GENERALIZED NEUTROSOPHIC HYPERGRAPHS

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ABSTRACT. The generalization of the concept of single valued neutrosophic hypergraph (SVNHG), strong SVNHG by considering SVN-Vertex instead of crisp vertex set and interrelations between SVN-Vertices and family of SVN-Edges are introduced here. A few properties and operations of such graphs are established here.

Keywords: Generalized SVNHG, generalized strong SVNHG, generalized SVN sub hypergraph, spanning generalized SVN sub-hyper graph.

1. INTRODUCTION

Neutrosopic sets were introduced by Smarandache [2] which are the generalization of fuzzy sets and intuitionistic fuzzy sets. The Neutrosophic sets have many applications in medical, management sciences, life sciences and engineering, graph theory, robotics, automata theory and computer science. The single valued neutrosophic graphs were introduced by Broumi, Talea, Bakali and Smarandache [5]. Recently in [9, 10, 6] proposed some algorithms dealt with shortest path problem in a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers.
Hypergraphs and various properties that we can prove about them are the basis of many techniques that are used in modern mathematics. While graph edges are pairs of nodes, hyperedges are arbitrary sets of nodes, and can therefore contain an arbitrary number of nodes. However, it is often desirable to study hypergraphs where all hyperedges have the same cardinality. Hyperedges are absurdly general. Likewise, the notion of data. To make this useful, one needs to constrain the form the hyper edges take. There are many research papers on fuzzy hypergraph in [7, 8] based on vertex set as a crisp set. In fact, in the definition of fuzzy graph, both the concepts of vertices and edges are fuzzy and there is an interrelation between the fuzzy vertices and fuzzy edges. The generalized strong intuitionistic fuzzy hypergraphs were discussed by Samanta and Mohinta [1]. In this paper, we generalize the concept of SVNHG by considering SVN-Vertex instead of crisp vertex set and interrelation between SVN-Vertices and family of SVN-Edges. The GSVNHG, generalized strong SVNHG and a few operations on them are defined here. Also some of their properties are studied.

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2. Preliminaries

Definition 2.1. [2] *Let* X *be a crisp set, the single valued neutrosophic set (SVNS)* Z *is characterized by three membership functions* $T_Z(x), I_Z(x)$ *and* $F_Z(x)$*, which are truth, indeterminacy and falsity membership functions, i.e* $\forall x \in X,\ T_Z(x), I_Z(x), F_Z(x) \in [0,1]$.

Definition 2.2. [2] *Let* A *be a SVNS on* X *then support of* A *is denoted and defined by* $Supp(A) = \{x : x \in X, T_A(x) > 0, I_A(x) > 0, F_A(x) > 0\}$.

Definition 2.3. [7, 8] *A hypergraph is an ordered pair* $H = (Z, \Theta)$*, where*
(**1**) $Z = \{\eta_1, \eta_2, \ldots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_m\}$ *be a family of subsets of* Z.
(**3**) $\Theta_j \neq \phi$, $\forall j = 1, 2, 3, \ldots, m$ *and* $\bigcup_j \Theta_j = Z$.
A hypergraph is also called a set system or a family of sets drawn from the universal set X.

3. Generalized strong SVNHGs

We introduce the concept of GSVNHG and generalized strong SVNHG and its properties and a few operations on GSVNHGs and GSSVNHGs.

Definition 3.1. *The single valued neutrosophic hypergraph (SVNHG) be a* $H = (Z, \Theta)$*, where*
(**1**) $Z = \{\eta_1, \eta_2, \ldots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_m\}$ *be a family of SVNSs of* Z.
(**3**) $\Theta_j \neq O = (0,0,0)$ $\forall j = 1, 2, 3, \ldots, m$ *and* $\bigcup_j Supp(\Theta_j) = Z$.

Definition 3.2. *A generalized single valued neutrosophic hypergraph (GSVNHG)* $H = (Z, \Theta)$*, where*
(**1**) $Z = \{\eta_1, \eta_2, \ldots, \eta_n\}$ *be a finite set of vertices.*
(**2**) $A, B, C : Z \to [0,1]$ *be the SVNS of vertices.*
(**3**) $\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_m\}$ *be set of SVNSs of* Z*, where*

$$\Theta_j = \{(\eta_i, T_{\Theta_j}(\eta_i), I_{\Theta_j}(\eta_i), F_{\Theta_j}(\eta_i)) : T_{\Theta_j}(\eta_i), I_{\Theta_j}(\eta_i), F_{\Theta_j}(\eta_i) : Z \to [0,1]\}$$

with

$$\bigvee_{j=1}^{m} T_{\Theta_j}(\eta_i) \leq A(\eta_i),\ \bigwedge_{j=1}^{m} I_{\Theta_j}(\eta_i) \geq B(\eta_i),\ \bigwedge_{j=1}^{m} F_{\Theta_j}(\eta_i) \geq C(\eta_i)$$

$\forall i = 1, 2, 3, \ldots, n$ *and* $\forall j = 1, 2, 3, \ldots, m$.
(**4**) $\Theta_j \neq O = (0,0,0)$, $j = 1, 2, 3, \ldots, m$ *and* $\bigcup_j Supp(\Theta_j) = Z$.

Remark 3.1. *The generalized single valued neutrosophic hypergraph is the generalization of generalized intuitionistic fuzzy hypergraph.*

Example 3.1. *Consider the* $H = (X, E)$*, where* $X = \{\alpha, \beta, \gamma, \delta\}$ *and* $E = \{E_1, E_2, E_3, E_4\}$. *aLSO* $A, B, C : X \to [0,1]$ *defined by* $A(\alpha) = .5$, $A(\beta) = .9$, $A(\gamma) = .8$, $A(\delta) = .6$, $B(\alpha) = .0$, $B(\beta) = .1$, $B(\gamma) = .1$, $B(\delta) = .0$, $C(\alpha) = .1$, $C(\beta) = .1$, $C(\gamma) = .2$, $C(\delta) = .3$,

$$\begin{aligned} E_1 &= \{(\alpha, .2, .3, .4), (\beta, .5, .3, .6), (\gamma, .5, .3, .2), (\delta, .0, .1, .3)\}, \\ E_2 &= \{(\alpha, .5, .0, .2), (\beta, .6, .7, .4), (\gamma, .1, .6, .9), (\delta, .2, .3, .6)\}, \\ E_3 &= \{(\alpha, .1, .3, .5), (\beta, .8, .1, .3), (\gamma, .3, .8, .9), (\delta, .5, .0, .9)\}, \\ E_4 &= \{(\alpha, .1, .6, .2), (\beta, .2, .1, .6), (\gamma, .6, .1, .3), (\delta, .3, .2, .6)\}. \end{aligned}$$

Then by routine calculations H *is GSVNHG.*

Definition 3.3. *The GSVNHG $H = (X, E)$ is said to be generalized strong single valued neutrosophic hypergraph (GSSVNHG), if*

$$\bigvee_{j=1}^{m} T_{E_j}(x_i) = A(x_i),\ \bigwedge_{j=1}^{m} I_{E_j}(x_i) = B(x_i),\ \bigwedge_{j=1}^{m} F_{E_j}(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \dots, n$ *and* $j = 1, 2, 3, \dots, m$.

Example 3.2. *Consider the GSVNHG $H = (X, E)$, where $X = \{\alpha, \beta, \gamma\}$ and $E = \{E_1, E_2, E_3, E_4\}$. Also $A, B, C : X \to [0, 1]$ defined by $A(\alpha) = .5$, $A(\beta) = .6$, $A(\gamma) = .8$, $B(\alpha) = .2$, $B(\beta) = .2$, $B(\gamma) = .0$, $C(\alpha) = .3$, $C(\beta) = .2$, $C(\gamma) = .1$,*

$$\begin{aligned} E_1 &= \{(\alpha, .5, .2, .3), (\beta, .5, .2, .9), (\gamma, .3, .9, .1)\}, \\ E_2 &= \{(\alpha, .1, .6, .5), (\beta, .3, .2, .6), (\gamma, .0, .3, .2)\}, \\ E_3 &= \{(\alpha, .3, .6, .9), (\beta, .1, .3, .2), (\gamma, .1, .0, .9)\}, \\ E_4 &= \{(\alpha, .2, .3, .6), (\beta, .6, .5, .2), (\gamma, .8, .6, .4)\}. \end{aligned}$$

Then by routine calculations H is GSSVNHG.

Definition 3.4. *Let $H = (X, E)$ be a GSVNHG, where $A, B, C : X \to [0, 1]$,*

$$E = \{(T_{E_j}, I_{E_j}, F_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \dots, m\}$$

and let $H' = (X, E')$, where $A', B', C' : X \to [0, 1]$,

$$E' = \{(T'_{E_j}, I'_{E_j}, F'_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \dots, m\}$$

H' is said to be a generalized single valued neutrosophic sub hypergraph (GSVNSHG) of H, whenever

$$\bigvee_{j=1}^{m} T'_{E_j}(x_i) \le \bigvee_{j=1}^{m} T_{E_j}(x_i),\ \bigwedge_{j=1}^{m} I'_{E_j}(x_i) \ge \bigwedge_{j=1}^{m} I_{E_j}(x_i),\ \bigwedge_{j=1}^{m} F'_{E_j}(x_i) \ge \bigwedge_{j=1}^{m} F_{E_j}(x_i)$$

$$A'(x_i) \le A(x_i),\ B'(x_i) \ge B(x_i),\ C'(x_i) \ge C(x_i)$$

$\forall i = 1, 2, 3, \dots, n$. *The GSVNHG $H' = (X, E')$ is said to be a spanning generalized single valued neutrosophic sub hypergraph (SGSVNSHG) of $H = (X, E)$, if*

$$A'(x_i) = A(x_i),\ B'(x_i) = B(x_i),\ C'(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \dots, n$.

Definition 3.5. *Let $H = (X, E)$ be a GSSVNHG, where $A, B, C : X \to [0, 1]$,*

$$E = \{(T_{E_j}, I_{E_j}, F_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \dots, m\}$$

and let $H' = (X, E')$, where $A', B', C' : X \to [0, 1]$, and

$$E' = \{(T'_{E_j}, I'_{E_j}, F'_{E_j}) : X \to [0, 1]^3 : j = 1, 2, 3, \dots, m\}$$

H' is is said to be a generalized strong single valued neutrosophic sub hypergraph (GSSVNSHG) of H, whenever

$$\bigvee_{j=1}^{m} T'_{E_j}(x_i) = \bigvee_{j=1}^{m} T_{E_j}(x_i),\ \bigwedge_{j=1}^{m} I'_{E_j}(x_i) = \bigwedge_{j=1}^{m} I_{E_j}(x_i),\ \bigwedge_{j=1}^{m} F'_{E_j}(x_i) = \bigwedge_{j=1}^{m} F_{E_j}(x_i)$$

$$A'(x_i) = A(x_i),\ B'(x_i) = B(x_i),\ C'(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \dots, n$. *The GSVNHG* $H^{'} = (X, E^{'})$ *is said to be a spanning generalized strong single valued neutrosophic sub hypergraph (SGSSVNSHG) of* $H = (X, E)$, *if*

$$A^{'}(x_i) = A(x_i),\ B^{'}(x_i) = B(x_i),\ C^{'}(x_i) = C(x_i)$$

$\forall i = 1, 2, 3, \dots, n$.

Example 3.3. *Consider the GSVNHGs* $G = (X, E)$, $H = (X, E^{'})$ *and* $S = (X, E^{''})$, *where* $X = \{\alpha, \beta, \gamma\}$, $E = \{E_1, E_2\}$, $E^{'} = \{E_1^{'}, E_2^{'}\}$ *and* $E^{''} = \{E_1^{''}, E_2^{''}\}$. *Also* $A, B, C : X \to [0, 1]$ *defined by* $A(\alpha) = .4$, $A(\beta) = .5$, $B(\alpha) = .2$, $B(\beta) = .2$, $C(\alpha) = .3$, $C(\beta) = .0$, $A^{'}(\alpha) = .4$, $A^{'}(\beta) = .4$, $B^{'}(\alpha) = .1$, $B^{'}(\beta) = .1$, $C^{'}(\alpha) = .3$, $C^{'}(\beta) = .0$, $A^{''}(\alpha) = .4$, $A^{''}(\beta) = .5$, $B^{''}(\alpha) = .2$, $B^{''}(\beta) = .2$, $C^{''}(\alpha) = .3$, $C^{''}(\beta) = .0$,

$$E_1 = \{(\alpha, .2, .3, .6), (\beta, .5, .6, .2)\},\ E_2 = \{(\alpha, .4, .2, .3), (\beta, .3, .2, .5)\},$$
$$E_1^{'} = \{(\alpha, .2, .3, .5), (\beta, .4, .3, .5)\},\ E_2^{'} = \{(\alpha, .3, .2, .3), (\beta, .3, .4, .3)\},$$
$$E_1^{''} = \{(\alpha, .2, .3, .5), (\beta, .5, .3, .5)\},\ E_2^{''} = \{(\alpha, .4, .2, .3), (\beta, .3, .4, .3)\}.$$

Then by routine calculations H *is GSVNSHG of* G *but* S *is SGSVNSHG of* G.

Definition 3.6. *Let* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, where* $X_1 = \{x_1, x_2, \dots, x_n\}$, $X_2 = \{y_1, y_2, \dots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0, 1]$, $A_2, B_2, C_2 : X_2 \to [0, 1]$ *and*

$$\begin{aligned} E_1 &= \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \dots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\} \\ E_2 &= \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \dots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\} \end{aligned}$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0, 1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0, 1],$$

$\forall i = 1, 2, 3, \dots, k$ *and* $j = 1, 2, 3, \dots, p$. *The union* $H_1 \cup H_2 = (X_1 \cup X_2, E_1 \cup E_2)$ *of* H_1 *and* H_2 *is defined by*

$$(A_1 \cup A_2)(x) = \begin{cases} A_1(x) & x \in X_1 - X_2 \\ A_2(x) & x \in X_2 - X_1 \\ \max(A_1(x), A_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(B_1 \cup B_2)(x) = \begin{cases} B_1(x) & x \in X_1 - X_2 \\ B_2(x) & x \in X_2 - X_1 \\ \min(B_1(x), B_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(C_1 \cup C_2)(x) = \begin{cases} C_1(x) & x \in X_1 - X_2 \\ C_2(x) & x \in X_2 - X_1 \\ \min(C_1(x), C_2(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(T_{E_{1i}} \cup T_{E_{2j}})(x) = \begin{cases} T_{E_{1i}}(x) & x \in X_1 - X_2 \\ T_{E_{2j}}(x) & x \in X_2 - X_1 \\ \max(T_{E_{1i}}(x), T_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(I_{E_{1i}} \cup I_{E_{2j}})(x) = \begin{cases} I_{E_{1i}}(x) & x \in X_1 - X_2 \\ I_{E_{2j}}(x) & x \in X_2 - X_1 \\ \min(I_{E_{1i}}(x), I_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

$$(F_{E_{1i}} \cup F_{E_{2j}})(x) = \begin{cases} F_{E_{1i}}(x) & x \in X_1 - X_2 \\ F_{E_{2j}}(x) & x \in X_2 - X_1 \\ \min(F_{E_{1i}}(x), F_{E_{2j}}(x)) & x \in X_1 \cap X_2 \end{cases}$$

Remark 3.2. *If* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, then* $H_1 \cup H_2$ *is also GSVNHG.*

Remark 3.3. *If* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSSVNHGs, then* $H_1 \cup H_2$ *is also GSSVNHG.*

Definition 3.7. *Let* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *be two GSVNHGs, where* $X_1 = \{x_1, x_2, \ldots, x_n\}$, $X_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0,1]$, $A_2, B_2, C_2 : X_2 \to [0,1]$,

$$E_1 = \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \ldots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\},$$
$$E_2 = \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \ldots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\},$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0,1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0,1],$$

$\forall i = 1, 2, 3, \ldots, k$ *and* $j = 1, 2, 3, \ldots, p$. *The cartesian product* $H_1 \times H_2$ *of* H_1 *and* H_2 *is defined by an ordered pair* $H_1 \times H_2 = (X_1 \times X_2, E_1 \times E_2)$, *where*

$$\begin{aligned}
(A_1 \times A_2)(x,y) &= \min(A_1(x), A_2(x)) \\
(B_1 \times B_2)(x,y) &= \max(B_1(x), B_2(x)) \\
(C_1 \times C_2)(x,y) &= \max(C_1(x), C_2(x))
\end{aligned}$$

$$\begin{aligned}
(T_{E_{1i}} \times T_{E_{2j}})(x,y) &= \min(T_{E_{1i}}(x), T_{E_{2j}}(y)) \\
(I_{E_{1i}} \times I_{E_{2j}})(x,y) &= \max(I_{E_{1i}}(x), I_{E_{2j}}(y)) \\
(F_{E_{1i}} \times F_{E_{2j}})(x,y) &= \max(F_{E_{1i}}(x), F_{E_{2j}}(y))
\end{aligned}$$

$\forall x \in X_1$, $y \in X_2$, $i = 1, 2, 3, \ldots, k$ *and* $j = 1, 2, 3, \ldots, p$.

Remark 3.4. *If both* H_1 *and* H_2 *are not GSSVNHGs, then* $H_1 \times H_2$ *may or may not be GSSVNHG.*

Example 3.4. *Consider a GSVNHGs* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *where* $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$ $E_2 = \{P', Q'\}$. *Also* $A_1, B_1, C_1 : X_1 \to [0,1]$ *defined by* $A_1(a) = .3$, $A_1(b) = .5$, $B_1(a) = .2$, $B_1(b) = .4$, $C_1(a) = .5$, $C_1(b) = .5$ *and* $A_2, B_2, C_2 : X_2 \to [0,1]$ *defined by* $A_2(p) = .5$, $A_2(q) = .9$, $B_2(p) = .1$, $B_2(q) = .5$, $C_2(p) = .5$, $C_2(q) = .5$,

$$P = \{(a, .1, .2, .5), (b, .5, .4, .5)\}, \ Q = \{(a, .3, .4, .5), (b, .4, .6, .5)\},$$
$$P' = \{(p, .5, .3, .5), (q, .8, .5, .5)\}, \ Q' = \{(p, .4, .6, .5), (q, .1, .5, .5)\}.$$

Then by routine calculations H_1 *is GSSVNHG and* H_2 *is GSVNHG. Let* $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$. *Then by routine calculations,* $A((a,p)) = .3$, $A((a,q)) = .3$, $A((b,p)) = .5$, $A((b,q)) = .5$, $B((a,p)) = .2$, $B((a,q)) = .5$, $B((b,p)) = .4$, $B((b,q)) = .5$, $C((a,p)) = .5$ $C((a,q)) = .5$, $C((b,p)) = .5$, $C((b,q)) = .5$,

$$\begin{aligned}
P \times P' &= \{((a,p), .1, .3, .5), ((a,q), .1, .5, .5), ((b,p), .5, .4, .5), ((b,q), .5, .5, .5)\}, \\
P \times Q' &= \{((a,p), .1, .6, .5), ((a,q), .1, .5, .5), ((b,p), .4, .6, .5), ((b,q), .1, .5, .5)\}, \\
Q \times P' &= \{((a,p), .3, .4, .5), ((a,q), .3, .5, .5), ((b,p), .4, .6, .5), ((b,q), .4, .6, .5)\}, \\
Q \times Q' &= \{((a,p), .3, .6, .5), ((a,q), .1, .5, .5), ((b,p), .4, .6, .5), ((b,q), .1, .6, .5)\}.
\end{aligned}$$

By calculations H *is not GSSVNHG.*

Example 3.5. *Consider the GSVNHGs* $H_1 = (X_1, E_1)$ *and* $H_2 = (X_2, E_2)$ *where* $X_1 = \{a, b\}$, $X_2 = \{p, q\}$, $E_1 = \{P, Q\}$, $E_2 = \{P^{'}, Q^{'}\}$. *Also* $A_1, B_1, C_1 : X_1 \to [0, 1]$ *defined by* $A_1(a) = .3$, $A_1(b) = .5$, $B_1(a) = .3$, $B_1(b) = .4$, $C_1(a) = .5$, $C_1(b) = .5$ *and* $A_2, B_2, C_2 : X_2 \to [0, 1]$ *defined by* $A_2(p) = .5$, $A_2(q) = .9$, $B_2(p) = .1$, $B_2(q) = .5$, $C_2(p) = .5$, $C_2(q) = .5$,

$$P = \{(a, .1, .3, .5), (b, .5, .4, .5)\},\ Q = \{(a, .3, .4, .5), (b, .4, .6, .5)\},$$
$$P^{'} = \{(p, .5, .3, .5), (q, .8, .5, .5)\},\ Q^{'} = \{(p, .4, .6, .5), (q, .1, .5, .5)\}.$$

Then by routine calculations H_1 *is GSSVNHG and* H_2 *is GSVNHG. Let* $H = (X_1 \times X_2, E_1 \times E_2)$, $A = A_1 \times A_2$, $B = B_1 \times B_2$, $C = C_1 \times C_2$, *then by routine calculations,* $A((a, p)) = .3$, $A((a, q)) = .3$, $A((b, p)) = .5$, $A((b, q)) = .5$, $B((a, p)) = .3$, $B((a, q)) = .5$, $B((b, p)) = .4$, $B((b, q)) = .5$, $C((a, p)) = .5$, $C((a, q)) = .5$, $C((b, p)) = .5$, $C((b, q)) = .5$,

$$P \times P^{'} = \{((a, p), .1, .3, .5), ((a, q), .1, .5, .5), ((b, p), .5, .4, .5), ((b, q), .5, .5, .5)\},$$
$$P \times Q^{'} = \{((a, p), .1, .6, .5), ((a, q), .1, .5, .5), ((b, p), .4, .6, .5), ((b, q), .1, .5, .5)\},$$
$$Q \times P^{'} = \{((a, p), .3, .4, .5), ((a, q), .3, .5, .5), ((b, p), .4, .6, .5), ((b, q), .4, .6, .5)\},$$
$$Q \times Q^{'} = \{((a, p), .3, .6, .5), ((a, q), .1, .5, .5), ((b, p), .4, .6, .5), ((b, q), .1, .6, .5)\}.$$

By calculations H *is GSSVNHG.*

Proposition 3.1. *If both* H_1 *and* H_2 *are GSVNHGs, then* $H_1 \times H_2$ *is also GSVNHG.*

Proof. Let $H_1 = (X_1, E_1)$ and $H_2 = (X_2, E_2)$ be two GSVNHGs, where $X_1 = \{x_1, x_2, \dots, x_n\}$, $X_2 = \{y_1, y_2, \dots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0, 1]$, $A_2, B_2, C_2 : X_2 \to [0, 1]$,

$$\begin{aligned} E_1 &= \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \dots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\} \\ E_2 &= \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \dots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\} \end{aligned}$$

where

$$T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0, 1],$$
$$T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0, 1],$$

$\forall i = 1, 2, 3, \dots, k$ and $j = 1, 2, 3, \dots, p$. Then the cartesian product $H_1 \times H_2 = (X_1 \times X_2, E_1 \times E_2)$, where

$$E_1 \times E_2 = \{((T_{E_{11}} \times T_{E_{21}}), (I_{E_{11}} \times I_{E_{21}}), (F_{E_{11}} \times F_{E_{21}})), \dots, ((T_{E_{11}} \times T_{E_{2p}}), (I_{E_{11}} \times I_{E_{2p}}), (F_{E_{11}} \times F_{E_{2p}})), \dots, ((T_{E_{1k}} \times T_{E_{2p}}), (I_{E_{1k}} \times I_{E_{2p}}), (F_{E_{1k}} \times F_{E_{2p}}))\}$$

with

$$\bigvee_{r=1}^{k} T_{E_{1r}}(x_i) \le A_1(x_i), \bigvee_{s=1}^{p} T_{E_{2s}}(y_j) \le A_2(y_j)$$
$$\bigwedge_{r=1}^{k} I_{E_{1r}}(x_i) \ge B_1(x_i), \bigwedge_{s=1}^{p} I_{E_{2s}}(y_j) \ge B_2(y_j)$$
$$\bigwedge_{r=1}^{k} F_{E_{1r}}(x_i) \ge C_1(x_i), \bigwedge_{s=1}^{p} F_{E_{2s}}(y_j) \ge C_2(y_j)$$

$\forall i = 1, 2, 3, \ldots, n$ and $j = 1, 2, 3, \ldots, m$. Now consider

$$\begin{aligned}
\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}} \times T_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}}(x_i), T_{E_{2s}}(y_j)) \\
&= (\bigvee_{r=1}^{k} T_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T_{E_{2s}}(y_j)) \\
&\leq A_1(x_i) \wedge A_2(y_j) = (A_1 \times A_2)(x_i, y_j)
\end{aligned}$$

$\forall i$ and j. Similarly

$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(I_{E_{1r}} \times I_{E_{2s}})(x_i, y_j) \geq (B_1 \times B_2)(x_i, y_j)$$

$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(F_{E_{1r}} \times F_{E_{2s}})(x_i, y_j) \geq (C_1 \times C_2)(x_i, y_j)$$

$\forall i$ and j. Thus $H_1 \times H_2$ is the GSVNHG. □

Proposition 3.2. *If both H_1 and H_2 are GSSVNHGs then $H_1 \times H_2$ is also GSSVNHG.*

Proof. Similar as Proposition 3.1 is proved. □

Proposition 3.3. *If $H_1 \times H_2$ is GSSVNHG, then at least H_1 or H_2 must be GSSVNHG.*

Proof. Let $H_1 = (X_1, E_1)$ and $H_2 = (X_2, E_2)$ be two GSVNHGs, where $X_1 = \{x_1, x_2, \ldots, x_n\}$, $X_2 = \{y_1, y_2, \ldots, y_n\}$, $A_1, B_1, C_1 : X_1 \to [0, 1]$, $A_2, B_2, C_2 : X_2 \to [0, 1]$ and

$$\begin{aligned}
E_1 &= \{(T_{E_{11}}, I_{E_{11}}, F_{E_{11}}), (T_{E_{12}}, I_{E_{12}}, F_{E_{12}}), \ldots, (T_{E_{1k}}, I_{E_{1k}}, F_{E_{1k}})\}, \\
E_2 &= \{(T_{E_{21}}, I_{E_{21}}, F_{E_{21}}), (T_{E_{22}}, I_{E_{22}}, F_{E_{22}}), \ldots, (T_{E_{2p}}, I_{E_{2p}}, F_{E_{2p}})\},
\end{aligned}$$

where

$$\begin{aligned}
&T_{E_{1i}}, I_{E_{1i}}, F_{E_{1i}} : X_1 \to [0, 1], \\
&T_{E_{2j}}, I_{E_{2j}}, F_{E_{2j}} : X_2 \to [0, 1],
\end{aligned}$$

$\forall i = 1, 2, 3, \ldots, k$ and $j = 1, 2, 3, \ldots, p$. Then the cartesian product $H_1 \times H_2 = (X_1 \times X_2, E_1 \times E_2)$, where

$$\begin{aligned}
E_1 \times E_2 = \{&((T_{E_{11}} \times T_{E_{21}}), (I_{E_{11}} \times I_{E_{21}}), (F_{E_{11}} \times F_{E_{21}})), \ldots, ((T_{E_{11}} \times T_{E_{2p}}), (I_{E_{11}} \times \\
&I_{E_{2p}}), (F_{E_{11}} \times F_{E_{2p}})), \ldots, ((T_{E_{1k}} \times T_{E_{2p}}), (I_{E_{1k}} \times I_{E_{2p}}), (F_{E_{1k}} \times F_{E_{2p}}))\},
\end{aligned}$$

next suppose that $H_1 \times H_2$ is GSSVNHG, but H_1 and H_2 are not GSSVNHGs, then by definition

$$\begin{aligned}
&\bigvee_{r=1}^{k} T_{E_{1r}}(x_i) < A_1(x_i), \quad \bigvee_{s=1}^{p} T_{E_{2s}}(y_j) < A_2(y_j) \\
&\bigwedge_{r=1}^{k} I_{E_{1r}}(x_i) > B_1(x_i), \quad \bigwedge_{s=1}^{p} I_{E_{2s}}(y_j) > B_2(y_j) \\
&\bigwedge_{r=1}^{k} F_{E_{1r}}(x_i) > C_1(x_i), \quad \bigwedge_{s=1}^{p} F_{E_{2s}}(y_j) > C_2(y_j)
\end{aligned}$$

$\forall i = 1, 2, 3, \ldots, n$ and $j = 1, 2, 3, \ldots, m$. Therefore

$$\begin{aligned}\bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}} \times T_{E_{2s}})(x_i, y_j) &= \bigvee_{s=1}^{p}\bigvee_{r=1}^{k}(T_{E_{1r}}(x_i), T_{E_{2s}}(y_j)) \\ &= (\bigvee_{r=1}^{k} T_{E_{1r}}(x_i)) \wedge (\bigvee_{s=1}^{p} T_{E_{2s}}(y_j))) \\ &< A_1(x_i) \wedge A_2(y_j) = (A_1 \times A_2)(x_i, y_j)\end{aligned}$$

$\forall i$ and j. Similarly

$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(I_{E_{1r}} \times I_{E_{2s}})(x_i, y_j) > (B_1 \times B_2)(x_i, y_j)$$

$$\bigwedge_{s=1}^{p}\bigwedge_{r=1}^{k}(F_{E_{1r}} \times F_{E_{2s}})(x_i, y_j) > (C_1 \times C_2)(x_i, y_j)$$

$\forall i$ and j. Therefore $H_1 \times H_2$ is not GSSVNHG, hence at least one of H_1 or H_2 must be GSSVNHG. □

4. Conclusion

In this paper, the concept of single valued neutrosophic hypergraph has been generalized by considering single valued neutrosophic vertex set instead of crisp vertex set and also considering interrelation between single valued neutrosophic vertices and family of single valued neutrosophic edges. Further one can use this concept to analyze the structure of a system and to represent a partition, covering and clustering.

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