The covariance of the wave equation does not support the constant hypothesis of light speed

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Abstract

On the basis of Maxwell's electromagnetic theory, the relationship between the speed of light and source is obtained for the first time according to the conclusion that the form of wave equation has nothing to do with the moving velocity of light source. Under the condition to do not make any amendments to the classical absolute space-time theory, all the current dynamic electromagnetic measurement results are easily explained by using this relationship.

Keywords: Electromagnetic wave, Phase, Covariant Transformation, Velocity of light, Light source.

As we all know, regardless of the light source movement or not, the radiation of electromagnetic waves is in accordance with the law of cosine cycle changes. This means that the form of the field equations and the electromagnetic wave equations will not change due to the change of the light source's motion state. In discussing the electromagnetic waves propagated by static and moving sources along the same direction, it is found that, if the wave functions satisfied by the wavefront of the same wave number (the same phase) of the two electromagnetic have the same form, the propagation velocity of the wavefront in space can not be independent of the state of motion of the light source.

In the reference system S, a stationary light source emits a monochromatic wave along the direction of the unit wave vector $\boldsymbol{k}_0$, and its intrinsic circle frequency and wave velocity are ω and $\boldsymbol{c}$ respectively (shown in Fig. 1). If the light source is moving at speed $\boldsymbol{u}$ and also emits a monochromatic wave in the same direction ($\boldsymbol{k}_0$ direction), frequency and wave velocity are ω' and $\boldsymbol{c}'$ respectively (shown in Fig. 2). The time τ required for the stationary light source to radiate out N complete waveforms and the vector distance $\boldsymbol{r}$ from the wavefront to the light source respectively are

$$\tau = 2\pi N/\omega \tag{1}$$

$$\boldsymbol{r} = \tau c \boldsymbol{k}_0 \tag{2}$$

The time τ' required for the moving light source to radiate out N' complete waveforms and the vector distance $\boldsymbol{r}'$ from the wavefront to the light source respectively are

$$\tau' = 2\pi N'/\omega' \tag{3}$$

$$\boldsymbol{r}' = \tau'\left(c'\boldsymbol{k}_0 - \boldsymbol{u}\right) \tag{4}$$

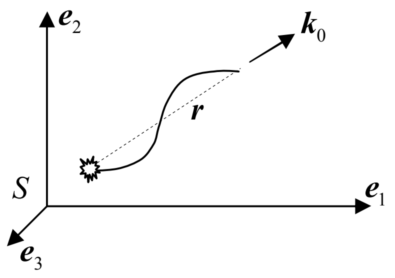

Figure 1

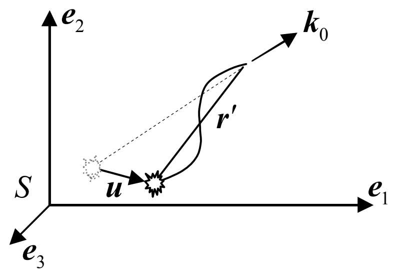

Figure 2

Assuming that $\boldsymbol{r}$ and $\boldsymbol{r}'$ contain the same wave number (that is, N=N'), comparing Eq. (1) and Eq. (3) we can obtain

$$\tau' = \left(\omega / \omega'\right)\tau \tag{5}$$

Substituting Eq. (5) and Eq. (2) into Eq. (4), result in

$$\boldsymbol{r}' = \left(\omega / \omega'\right)\left[\left(c'/c\right)\boldsymbol{r} - \boldsymbol{u}\tau\right] \tag{6}$$

We dot-multiply both sides of Eq. (6) by $\boldsymbol{k}_0$ and $\boldsymbol{k}_0(\boldsymbol{k}_0 \bullet \boldsymbol{e}_j)$, respectively, and the projection components of $\boldsymbol{k}_0 \bullet \boldsymbol{r}'$ and $\boldsymbol{k}_0 \bullet \boldsymbol{r}$ on the coordinate axes are

$$\begin{aligned} x_j' &= \left(\boldsymbol{k}_0 \cdot \boldsymbol{r}'\right)\boldsymbol{k}_0 \cdot \boldsymbol{e}_j \\ x_j &= \left(\boldsymbol{k}_0 \cdot \boldsymbol{r}\right)\boldsymbol{k}_0 \cdot \boldsymbol{e}_j \end{aligned}, \tag{7}$$

and where,

$$\begin{aligned} \boldsymbol{k}_0 \cdot \boldsymbol{r}' &= c't' \\ \boldsymbol{k}_0 \cdot \boldsymbol{r} &= ct \end{aligned}, \tag{8}$$

result in

$$\begin{aligned} x_j' &= \left(\omega / \omega'\right)\left[\left(c'/c\right)x_j - \left(\boldsymbol{k}_0 \cdot \boldsymbol{u}/c\right)\boldsymbol{k}_0 \cdot \boldsymbol{e}_j ct\right] \\ c't' &= \left(\omega / \omega'\right)\left[\left(c'/c\right)ct - \left(\boldsymbol{k}_0 \cdot \boldsymbol{u}/c\right)\boldsymbol{k}_0 \cdot \boldsymbol{e}_j x_j\right] \end{aligned} \tag{9}$$

Where, $\boldsymbol{e}_j$ denotes the 3D coordinate axis forward unit vector of the reference coordinate system S: t=$\boldsymbol{k}_0 \bullet \boldsymbol{r}/\boldsymbol{c}$ =τ; j=1, 2, 3. It is easily verified that, if ω' and $\boldsymbol{c}'$ satisfy Eq. (10),

$$\left(\omega / \omega'\right)^2\left[\left(c'/c\right)^2 - \left(\boldsymbol{k}_0 \cdot \boldsymbol{u}/c\right)^2\right] = 1, \tag{10}$$

Eq. (9) satisfy Eq. (11)

$$x_j'x_j' - \left(c't'\right)^2 = x_jx_j - \left(ct\right)^2. \tag{11}$$

According to above results, Eq. (12) can be proved.

$$\frac{\partial^2}{\partial x_j'^2}-\frac{\partial^2}{\partial\left(c't'\right)^2}=\frac{\partial^2}{\partial x_j^{\,2}}-\frac{\partial^2}{\partial\left(ct\right)^2}. \tag{12}$$

We have theoretically proved that the form of the electromagnetic wave equation is independent of the motion state of the light source.

A large number of experimental observations confirm that, when $\boldsymbol{k_0}\bullet\boldsymbol{u} = \pm\boldsymbol{u}$, there is constant $\boldsymbol{c'}=\boldsymbol{c}$. Substituting the result into Eq. (10), Eq. (13) can be obtained

$$\omega' = \omega\sqrt{1-\left(u/c\right)^2}. \tag{13}$$

Since the equation (13) is also workable under the condition of $\boldsymbol{k_0}\bullet\boldsymbol{u} \neq \pm\boldsymbol{u}$, thus, Substituting Eq. (13) into Eq. (10), we can obtain Eq. (14): the general relationship between the light speed and the light source speed. It can be seen from the Eq. (14) that velocity of light both is independent of light source speed ($\boldsymbol{c'}\equiv\boldsymbol{c}$) and not the vector sum between $\boldsymbol{c}$ and the velocity of the light source ($\boldsymbol{c'}=\boldsymbol{c}+\boldsymbol{k_0}\bullet\boldsymbol{u}$).

$$c' = c\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}. \tag{14}$$

Otherwise, the Eq. (13) can not workable. If the influence of light source motion on the energy density of electromagnetic field is considered, Eq. (14) can be derived from Maxwell's field equation [1].

In this way, Eq. (15): the wave number invariant transformation of two electromagnetic waves can be obtained by substituting Eqs. (13) - (14) into Eq. (9).

$$\begin{aligned} x_j' &= \gamma\left[x_j\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}-\left(\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)\boldsymbol{k}_0\cdot\boldsymbol{e}_j ct\right] \\ c't' &= \gamma\left[ct\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}-\left(\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)\boldsymbol{k}_0\cdot\boldsymbol{e}_j x_j\right] \end{aligned} \tag{15}$$

The general expression of the transformation can be obtained by multiplying the two sides of the first equation in Eq. (15) by $\boldsymbol{k_0}\bullet\boldsymbol{e_j}$.

$$\begin{aligned} x' &= \gamma\left[x\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}-\left(\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)ct\right] \\ c't' &= \gamma\left[ct\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}-\left(\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)\boldsymbol{k}_0\cdot\boldsymbol{x}\right] \end{aligned} \tag{16}$$

Where, $\gamma=1/\sqrt{1-u^2/c^2}$, $x=k_0\bullet e_j x_j$, $x'= k_0\bullet e_j x_j'$. The following formula is known as the

Lorenz transform.

$$\begin{aligned} x' &= \gamma\left(x-ut\right) \\ t' &= \gamma\left(t-u\boldsymbol{x}/c^2\right) \end{aligned} \tag{17}$$

Eq. (17) is only a result of Eq. (15) in the special case of $\boldsymbol{k}_0\times\boldsymbol{u}=\boldsymbol{k}_0\times\boldsymbol{e}_1=0$.

It can be proved that Eq. (15) satisfies the following two important relations

$$k'\lambda' = k\lambda = 2\pi \tag{18}$$

and

$$k'_+k'_- = k^2\,, \qquad \lambda'_+\lambda'_- = \lambda^2\,. \tag{19}$$

Where, (k_+', k_-') and (λ_+', λ_-') are the modules and wavelengths of the wave vectors radiated by the moving light source along the two opposite directions of $\pm k_0$. In the reference coordinate system S, the module of the wave vector of the electromagnetic wave radiated by the moving light source is k', and the module of the wave vector of the electromagnetic wave radiated by the stationary light source is k. Substituting Eq. (15) into the following formula,

$$\boldsymbol{k}'\cdot\boldsymbol{e}_j x'_j - \left(f'/c'\right)c't' = \boldsymbol{k}\cdot\boldsymbol{e}_j x_j - \left(f/c\right)ct \tag{20}$$

Eq. (21) can be satisfied by the relation between k' and k.

$$\boldsymbol{k}'\cdot\boldsymbol{e}_j = \gamma\left(\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}+\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)\boldsymbol{k}\cdot\boldsymbol{e}_j\,. \tag{21}$$

The both sides of Eq. (21) multiplied by $\boldsymbol{k_0}\cdot\boldsymbol{e_j}$, we can obtain the general expression of wave vector module

$$k' = \gamma\left(\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}+\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)k\,. \tag{22}$$

Substituting $k'=2\pi/\lambda'$ and $k=2\pi/\lambda$ into Eq. (22), the expression of electromagnetic wave length can be obtained

$$\lambda' = \gamma\left(\sqrt{1-\left|\boldsymbol{k}_0\times\boldsymbol{u}/c\right|^2}-\boldsymbol{k}_0\cdot\boldsymbol{u}/c\right)\lambda\,. \tag{23}$$

According to Eqs. (18) - (19), it can be concluded that the Eq. (23) derived from Eq. (15) is the correct mathematical expression of Doppler shift.

In short, according to the fact that wave equation has covariance relative to the state of light movement, not only the wave number invariant transformation can be derived, but also Eq. (14) — the general relation between the velocity of light and the velocity of light source can be derived. Eq. (14) is the relationship that people dream of. Derivation of it is a landmark thing.

References

[1] Xiao Jun. Super unified field and covariant electromagnetism introduction. *Harbin Engineering University Press*. Harbin: 2016, pp. 123-124.