taking as reference a state where it is not affected by the gravitational field which would have associated  $dt$ ) changes with the radial coordinate, taking as value  $dt/dt$ , reaching a value Null at the event horizon of a black hole.

Taking into account the proposed approach, there would be an alternative variant. The imbalance between the number of tests and the time elapsed in addition to the fact that the distribution factor decreases the value of the probability assigned to each test, would correspond to the physical phenomenon of gravitational waves altering the mass-energy, and the value of the  $p$  factor. However, something more may happen. The hypothesis proposed raises a scenario where gravitational waves interact with mass-energy and energy  $q' \cdot P'$  is required, as a result instead of having  $p \cdot P$  we would have  $p' \cdot P' + q' \cdot P'$

Now, instead of diluting the probability at each of those  $n'$  tests (as the factor  $p'$  decreases its value), being  $q' = 1 - p'$ , this way the total value of the probability remains a constant value after adding the term  $q' \cdot P'$  ( $P'$  the generic value of the probability linked to the altered state), that is to say the value changes from  $P$  to  $P'$ .

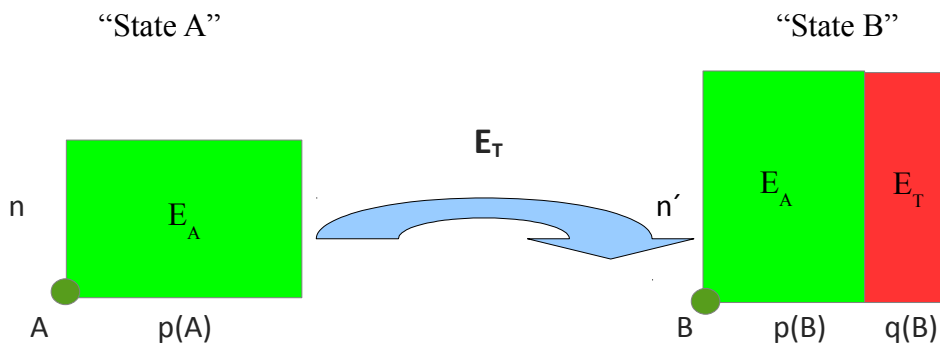
It should be noticed that considering the probabilistic approach,  $P'$  represents a variation of the probability, meanwhile when dealing with energies,  $E'$  represents the value  $E_B = E_A + E_T$

However, adding the term  $q' \cdot P'$  implies or requires an input of energy, which corresponds to the value  $E_T$

This would correspond to a scenario where occur phenomena of the type

Gravitational waves interact with mass-energy, part of the energy would be consumed in the process ( $E_T$ ) and the rest of the energy would be used as Kinetic.

**Considering energies it was used the following scheme:**



the probabilistic approach we have at "State B"  $p' \cdot P' + q' \cdot P'$

where  $q' \cdot P'$  would correspond to the probability associated with the " **$\beta$  state**" of mass-energy while  $p' \cdot P'$  would correspond to the probability associated with the " **$\alpha$  state**".

Mass-energy passing from " **$\alpha$  state**" to " **$\beta$  state**" requires the contribution of  $E_T$

What has been generically defined as "State A" and "State B" correspond to different values for those " **$\alpha$  state**" and " **$\beta$  state**" (the amount of energy at " **$\beta$  state**" increases as the distortion of time increases which generically is characterized by the proper time  $d\tau$  and generically has been used the notation "State B")

Considering Special Relativity, the different states would correspond to the relativistic velocities, the relation between times corresponds to the inverse of the Lorentz factor  $\Upsilon$  and the  $E_T$  value coincides with the kinetic energy.

If the proposed process depends on the interaction between gravitational waves and mass-energy. That interaction is affected by the velocity of mass-energy, so that, bodies with different velocities would have associated different states, those different states depend on the inverse of the Lorentz factor  $\Upsilon$  (the hypothetically pure Special Relativity from a practical point of view would not change), if we have the presence of a gravitational field and a relative velocity, there is a combination of both,

## METHODOLOGY

Note: this methodology section would not be part of the paper. The proposal is presented as a hypothesis, adding a new condition, giving different results comparing to the officially accepted model, the hypothesis can not be deduced from the established model, requires to add the additional condition, so this methodology section has not the goal to deduce the hypothesis, but to define a criteria (which differs from the established one) that drives to the proposal.

The proposal adds a new condition to the established model. It will be considered two approaches, an energy related approach and a probabilistic approach, analyzing the consequences of the additional condition on both perspectives. Considering the probabilistic approach, following will be defined an example to better understand the proposal and its implications.

Considering the Schwarzschild metric and two space-time positions corresponding to that metric, one linked to a space-time position where the gravitational field is negligible, linking it to time  $dt$ , and the other one with proper time  $d\tau$ , denoting them "State A" and "State B" respectively. Time runs at more slowly rate at "State B" (closer to the source of gravity) than at "State A" and the metric might be used in order to obtain the ratio between both  $dt/d\tau$ .



Representing quite an extreme curved space-time scenario (to see the different values at the scheme (1)) Note: red text would not be included at the paper, is just to clarify some topics better. where  $dt/d\tau=3$ , time runs three times more slowly at B than at A.

Considering now the Poisson distribution  $\mathcal{P}(\lambda, x)$  where  $\lambda$  represents the frequency of occurrence of a given event and  $x$  would represent the amount or number of events in an interval.

The Poisson distribution is related to another discrete probability distribution, the binomial distribution. Considering  $n$  statistical trials, each of them with probability  $p \cdot P$  that a certain event takes place, fulfilling the following conditions:

$0 < p \cdot P \ll 1$  very small probability of success.

$n \uparrow \uparrow$  very high number of statistical trials.

$n \cdot p \cdot P = \lambda$  The product of the number of statistical trials multiplied by the probability associated with each of the trials is equal to the frequency of occurrence  $\lambda$

If these three conditions are met, both distributions give very similar values, at the limit when  $n \rightarrow \infty$  are equivalent ones.

This leads to a proposal relating both so that at the Physical System is taking place the occurrence of statistical trials or tests each one with probability  $p \cdot P$  of being successful.

$P$  representing the probability of an event taking place at a particular trial

$p$  is a factor (which depends on distortion of time) that modifies the value of  $P$

$p$  and  $n$  are dimensionless factors that depend on distortion of time, the product of both is a constant value. At "State A" ( $p, n$ ) At "State B" ( $p', n'$ ) with  $p \cdot n = p' \cdot n'$   
The value of  $n$  is linked to distortion of time, so that the higher the distortion of time, the higher the value of  $n$ .

The Physical System will comply  $dt \cdot n = d\tau \cdot n'$

That would be an invariant value for all the corresponding states whatever the distortion of time.

Considering the number of statistical trials, it is obtained the inverse relation:



(Now the lines represent a different feature, not the distortion of time but the ratio of the number of trials), keeping the metric where time runs more slowly at B. Those two features: number of statistical trials and time distortion, have an inverse relation.

“State B” is linked to three times the number of statistical trials in relation to “State A” So that if clock runs three seconds at “State A” while one at B, each second at “State B” is configured by three times the number of trials corresponding to one second for the “State A”

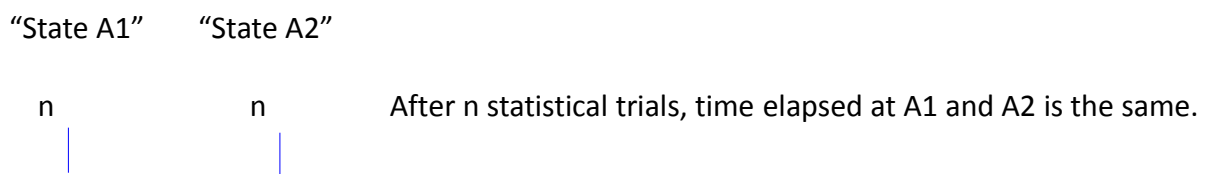
Combining (1) and (2) results the invariant value:  $dt \cdot n = d\tau \cdot n'$



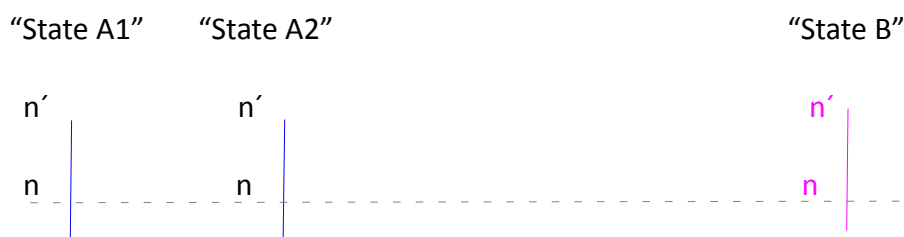
**The number of trials corresponding to the interval  $dt$  at “State A” is equal to the number of trials corresponding to  $d\tau$  at “State B”.**

Considering the Physical System, it would take place the same number of trials, but while that number of trials corresponds to  $dt$  time at A, it will correspond to  $d\tau$  at B. For example, if  $n$  is the number of trials corresponding to one second at “State A”, after  $3n$  trials the clock has run three seconds at A, while one at B. These features hint that the Physical System might behave taking the number of trials as an invariant value.

Two space-time positions denoted as States A1 and A2 where the gravitational field is negligible (with no relative velocity) clock will run at the same rate.



Adding to the scheme “State B” with proper time  $d\tau$



After  $n'$  statistical trials, time corresponding to A1 and A2 is  $dt$  while at B is  $d\tau$

(After  $n$  statistical trials clock will run one second at A1 and A2 but  $1/3$  of a second at B, after  $n'$  trials with  $n'=3n$  time will run three seconds at A1 and A2 while one second at B).

Previously has been stated that the number of trials taking place at the Physical System might behave as an invariant value. But that behaviour would have some problems.

I) Considering the same number of trials on both states:

mass-energy at "State B" =  $(p'/p) * \text{mass-energy at "State A"}$

The value of the probability and consequently the mass-energy corresponding to the same number of trials would be reduced at "State B". Because  $p \cdot n = p' \cdot n'$

II) If "State B" corresponds to the event horizon, then then number of trials would reach an infinite value. Because  $n'/n = dt/d\tau$  time elapsed at "State A" is infinite comparing to B

Those two problems might be solved by taking into account a process linked to gravity as it will be defined. The introduction of the process is the hypothesis proposed at this paper, which implies an additional condition to the officially accepted model.

Taking as reference "State A", the process between A and B: "State A"  $\rightarrow$  "State B"  
Mass-Energy  $mc^2$  ( $\alpha$ -state) **interaction with GW + Energy  $((dt/d\tau)mc^2 - mc^2)$**   $\rightarrow$  Mass-Energy ( $\alpha$ -state) + Mass-Energy ( $\beta$ -state) =  $(dt/d\tau)mc^2$

Mass-energy interacts with Gravitational Waves, obtaining as result Mass-Energy ( $\alpha$ -state) + Mass-Energy ( $\beta$ -state). **That process requires energy  $((dt/d\tau)mc^2 - mc^2)$  to take place.**

Mass-Energy at  $\beta$ -state depends on the factor  $q' = (1 - p')$ . where  $p' = dt/d\tau$ .

considering the probabilistic approach:

$\alpha$ -state corresponds to  $p'P'$  being  $P'$  the probability of a given event taking place. The interaction with gravitational waves generates the  $\beta$ -state ( $q'P'$ ), that process requires energy  $((dt/d\tau)mc^2 - mc^2)$ , depending on the factor  $q'$  the probability at  $\beta$ -state would increase and consequently the amount of mass-energy at the  $\beta$ -state would increase, that is what is called generically "State B" which is characterized by the quantity of mass-energy at  $\beta$ -state

Relative velocities will affect the process as well, so that bodies situated at the same space-time position of the gravitational field but with different relative velocities would correspond to different states, the energy differentiating those states corresponds to the Kinetic energy (note: taking into account that velocity is not invariant, as it depends on the reference).

### Concerning Special Relativity

Taking as reference State A, the process between A and B: "State A" → "State B"

Mass-Energy  $mc^2$  ( $\alpha$ -state) + Energy ( $\gamma mc^2 - mc^2$ ) → Mass-Energy ( $\alpha$ -state) + Mass-Energy ( $\beta$ -state)  
=  $\gamma mc^2$  Being  $\gamma$  The Lorentz factor

Concerning Special Relativity (hypothetically in the absence of GF, this expression is an idealistic scenario because bodies are immersed on gravitational fields and would be required an infinite distance from the source to avoid being affected, so the process is taking place even if it is a weak interaction, different velocities would alter that interaction with the kinetic energy being associated to those different states, so the meaning of hypothetically in the absence of gravitational fields means that the focus is now on analyzing how the relative velocity produces a change of the State):

"State A" reference

"State B" relative velocity (repect to A)=0 then  $p'=1$ ;  $q'=0$

Note: although it is used the same notation ( $p', q'$ ) now they are referred to the phenomenon corresponding to relative velocity, now those factors will define the distinction between states corresponding to relative velocity.

"State B" relative velocity (repect to A)= $v$  then  $p'=1/\gamma$ ;  $q'=1-1/\gamma$

Energy between both states (energy required to pass from "State A" to "State B")=  $\gamma mc^2 - mc^2$

values of  $p'$  between 0 and 1 ;  $p'=1$  when relative velocity=0 and  $p'=0$  when relative velocity= $c$

values of  $q'$  between 0 and 1 ;  $q'=0$  when relative velocity=0 and  $q'=1$  when relative velocity= $c$

Concerning General Relativity (hypothetically with no relative velocities between states):

"State A" as reference, with associated time  $dt$

"State B" proper time  $d\tau$  (repect to A) if State A and B are the same then  $p'=1$ ;  $q'=0$

"State B" proper time  $d\tau$  (repect to A)  $p'=d\tau/dt$ ;  $q'=1- d\tau/dt$

Energy between both states (energy required to pass from State A to State B)=  $(dt/d\tau)mc^2 - mc^2$

values of  $p'$  between 0 and 1 ;  $p'=1$  when State A and B are the same and  $p'=0$  when state B corresponds to the event horizon

values of  $q'$  between 0 and 1 ;  $q'=0$  when State A and B are the same and  $q'=1$  when state B corresponds to the event horizon

Considering a combination of both GR and SR

"State A" reference time  $dt$

"State B" with proper time  $d\tau$  and velocity  $v$  relative to State A

Note: taking into account that Kinetic energy is not invariant, as it depends on the reference.

If we consider:

"State A" as reference, with associated time  $dt$

"State B1" with proper time  $d\tau$  and velocity  $v_1$  relative to State A

We would have distortion of time between both states due to GR and SR with velocity  $v_1$

If we consider:

"State A" as reference, with associated time  $dt$

"State B2" with proper time  $d\tau$  and velocity  $v_2$  ( $<v_1$ ) relative to State A

We would have distortion of time between both states due to GR and SR with velocity  $v_2$

The distinction between both B1 and B2 is the energy required to decelerate the object from  $v_1$  to  $v_2$

The proposal implies that the value of  $(dt/d\tau)mc^2 - mc^2$  **corresponds to a negative acceleration in the trajectory from A to B, so instead of velocity  $v_1$  would be  $v_2$**

That is why it is taken this approach, in order to get a better understanding of the phenomenon. This approach is useful to obtain certain data, for example we can calculate the velocity at State B or time distortions. We could force the object to follow the geodesic if we apply the energy required by the process, between the initial position and each space-time position.

The proposed process follows a pattern similar to the phenomenon corresponding to the kinetic energy.

Concerning SR:

Reference "State A" and "State B" with velocity  $v$  relative to "State A".

Mass-energy of a body passing from A to B increases by the factor  $\Upsilon$  (relative to an observer at State A), that increase is at the expense of kinetic energy.

Concerning the proposed process:

Reference "State A" ( $dt$ ) and "State B" with proper time  $d\tau$ .

Mass-energy of a body passing from A to B increases by the factor  $(dt/d\tau)$  (relative to an observer at State A), that increase is at the expense of the velocity term of the kinetic energy.

Taking up problems I) and II)

I)  $p \cdot P' = p' \cdot P' + q' \cdot P' = (dt/d\tau)mc^2$  The hypothesis adds the term  $q' \cdot P'$

The value of the probability and consequently the mass-energy corresponding to the same number of trials is the same in both States A and B.

If the Physical System behaves as proposed would satisfy two demands

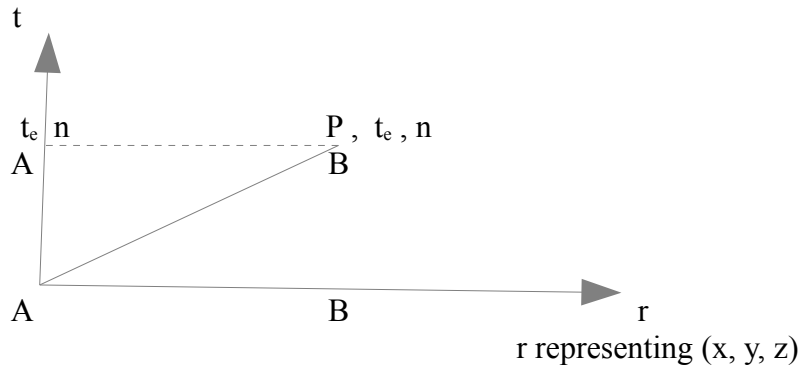
The corresponding to Time, preserving the Noether Theorem, conservation of energy with time. The counterpart of the energy  $((dt/d\tau)mc^2 - mc^2)$  would account for that.

The corresponding to number of trials as an invariant reference. Mass-Energy ( $\beta$ -state) would account for that.

II) If "State B" corresponds to the event horizon, then the energy required to reach that state would be infinite,  $((dt/d\tau)mc^2 - mc^2)$  where  $(dt/d\tau)$  representing the ratio between time elapsed at A and B takes an infinite value.

$$\psi(r, t)$$

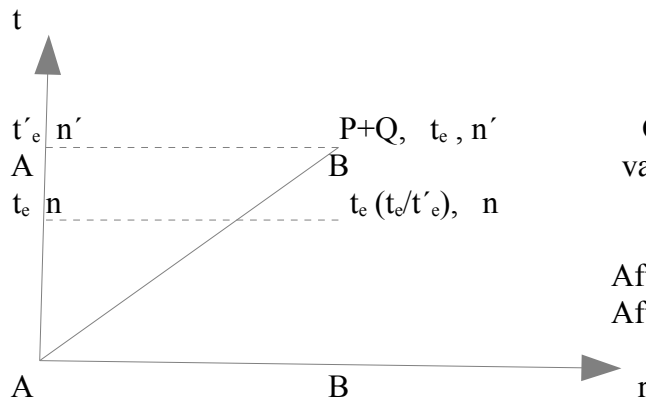
Absence of GF and no relative velocity between A and B



Observer at reference A

GF and no relative velocity between A and B

just taking into account space-time distortion effect and not the effect on values of the wave function.



$Q = (t'_e / t_e)P - P$  in order to keep the same value of probability per number of trials  $n$

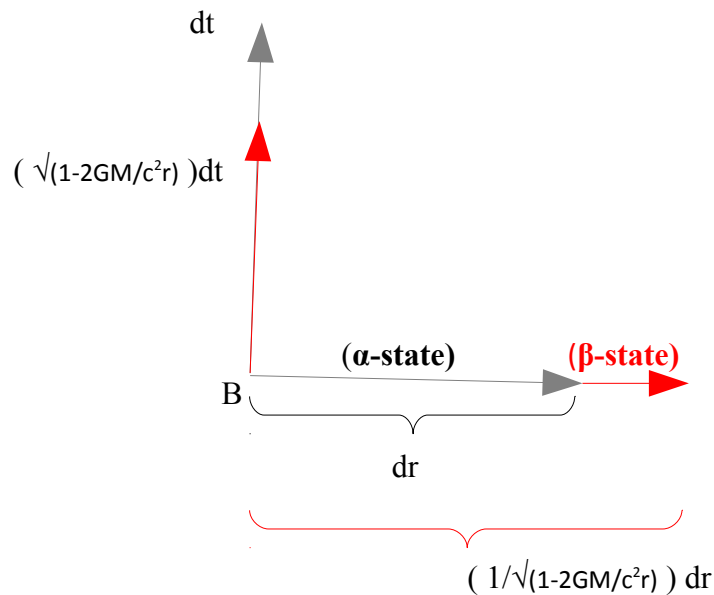
After  $n'$  Clock has run  $t'_e$  at A but  $t_e$  at B

After  $n$  Clock has run  $t_e$  at A but  $t_e (t_e/t'_e)$  at B



The effect produced by the gravitational field affects space-time position B and locally to its space-time surroundings, as defined by the Einstein Field Equations.

Considering the Schwarzschild metric



$r$  representing now the radial coordinate, distortion of time coordinate corresponding to clock running more slowly at B and space contraction on radial direction

Scheme represents the proportion between  $\alpha$ -state and  $\beta$ -state, meaning that interaction of gravitational waves and mass-energy on the radial space direction produces as result  $\beta$ -state of mass-energy on that proportion, which corresponds to the distortion of space on that direction. Inversely to the distortion of space, is the distortion of time. So knowing the value of the distortion of time is possible to know the value of mass-energy at  $\beta$ -state linked to the space-time position B (or "State B").

Locally, for the radial coordinate on the surroundings of B, the chances of a quantum of energy to transform into  $\beta$ -state will increase, the higher the value of  $2GM/c^2R$ . On the other hand, time-like coordinate is distorted as previously defined. Similarly, this process will take place on a general expression of space-time curvature considering gravitational fields and energies corresponding to General Relativity. **The Schwarzschild metric does not include space-time like coordinates, metrics with space-time distortion including them, have to be taken into account to obtain the value of the energy linked to the process, which ultimately will depend on the corresponding distortion of space.**

The probabilistic approach as defined, establishes a discrete quantity for space-time distortion, if the interaction between gravitational waves and mass-energy giving as result  $\beta$ -state of mass-energy is quantized and the distortion of space corresponds to that proportion.

Another quantum like property to take into account is the Heisenberg's uncertainty principle, because now the energy required to confine the particle on a region of space-time increases with the value of the energy linked to the proposed process.

If we consider a sphere, and uniformly fill that sphere with mass, the energy required to confine the particle up to the surface of the sphere increases reaching an infinite value when the mass of the sphere corresponds to Schwarzschild value and the surface corresponds to the event horizon of a black-hole.

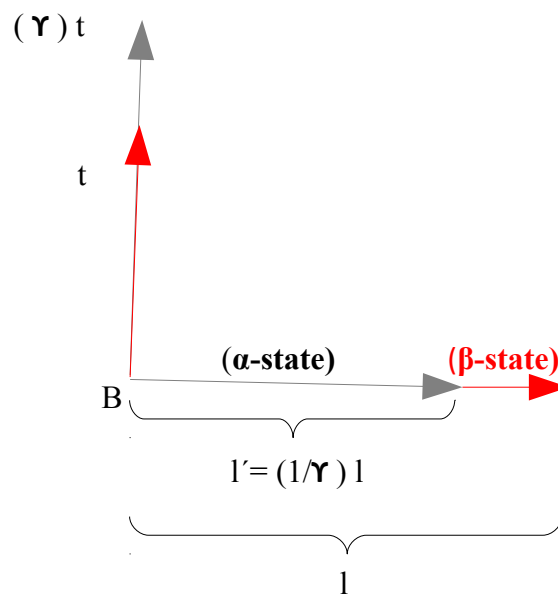
So the Einstein Field Equations give us quantum like information about the interactions taking place locally on the surroundings of any given space-time position and the energy required for that process to take place and consequently the energy linked to that space-time position.

The process, in order to take place as defined, requires energy, and it has to be taken into account the conundrum of General Relativity and Special Relativity.

## Special Relativity

Considering the same interval of time, and the same space-time position B (same “State B” of the Gravitational Field), bodies with different relative velocities would suffer a different interaction process between gravitational waves and mass-energy, corresponding to different states. Note: the pure idealistic “Special Relativity” scenario in absence of gravitational fields is not a realistic scenario, bodies are immersed in gravitational fields, even in an area of extremely low effect produced by gravitational fields, the proposed process would take place, bodies with different relative velocities would affect the interactions and correspond to different states.

Considering the same interval of time, or a particular instant of time, interactions will differ depending on the velocity of the body.



Velocity on x direction.

A case of negligible distortion of space-time corresponding to General Relativity, the energy linked to the proposed process is the Kinetic Energy, so that energy is already taken into account as a body passes from “State A” to “State B”.

$$Q = E_K = \gamma mc^2 - mc^2$$

Q energy linked to the proposed process concerning Special Relativity

When General Relativity has a significant effect and it is needed to take into account both effects, for example a free fall body at space-time position B relative to another position A, between them there is different space-time curvature due to General Relativity and Special Relativity in this case the energy required for the process corresponding to General Relativity will be at the expense of the velocity, so that the energy available will satisfy the process corresponding to General and Special Relativity.

## CONCLUSIONS

Quantum mechanics is characterized by processes where particles interact, passing from an “ $\alpha$  state” to a “ $\beta$  state”.

Considering the phenomenon corresponding to the photoelectric effect :

Photons interact with electrons, part of the energy is absorbed by the process and the rest goes to kinetic energy.

The hypothesis at this paper proposes that the interaction between Gravitational waves and mass-energy requires a contribution of energy,

The proposal defines a process linked to gravity where mass-energy would be affected changing its state from A to B. Gravitational waves would interact with mass-energy, part of the energy is absorbed by the process and the rest goes to the velocity term of the kinetic energy.

Factor  $p$  (which depends on time distortion) relates “State B” to “State A”

The energies linked to Special Relativity and General Relativity would follow a similar pattern:  $E_B = E_A + E_T$

$$E_A = mc^2 \qquad E_T = (1/p)mc^2 - mc^2 \qquad E_B = (1/p)mc^2$$

Considering the process linked to gravitational fields, both observers do agree on the values of  $dt$  and  $d\tau$  corresponding to “State A” and “State B” respectively.

The value of the energy at B with reference A:  $E_B^A = (dt/d\tau)mc^2$

The value of the energy at A with reference B:  $E_A^B = (d\tau/dt)mc^2$

The process linked to gravitational fields is endothermic from A to B and exothermic from B to A. the endothermic process would be at the expense of reducing velocity of the kinetic energy, meanwhile the reverse process would be an exothermic one, increasing the velocity of the body that passes from “State B” to “State A”. Expansive scenarios would show velocities higher than expected.

A free fall body follows a trajectory in a curved space-time framework towards the source of the gravitational field. The trajectory is defined by applying the Euler-Lagrange equations to Einstein field equations. The effect proposed implies that the body would not follow the geodesic of a “free fall body” what we consider as a “free fall body” in fact would be forced by the effect proposed. The effect is negligible insofar the distortion of time does not reach a significant value. Considering the officially accepted model, nothing prevents from a free fall body reaching the horizon event and inevitably ending in a Singularity. The proposal forces the body out of that geodesic, the energy required to follow that geodesic at the event horizon would be infinite.

The proposal allows to mathematically calculate the discrepancy between both scenarios.

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