Under consideration for publication in Math. Struct. in Comp. Science

A derivation of special and general relativity from algorithmic thermodynamics

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Received 1 September 2017; Last revision 21 November 2017

In this paper, I investigate a prefix-free universal Turing machine (UTM) running multiple programs in parallel, orchestrated by a scheduler. I found that if, over the course of the computation, the scheduler adjusts the work done so as to maximize the entropy in the calculation of the halting probability Ω, the system will follow many laws analogous to the laws of physics. As the scheduler maximizes entropy, the result relies on algorithmic thermodynamics, which connects the halting probability of a prefix-free UTM to the Gibbs ensemble of statistical physics (which also maximizes entropy). My goal with this paper is to show that special relativity, general relativity, and an arrow of time can be derived from algorithmic thermodynamics under a certain choice of thermodynamic observables applied to the halting probability.

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0.1. *Notation*

I will use the following notations: The double vertical lines $|X|$ means the length of the string X. The suffix b, for example in 110_b, refers to the binary notation.

1. Introduction

A connection between algorithmic information theory and statistical physics was established by (Tadaki(2002)) and extended by (Baez and Stay(2012)). Indeed, Tadaki found that augmenting the halting probability of a prefix-free universal Turing machine (UTM) with a multiplication constant, applicable to the program length, is enough to recover the Gibbs ensemble of statistical physics. John C. Baez and Mike Stay took it a step further by introducing into the halting probability additional terms interpreted as thermodynamic observable equivalents. This was sufficient for them to construct a thermodynamic cycle applicable to algorithmic information theory.

I have found that introducing two new thermodynamic observables, each having a dual interpretation (both as a physical system and as an algorithmic system), is sufficient to connect Ω, the halting probability of a prefix-free UTM, to many unsolved problems of theoretical physics. The first observable, let's call it entropic time, is the action $\mathcal{S}$ conjugated with the frequency f and accounts for the change of entropy in time. It can be interpreted as an internal clock used by the UTM to synchronize the calculation. The second observable, let's call it entropic space, is a "decompression" factor D and is conjugated with the program length $|p|$. The pair $D|p|$ can be interpreted as a function that assigns a prefix-free code to each program according to an arbitrary algorithm.

Adding a time variable to a Gibbs ensemble (such as the frequency f) adds a whole new dynamic to a thermodynamic system. The system now becomes aware of future, past and present entropy and can translate from time to space and from space to time for an entropic cost (provided that various limits are respected). By studying thermodynamic cycles involving space and time, I was able to investigate what happens to the entropy when a system is translated forward or backward in time and draw conclusions in regards to the arrow of time. In the model presented, space serves as an entropy sink that encourages a forward arrow of time, the future is non-computable and the past is singular.

As these new observables can be added to any Gibbs ensemble and will produce a similar system, the bits of Ω serve as the microscopic interpretation of the entropy for the system. A forward translation in time increases the number of bits of Ω that are known, which simultaneously lowers the entropy and increases the quantity of information encoded by the past. As multiple choices of prefix-free code exist to encode Ω for a given

UTM, this provides an entropy sink available to offset the reduction in entropy produced by forward translation in time - this is the role of the second observable. The main result is that for an arbitrary prefix-free encoding, the limitations and costs associated with this sink are identical to those required to derive special and general relativity, various holographic principles and the arrow of time from an entropic perspective.

1.1. *Algorithmic information theory*

In accordance with Gregory Chaitin's Ω construction (Chaitin(1975)), I can define a sum which encodes the solution to the halting problem for a prefix-free UTM as a probability;

$$\Omega = \sum_p 2^{-E(p)-|p|} \qquad \textit{where, } E(p) = \begin{cases} 0 & p \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{1}$$

I will now unpack this sum and use example values to help fix the idea. Consider the following example values for $E(p)$ and $|p|$,

$$= 2^{-\infty}2^{-1} + 2^{-0}2^{-2} + 2^{-0}2^{-3} + 2^{-0}2^{-4} + 2^{-\infty}2^{-5} + ... \tag{2}$$

The presence of the negative infinity in the term of the exponential causes some terms to vanish to zero. Note that the suffix b indicates the binary notation.

$$= 0_b + 0.01_b + 0.001_b + 0.0001_b + 0_b + ... \tag{3}$$

$$= 0.01110..._b \tag{4}$$

which recovers Ω for the example values.

$$\tag{5}$$

1.2. *Algorithmic thermodynamics*

The halting probability (1) is similar to a Gibbs ensemble of statistical physics. In fact, this similarity has been noted by other authors (Li and Vitanyi(2008); Calude and Stay(2006); Baez and Stay(2012); Tadaki(2002); Tadaki(2008)). Indeed, the Gibbs ensemble compares to the halting probability as follows;

$$\begin{array}{cc} \text{Gibbs ensemble} & \text{Halting probability} \\ Z = \sum_x e^{-\beta(E+pV+Fx)} & \Omega = \sum_p 2^{-E(p)-|p|} \end{array} \tag{6}$$

To be upgraded to a full-fledge Gibbs ensemble, I only need to add a conjugate variable to the halting probability analogous to the temperature β. As suggested by Tadaki, I multiply the terms of the exponential by a decompression factor D adjusting the packing density of the bits of Ω. I get

Observable	Conjugate variable
Energy E	Temperature $\beta = 1/(k_b T)$
Volume V	Pressure $\gamma = p/(k_b T)$
Number of particles N	Chemical potential $\delta = -\mu/(k_b T)$

Table 1. *Typical observables of statistical mechanics.*

$$Z_\Omega^{'} = \sum_p 2^{-\beta[E(p)+D|p|]} \tag{7}$$

$E(p)$, as it is either 0 or ∞ will absorb β, so its contribution to Ω remains the same. For $\beta D|p|$, the effect is to "decompress" the bits of Ω. If $\beta D > 1$, no bit erasure take places. To fix the idea, I unpack the sum taking $\beta D = 2$ as an example.

$$= 2^{-2\times 1} + 2^{-2\times 2} + 2^{-2\times 3} + 2^{-2\times 4} + 2^{-2\times 5} + ... \tag{8}$$

$$= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-10} + ... \tag{9}$$

$$= 0.01_b + 0.0001_b + 0.000001_b + 0.000000001_b + ... \tag{10}$$

$$= 0.0101010101..._b \tag{11}$$

The result is that some zero-valued bits have been injected between the bits of Ω. To recover Ω, it suffices to eliminate the extra bits. No halting information is lost.

1.3. *Statistical physics*

Before continuing to the next section, I will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy

$$S = -k_b \sum_{x\in X} p(x) \ln p(x) \tag{12}$$

subject to the fixed macroscopic observables. The solution for this is the Gibbs ensemble. Taking the observables listed in Table 1 as examples, the partition function becomes

$$Z = \sum_{x\in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{13}$$

The probability of occupation of a micro-state is;

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{14}$$

The average values and their variance for the observables are;

$$\overline{E} = \sum_{x \in X} p(x)E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{15}$$

$$\overline{V} = \sum_{x \in X} p(x)V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{16}$$

$$\overline{N} = \sum_{x \in X} p(x)N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{17}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial E}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial N}\right|_{E,V} = -\frac{\mu}{T} \tag{18}$$

which can be summarized as

$$dE = TdS - pdV + \mu dN \tag{19}$$

This is known as the state equation of the thermodynamic system.

1.4. *An "entropic UTM"*

A UTM can attempt to calculate Ω by starting each program in dovetail, and as they halt, add their contribution to Ω. After an infinite amount of time, Ω will indeed be recovered. However, the calculation does not converge towards Ω as it progresses and discontinuously yields Ω only at infinity. To see why, consider the case where the first zero-valued bit of Ω is at position i. Since the general non-halting problem is unsolvable, at most the calculation of Ω differs from the real value of Ω by 2^{-i}. The error rate does not decrease during the calculation and only vanishes at infinity when all halting programs are known.

To make the laws of physics come out, I must adjust the calculation so that it converges towards Ω, even during the calculation. In other words, the error rate must be made to monotonically decrease during the calculation. This can be done with entropic dovetailing.

Definition 20 (Dovetailing). Dovetailing is a program execution strategy for a Turing machine to guarantee that progress will be made on arbitrarily-many programs even in the presence of non-halting programs.

Definition 21 (Standard dovetailing). Consider the case of standard dovetailing. First, we start the shortest program and perform one iteration. Then, we start the second program and perform one iteration on the first and second programs. Then, we start the third program and perform one iteration on the first, second and third programs, and so on. Using dovetailing, progress will eventually be made on every program and no program will cause the TM to hang.

To convert (1) into a dovetailing calculation of Ω, it suffices that I add to the sum the action observable $\mathcal{S}$ to the frequency conjugate f, which yields,

$$Z_\Omega = \sum_p e^{-\beta(\ln 2)[E(p)+2\pi\mathcal{S}f+D|p|]} \qquad \text{Partition function} \qquad (22)$$

Here, the 2π factor is added to recover the physical definition of $\mathcal{S}$ as related to the angular frequency, commonly used in physics. Having a partition function in base 2 instead of the natural base e is equivalent to performing a change in temperature from β' to $\beta \ln 2$. Hence $T' = T/\ln 2$. The state equation therefore is

$$dE = \frac{1}{\ln 2} T dS - 2\pi\mathcal{S}df - Dd|p| \qquad \text{State equation} \qquad (23)$$

What are these program observables and why am I allowed to add them? Recall that $Z_\Omega^{'}$ and Z_Ω are Gibbs ensembles. As a result, observables of program properties can be added. I will now look at $2\pi\mathcal{S}f$ in greater detail to understand the impact it has on the calculation of Z_Ω. Starting with an example, suppose the following values of $\mathcal{S}$ for the first three programs,

$$S_1 = \frac{5}{2\pi} \qquad S_2 = \infty \qquad S_3 = \frac{5}{2\pi} \qquad (24)$$

Note that I do not, nor am I trying to, escape the non-computability of Ω. Indeed, $\mathcal{S}$ is non-computable because $\mathcal{S}$ bears the solution to the general non-halting problem. Z_Ω simply shifts the non-computability from $E(p)$ to $\mathcal{S}$. In my example, the sum Z_Ω becomes

$$Z_\Omega = 2^{-1-\frac{5}{t}} + 2^{-2-\frac{\infty}{t}} + 2^{-3-\frac{5}{t}} + ... \qquad (25)$$

As an example, consider these values of $Z_\Omega(t)$ for specific values of t along with the error rate $\xi(t) = \Omega - Z_\Omega(t)$

estimation of Ω	error	bound (on error)	
$\Omega = 0.101..._b$	$\xi = 0$		(26)
$\lim_{t\to 0^+} Z_\Omega(t) = 0$	$\xi = \Omega$		(27)
$Z_\Omega(1) = 0.00000101..._b$	$\xi = 0.10011011..._b$	$\xi \le 2^{-0}$	(28)
$Z_\Omega(5) = 0.0101..._b$	$\xi = 0.0101..._b$	$\xi \le 2^{-1}$	(29)
$Z_\Omega(10) = 0.01110001001000..._b$	$\xi = 0.00101110110111..._b$	$\xi \le 2^{-2}$	(30)
$Z_\Omega(1000) = 0.10011010100011..._b$	$\xi = 0.00000101011100..._b$	$\xi \le 2^{-5}$	(31)
$\lim_{t\to\infty} Z_\Omega(t) = 0.101..._b$	$\xi = 0$	$\xi \le 2^{-\infty}$	(32)

As I grow t from 0 to ∞, the error rate monotonically diminishes until it eventually vanishes. I will now prove two theorems: 1) Z_Ω can recover Ω at $t \to \infty$; and 2) Z_Ω calculates Ω through time with a monotonically decreasing error rate.

Theorem 33. To prove that Z_Ω recovers Ω at $t \to \infty$, I will show that Ω is computable from $Z_\Omega^{'}$ and that

$$\lim_{t\to\infty} Z_\Omega \to Z_\Omega^{'}$$

Proof. A program p can have any value of $\mathcal{S}_p$ within $[0, \infty]$. If the program halts immediately, $\mathcal{S}_p = 0$. If it never halts, $\mathcal{S}_p = \infty$. If it halts after a certain time, $\mathcal{S}_p \in \mathbb{N}$. A program that never halts will not contribute to the halting partition. This will be the case if $\mathcal{S}_p = \infty$. This yields,

$$\lim_{f\to 0^+} 2\pi f S_p = \lim_{t\to\infty} \frac{2\pi \mathcal{S}_p}{t} = \begin{cases} 0 & \text{p halts} \\ \infty & \text{otherwise} \end{cases} \tag{34}$$

As this is the definition of $E(p)$ (equation 1) we obtain

$$\lim_{t\to\infty} \frac{2\pi \mathcal{S}_p}{t} = E(p) \tag{35}$$

Lemma 1.1. $E(p) + E(p) = E(p)$

Proof. $E(p)$ is either 0 or ∞. Since $0 + 0 = 0$ and $\infty + \infty = \infty$, the lemma holds. ☐

Therefore,

$$\lim_{t\to\infty} Z_\Omega = \lim_{t\to\infty} \left(\sum_p 2^{-\beta[E(p)+2\pi \mathcal{S} f + D|p|]} \right) \tag{36}$$

$$= \sum_p 2^{-\beta[E(p)+E(p)+D|p|]} \tag{37}$$

$$= \sum_p^\infty 2^{-\beta[E(p)+D|p|]} \qquad \text{(lemma 1.1)}$$

$$= Z_\Omega^{'} \tag{38}$$

Is knowing $Z_\Omega^{'}$ enough to compute Ω? Yes, because I only need to remove the zero-valued bits inserted between the bits of Ω. ☐

Theorem 39. To show that equation (22) dovetails programs, it suffices to show the following. For $0 < t < \infty$, the partition function Z_Ω is

$$Z_\Omega(t) = \Omega - 2^{-k(t)}$$

where $2^{-k(t)}$ is an error rate that is monotonically decreasing to 0 as $t \to \infty$. As a result of increasing the time, the calculation of Z_Ω produces an ever more precise estimation of Ω.

Proof. Using a similar argument as the one provided by John C. Baez and Mike Stay, I argue that as $\mathcal{S}$ exponentially suppresses programs with long halting time, there will always be a time t such that the contribution of programs that have not yet halted will be less than $2^{-k(t)}$. □

As a result, the partition function Z'_Ω produces a monotonically improving estimation of Ω over time. The fact that the error rate is able to decrease monotonically implies that the calculation does not hang. Hence it is a type of dovetailing. Furthermore, since I have defined the calculation with a Gibbs ensemble, I am guaranteed that the calculation maximizes the entropy during the calculation.

In the end, what I have described is a dovetailing algorithm which maximizes the entropy in the calculation of Ω.

1.5. *Prior and related work*

How then are the laws of physics recovered from (22)? To recover the laws of physics, I will make use of the properties of the Gibbs ensemble, principally by studying its state equation. Although not necessary, it helps to give (22) a physical interpretation of its observables. Let me start by considering the prior work.

Many authors(Bennett et al.(1998)Bennett, Gacs, Li, Vitanyi, and Zurek; Chaitin(1975); Fredkin and Toffoli(1982); Kolmogorov(1965); Zvonkin and Levin(1970); Solomonoff(1964); Szilard(1964); Tadaki(2002); Tadaki(2008)) have discussed the similarity between physical entropy $S = -k_B \sum p_i \ln p_i$ and the entropy in information theory $S = -\sum p_i \log_2 p_i$.

John C. Baez and Mike Stay suggest (Baez and Stay(2012)) an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be observables. Starting from Gregory Chaitin's Ω number, the halting probability

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{40}$$

is extended with algorithmic observables to obtain

$$\Omega' = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{41}$$

Noting the similarity between the Gibbs ensemble of statistical physics (13) and (41), these authors suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1 $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.

2 $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the tradeoff between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.
3 $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (41), they derive analogues of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*.

Other authors have suggested other somewhat arbitrary correspondence (Li and Vitanyi(2008); Tadaki(2008)).

1.6. *Physical interpretation*

I suggest to map the program-observables to physical-observables as follows. As I will show, this interpretation correctly maps the algorithmic thermodynamics interpretation of special relativity (and other laws) to its physical interpretation. Hence, it would appear to be the preferred mapping.

— The program-runtime is the number of *Iterations* a UTM needs to perform until a program halts. It is, therefore, natural to associate it with the physical *Time* in *seconds*. Indeed, a program requiring more iterations to halt will also require more time to terminate. If a system performs iterations at a faster or slower rate, the conjugate variable to time, the *Power* in *Watts*, can be adjusted to account for this variation.
— The program-frequency, is associated with the reverse of the second, s^{-1}, and its conjugate variable is the *Action* in *Joules-seconds*.
— The program-size is expressed in number of *bits*. Writing the bits one after the other on any medium (paper, disk drive, etc.) will require a certain physical size for each bit. As the line is the lowest dimensional geometry to spread bits, the program-size is naturally associated with the physical *length* as its simplest case. Furthermore, if an encoding medium would allow greater or lesser "packing-tightness" of the bits, it can be modelled with its conjugate variable, the *Force* in *Newtons*, pushing the bits together or pulling them apart. If one wishes instead to investigate geometries of higher dimensions, one can use different units. For the $2D$ case it can be mapped to an *Area* in m^2 and its conjugate variable will be the *Surface tension* in N/m. For the $3D$ case, the program-size can be mapped to a *Volume* in m^3 and its conjugate variable will be the *Pressure* in N/m^2. Even higher dimensions could be used, but their physical interpretation, if any, would be less clear.
— Only the halting event remains. As it is the only quantity with *no units*, it is natural to map it to the *Energy* in *Joules*. Indeed, in the Gibbs ensemble the energy is the only

Table 2. *The preferred correspondence between algorithmic thermodynamics and statistical physics.*

Observable	Variable	Units	Conjugate	Variable	Units
Halting event	E	J	Temperature	T	K
Program-size (length)	x	m	Force	F	N
Program-size (area)	A	m^2	Stiffness	k	N/m
Program-size (volume)	V	m^3	Pressure	p	N/m^2
Program-frequency	f	$1/s$	Action	$\mathcal{S}$	Js
Program-runtime	t	s	Power	P	W

observable not multiplied by a conjugate variable. Adding extra units to the halting event, only to have them cancelled out by a conjugate variable, would be futile.

Summarizing the points above, I obtain Table 2 as the mapping of choice between *algorithmic thermodynamics* and *physical thermodynamics*.

1.7. *State equation*

The state equation for the partition function (22) is,

Definition 42 (Algorithmic state equation).

$$dE = \frac{1}{\ln 2} T dS - 2\pi \mathcal{S} df - Dd|p|$$

This is analogous to the law of conservation of energy, interpreted as a law of conservation of halting information. I will now take the Taylor series of $Dd|p|$. To do so, I first pose $L(p) := |p|$. Then I obtain,

$$DL(p) = DL(0) + L'(0)p + \frac{1}{2}L''(0)p^2 + \frac{1}{6}L''(0)p^3 + ... \tag{43}$$

$$DdL(p) = DL'(0)dp + DL''(0)pdp + D\frac{1}{2}L'''(0)p^2dp + ... \tag{44}$$

switching the notation from p to x, I get

$$DdL(x) = DL'(0)dx + DL''(0)xdx + D\frac{1}{2}L'''(0)x^2dx + ... \tag{45}$$

then further posing $F := DL'(0)$, $k := DL''(0)$, $p := DL''(0)$ (here p means the pressure, not a program), I get

$$DdL(x) = Fdx + kxdx + px^2dx + ... \tag{46}$$

To recover the physical interpretation, it suffices that I replace $Dd|p|$ with its Taylor expansion. The state equation, in the physical interpretation, is:

Definition 47 (Physical state equation).

$$dE = \frac{1}{\ln 2} T dS - 2\pi \mathcal{S} df - \left(F + kx + px^2 + ...\right) dx$$

and if a three-dimensional simplification cutoff is desired, we get

$$dE = \frac{1}{\ln 2} T dS - 2\pi \mathcal{S} df - \left(F + kx + px^2\right) dx \tag{48}$$

Solutions to this state equation yields entropic and computed "universes" generated by the calculation of Ω with a prefix-free UTM. Note that as the Taylor expansion was taken, the physical state equation is only defined for smooth functions of $L(p) = |p|$. Therefore, a smoothness of the space of $L(p)$ is implicitly assumed. Furthermore, the smoothness approximation will apply in a domain where the lengths are large with respect to the program step size. In the case of the universe, where the program step length could be on the order of the Planck length, the smoothness approximation would apply for lengths $\gg$ Planck length.

2. Thermodynamics

2.1. *Time*

Theorem 49. The state equation (47) implies a halting entropy decreasing with time.

Proof.

$$dE = \frac{1}{\ln 2} T dS - 2\pi \mathcal{S} df - \left(F + kx + px^2 + ...\right) dx \qquad \text{state equation} \tag{50}$$

$$0 = (\ln 2)^{-1} T dS - 2\pi \mathcal{S} df \qquad \text{posing } dE = dx = 0 \tag{51}$$

$$0 = (\ln 2)^{-1} T dS + 2\pi t^{-2} \mathcal{S} dt \qquad df = -t^{-2} dt \tag{52}$$

$$0 = (\ln 2)^{-1} T dS + P dt \qquad P = 2\pi t^{-2} \mathcal{S} \tag{53}$$

$$\implies \frac{dS}{dt} = -(\ln 2) \frac{P}{T} \qquad \text{(decreasing entropy)}$$

□

Definition 54 (Halting entropy). The halting entropy is the entropy exclusively associated with the calculation of Ω over time. It is the entropy obtained when $dE = dx = 0$.

As time increases the entropy from the calculation of Ω decreases according to the term $-(\ln 2)P/T$. Why does it decrease over time? Consider that at the beginning of the calculation none of the bits of $\Omega(t)$ are known, hence the error rate is at its maximum. Each bit with an unknown value contributes $k_B \ln 2$ to the entropy. As the calculation progresses and the error rate is diminished, then each additional and correct bit that has been calculated becomes fixed and their entropy contributions are reduced to 0.

As a result, an arrow of time connected to the non-computability of Ω can be attributed to the system as follows. A forward translation in time is associated with an increase in halting information. Furthermore, since each bit of Ω is algorithmically random, then the future, which can only be described with more bits of Ω, is guaranteed to be non-computable. While the past, which holds less bits than the present, is guaranteed to be computable from the present. This corresponds more closely to our human experience, as we can remember and even deduce the past based on present evidence, but cannot precisely know the future until it happens.

Furthermore, as the entropy of the valid bits of Ω is exactly 0, then it means that the past of the system is fixed and cannot be changed. Again, this more closely matches our human experience as we cannot change our past, so why would its halting entropy be anything other than 0?

2.2. *Exfoliation*

As an entropy decreasing with time would violate the second law of thermodynamics, I suggest that an entropic exfoliation to space occurs so as to make the second law hold. In this scenario, the entropy reduction from the calculation of Ω is compensated by an increase in entropy associated with the exfoliation observables. Consider the following theorem.

Theorem 55. The state equation (47), the second law of thermodynamics and theorem (49) implies an entropic exfoliation to space.

Proof.

$$dE = (\ln 2)^{-1} T dS - 2\pi \mathcal{S} df - \left(F + kx + px^2 + ...\right) dx \qquad \text{state equation} \qquad (56)$$

$$0 = (\ln 2)^{-1} T dS - 2\pi \mathcal{S} df - \left(F + kx + px^2 + ...\right) dx \qquad \text{posing } dE = 0 \qquad (57)$$

$$0 = (\ln 2)^{-1} T dS + 2\pi t^{-2} \mathcal{S} dt - \left(F + kx + px^2 + ...\right) dx \qquad df = -t^{-2} dt \qquad (58)$$

$$0 = (\ln 2)^{-1} T dS + P dt - \left(F + kx + px^2 + ...\right) dx \qquad P = 2\pi t^{-2} \mathcal{S} \qquad (59)$$

$$\frac{dS}{dt} = (\ln 2) \frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right) - (\ln 2) \frac{P}{T} \qquad \text{(exfoliation)}$$

□

Definition 60 (Exfoliation entropy). The exfoliation entropy is the entropy contribution by the term $(\ln 2) \frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right)$ to the entropy.

To investigate this result, let us look at three cases;

$$\frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right) < \frac{P}{T} \implies \frac{dS}{dt} < 0 \qquad \text{decreasing entropy} \qquad (61)$$

$$\frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right) = \frac{P}{T} \implies \frac{dS}{dt} = 0 \qquad \text{constant entropy} \qquad (62)$$

$$\frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right) > \frac{P}{T} \implies \frac{dS}{dt} > 0 \qquad \text{increasing entropy} \qquad (63)$$

At (62), a shift occurs in the direction of the production of entropy over time. It is the point at which the exfoliation entropy overtakes and exceeds the reduction in halting entropy. The second law of thermodynamics, which states that $dS/dt \geq 0$ will hold for (62) and (63), but will be violated for (61). In any case, if $(\ln 2) \frac{1}{T} \frac{dx}{dt} \left(F + kx + px^2 + ...\right) > 0$ then the second law of thermodynamics applicable to the exfoliation observables will be observed.

This derivation more closely matches human experience. Indeed,

1 at the beginning of time the future of the system is un-actualized, hence the possibilities are endless. To reflect this, the halting entropy is at its maximum at $t = 0$, and the exfoliation entropy is equal to 0. This matches our current belief that the exfoliation entropy at the Big Bang is very low.

2 during the evolution the future becomes past which is "set in stone". As the past is "set in stone", the halting entropy of the bits defining it are equal to 0. This is because we "remember" or "observe" only one past. This reduction in halting entropy is offset by a growth in exfoliation entropy, which is related to the size and complexity of the space encoded by the exfoliation observables. This growth in space entropy obeys the second law of thermodynamic.

3 at the end of time there is no future. The value of Ω has been calculated, and the full history of the system is now "set in stone". The halting entropy is 0 and the exfoliation entropy is at its maximum. This matches the hypothesis of the heat death.

Note that contrary to the halting entropy, the exfoliation entropy of an observer's past does not need to be equal to 0 as multiple exfoliated micro-states could be compatible with an observer's present. Indeed, as per the second law of thermodynamics, the observer sees a monotonically increasing exfoliation entropy.

How then do we understand this result from the perspective of algorithmic information theory? The exfoliation variable represents the entropy in the choice of available prefix-free encodings for the programs of the UTM. When no bits of Ω are known, it doesn't make sense to speak of the ways to encode this information as there is nothing to encode. Hence, the entropy should be 0. As more bits of Ω are known then more ways to encode this information exist and the entropy associated with the possible encodings increases.

2.3. *Holographic principles*

Theorem 64. The state equation (47) implies a holographic principle in the area-dominant regime, where xdx is the dominant contributor to the exfoliation entropy.

Proof.

Under the approximation $kx \gg \left(F + px^2 + ...\right)$, the state equation is,

$$dE = (\ln 2)^{-1} TdS - 2\pi \mathcal{S} df - kxdx \qquad \text{state equation approx.} \tag{65}$$

$$TdS = (\ln 2) kxdx \qquad \text{posing } dE = dt = 0 \tag{66}$$

$$\int TdS = (\ln 2) \int kxdx \tag{67}$$

$$TS = (\ln 2) k \frac{1}{2} x^2 + C \tag{68}$$

$$\implies S \propto A \qquad \text{(holographic principle)}$$

□

The laws of physics, which will be derived from the area-dominant approximation $kx \gg \left(F + px^2 + ..\right)$ will necessarily contain a holographic principle linking the entropy to the area enclosing the volume. However, the holographic principle need not necessarily hold at other entropic growth scales, for example, where the volumetric entropy x^2dx is dominant. Indeed, state equation (47) would appear to suggest three different scales, each having a "holographic principle" of a different dimensional size.

Dimension	Dominant force	Approximation	Entropy	
$1D$	Fdx	$F \gg kx + px^2 + ...$	$S \propto L$	(69)
$2D$	$kxdx$	$kx \gg F + px^2 + ...$	$S \propto A$	(70)
$3D$	px^2dx	$px^2 \gg F + kx + ...$	$S \propto V$	(71)
...				

In this scenario, the universe would be dominated by the linear scale at short distances, which would then be overtaken by the area scale and, finally, by the volume scale. In the next section, I will show that special relativity and the law of inertia are derivable in the $S \propto L$ scale, that general relativity is derivable at the $S \propto A$ scale and will provide citations suggesting that dark energy might be derivable from the $S \propto V$ scale.

2.4. *Spacetime*

Theorem 72. The state equation (47) implies a relation between space and time.

Proof.

Under the approximation $F \gg \left(kx + px^2 + ...\right)$, the state equation is,

$$\begin{aligned}
dE &= (\ln 2)^{-1}TdS - 2\pi\mathcal{S}df - Fdx && \text{state equation approx.} && (73)\\
0 &= -2\pi\mathcal{S}df - Fdx && \text{posing } dE = dS = 0 && (74)\\
0 &= 2\pi t^{-2}\mathcal{S}dt - Fdx && df = -t^{-2}dt && (75)\\
0 &= Pdt - Fdx && P = 2\pi t^{-2}\mathcal{S} && (76)\\
Fdx &= Pdt && \text{add } Fdx && (77)\\
dx &= \frac{P}{F}dt && && \text{(fundamental relation of spacetime)}
\end{aligned}$$

□

The units of P/F are meters per second. This implies that any system described by (47) will have a characteristic power and force that relates time to space. Indeed, if the system is the universe, then by taking the characteristic Planck power and force we do recover the speed of light,

$$P\frac{1}{F} = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{78}$$

In lieu of an appeal to the Planck constant, we are permitted to pose $c := P/F$ as a definition and rewrite the result as

$$dx = cdt \tag{79}$$

which is the fundamental relation of special relativity and c is a constant connecting space to time.

2.5. *Limiting relations*

Theorem 80. The state equation (47) implies a maximum speed.

Proof.

Under the approximation $F \gg \left(kx + px^2 + ...\right)$, the state equation is,

$$\begin{aligned}
dE &= (\ln 2)^{-1} TdS - 2\pi \mathcal{S} df - Fdx && \text{state equation approx.} && (81)\\
0 &= (\ln 2)^{-1} TdS - 2\pi \mathcal{S} df - Fdx && \text{posing } dE = 0 && (82)\\
0 &= (\ln 2)^{-1} TdS + 2\pi t^{-2} \mathcal{S} dt - Fdx && df = -t^{-2} dt && (83)\\
0 &= (\ln 2)^{-1} TdS + Pdt - Fdx && P = 2\pi t^{-2} \mathcal{S} && (84)\\
Fdx - Pdt &= (\ln 2)^{-1} TdS && \text{add } Fdx - Pdt && (85)\\
\frac{dx}{dt} - \frac{P}{F} &= \frac{1}{\ln 2} \frac{T}{F} \frac{dS}{dt} && && \text{(maximum speed)}
\end{aligned}$$

□

To see why this implies a maximum speed, first consider that the units of this equation are meters per second. Second, consider the following three cases;

$$\frac{dx}{dt} = \frac{P}{F} \implies \frac{dS}{dt} = 0 \tag{86}$$

$$\frac{dx}{dt} < \frac{P}{F} \implies \frac{dS}{dt} < 0 \tag{87}$$

$$\frac{dx}{dt} > \frac{P}{F} \implies \frac{dS}{dt} > 0 \tag{88}$$

To prove that the speed P/F is a maximum, it suffices to note the presence of a reversal of the second law of thermodynamics at the P/F barrier. Furthermore, as the irreversibility of the second law of thermodynamics is well established, it follows that the barrier cannot be overcome. A system evolving faster than c will experience a reversal of the second law compared to a system slower than c (and vice-versa), but neither will be able to cross c and flip its direction.

Please note that in standard physics the speed of light is accepted as an axiom and is not derived from more fundamental principles. Here the speed of light is a direct consequence of (47).

Theorem 89. The following relations each characterize a maximum quantity.

approx.			
none	$\frac{1}{\ln 2} T \frac{dS}{dt} = \frac{dE}{dt} - P$	maximum power (J/s)	(90)
$S \propto L$	$\frac{1}{\ln 2} \frac{T}{F} \frac{dS}{dt} = \frac{dx}{dt} - \frac{P}{F}$	maximum speed (m/s)	(91)
$S \propto A$	$\frac{1}{\ln 2} \frac{T}{k} \frac{dS}{dt} = \frac{xdx}{dt} - \frac{P}{k}$	maximum viscosity (m^2/s)	(92)
$S \propto V$	$\frac{1}{\ln 2} \frac{T}{p} \frac{dS}{dt} = \frac{x^2dx}{dt} - \frac{P}{p}$	max. vol. flow rate (m^3/s)	(93)

Proof. Each relation can easily be obtained from (47) by posing the other observables to 0. To prove that the quantities are a maximum, it suffices to notice that they each correspond to the point at which the second law of thermodynamics is reversed. □

Theorem 94. The partition function (22) implies a discrete halting entropy with a minimum step.

This theorem has a stronger requirement than the previous two theorems on maximum quantities. It is not enough to just prove an extremum value, but a minimum value that is also discrete.

Proof. To prove it, recall how Z_Ω calculates an estimation of Ω valid within a monotonically deceasing error rate ξ. Knowing the precise value of ξ is equivalent to knowing Ω (as Ω can simply be recovered by adding ξ to Z_Ω). The implication is that the bits of ξ must also be non-computable. As one bit of ξ is enough to recover one bit of Ω, it follows that, as ξ is the error rate, no bits of ξ can be known beyond the position of its first one-valued bit.

Second, let us define a non-divergent entropy for the system $\Omega = Z_\Omega + \xi$. As the system is infinitely complex, its entropy will be convergent only for the first $N \in \mathbb{N}$ bits. The bits of Z_Ω have an entropy of 0, and the bits of ξ have an entropy of $N_\xi k_B \ln 2$.

$$S = N_{Z_\Omega} k_B \ln 1 + N_\xi k_B \ln 2 \quad (95)$$

$$= N_\xi k_B \ln 2 \quad (96)$$

As a result, the smallest entropy of the system S_0 is $k_B \ln 2$. Furthermore, the entropy increases by steps of $k_B \ln 2$, as N_ξ is a natural number. This proves the theorem. □

Theorem 97. Exfoliation observables and exfoliation conjugates are discrete as per the discrete halting entropy.

Proof. Consider the following relations connecting the halting entropy to the exfoliation variables, $dS = (\ln 2)dE$, $dS = (\ln 2)(F + kx + px^2 + ...)dx$ and the halting variables $dS = (\ln 2)2\pi \mathcal{S} df$ and $dS = -(\ln 2)Pdt$. If dS is discrete, then it implies that these variables dE, $\mathcal{S}$, F, k, p, df, P, dx and dt are also discrete.

Without loss of generality, consider the pair $\mathcal{S}df$. Since dS is discrete (94), then both

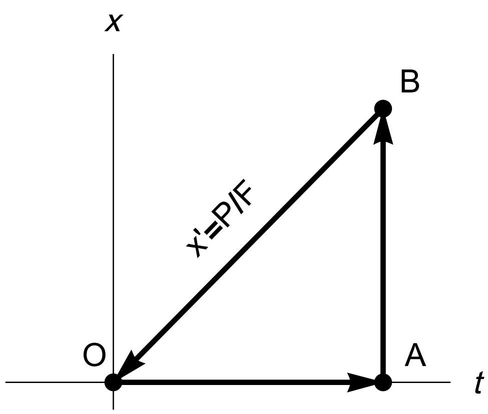

Fig. 1. A thermodynamic cycle through space, time and entropy as observables.

$\mathcal{S}$ and df must be discrete. This reasoning can be applied to all exfoliation and halting variables. Why must both $\mathcal{S}$ and df be discrete? Suppose that either $\mathcal{S}$ or df are real. Then a real multiplied by a real is a real, and a real multiplied by a whole number is also a real. Hence, for dS to be a whole number, both $\mathcal{S}$ and df must both be whole number.

Another way to see it is that the entropy of a real number can be infinite, but we are only allowed an entropy $Nk_B \ln 2$, hence none of these variables can be arbitrary real numbers. □

The discretization of observables, notably the action observable, has been used since the early days of quantum physics to justify and explain it. Indeed, the explanation of the photoelectric effect and the black body radiation, two early successes of quantum physics, were both explained via the discretization of what I refer in here as the exfoliation variables. Could the discretization of exfoliation variables imply a certain quantum character applicable to the partition function? Further connections to quantum physics are investigated by the author in another paper.

3. Relativity

3.1. *Light cones as thermodynamic cycles*

In this section, I look at the thermodynamic cycle of the system transiting through time and space starting at O to A to B and back to O, as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles, such as those involving pressure and volume but also of relativistic light cones.

I pose that $dE = 0$ and the $S \propto L$ approximation throughout the cycle.

$$\frac{1}{\ln 2} T dS = F dx - P dt \tag{98}$$

O to A: As O is translated forward in time to A while keeping the distance constant ($dx = 0$), the halting entropy must decrease over time to compensate.

$$\left(\frac{1}{\ln 2}TdS = Fdx - Pdt\right|_{dx=0} \tag{99}$$

$$\implies \frac{dS}{dt} = -(\ln 2)\frac{P}{T} \tag{100}$$

A forward translation in time causes the system to know more bits of Ω. This reduces the halting entropy. Conversely, a backward translation in time causes the system to erase bits from its pool of information so as to increase its halting entropy. A backward translation in time is equivalent to erasing halting information about the system's present.

A to B: As A is translated forward in space to B while keeping the time constant ($dt = 0$), the exfoliation entropy must increase over space to compensate.

$$\left(\frac{1}{\ln 2}TdS = Fdx - Pdt\right|_{dt=0} \tag{101}$$

$$\implies \frac{dS}{dx} = (\ln 2)\frac{F}{T} \tag{102}$$

I conclude that the further away from A a region is, the higher its exfoliation entropy will be. Since $dt = 0$, no change in time is experienced.

O to B: As O is translated forward both in time and in space to B while keeping the entropy constant ($dS = 0$), the system has a velocity at the speed c.

$$\left(\frac{1}{\ln 2}TdS = Fdx - Pdt\right|_{dS=0} \tag{103}$$

$$\implies \frac{dx}{dt} = \frac{P}{F} = c \tag{104}$$

I conclude that an object travelling at speed c is neither encouraged nor discouraged by entropy. However, the type of entropy changes. The rate P/F is the rate of conversion of halting entropy to exfoliation entropy. At O, the system is comprised exclusively of halting entropy as its future is not yet determined. As the system evolves towards B, its halting entropy is decreased over time as the system replaces its future entropy with a singular past. Its exfoliation entropy, however is increased over space to offset the reduction.

As a backward translation in time erases the most recently calculated bits of Ω, I conclude that the system "forgets its future" during the backward translation.

3.2. *Lorentz's transformation*

To recover the Lorentz's factor γ, let us consider figure 2. Two observers start at the origin S and travel in space-time respectively to O and O'. We regard O' as traveling

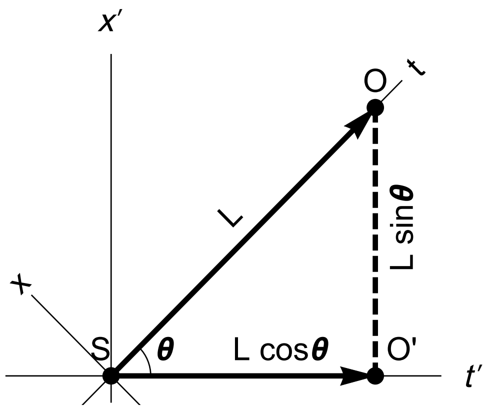

Fig. 2. The spacetime intervals between two observers. Here O' travels at speed $|v|$ in O's reference frame.

at speed $|v|$ in O's reference frame. From standard trigonometry, I derive the following values for the length of the segment;

Segment	Length	
$\left\lvert\overline{SO}\right\rvert$	L	(105)
$\left\lvert\overline{SO'}\right\rvert$	$L\cos\theta$	(106)
$\left\lvert\overline{O'O}\right\rvert$	$L\sin\theta$	(107)

I start with the Pythagorean theorem and solve for $\cos\theta$.

$$\left\lvert\overline{SO}\right\rvert^2 = \left\lvert\overline{SO'}\right\rvert^2 + \left\lvert\overline{O'O}\right\rvert^2 \tag{108}$$

$$L^2 = (L\cos\theta)^2 + (L\sin\theta)^2 \tag{109}$$

$$1 = (\cos\theta)^2 + (\sin\theta)^2 \tag{110}$$

$$\sqrt{1-(\sin\theta)^2} = \cos\theta \tag{111}$$

I consider that the distance between two observers moving at constant speed is simply vt. Hence, $\left\lvert\overline{O'O}\right\rvert = vt$. Solving for $\sin\theta$, I obtain

$$\left\lvert\overline{O'O}\right\rvert = vt = L\sin\theta \tag{112}$$

$$\implies \sin\theta = \frac{vt}{L} \tag{113}$$

From equation (111) and (113), I get the reciprocal of the Lorentz factor,

$$\sqrt{1-\frac{v^2t^2}{L^2}} = \cos\theta = \gamma^{-1} \tag{114}$$

$$\implies \gamma = \frac{1}{\sqrt{1-\frac{v^2t^2}{L^2}}} \tag{115}$$

Finally, I consider that L is the distance travelled in time by O in its own reference frame. This is given via the relation $dx = cdt$. Hence $L = ct$. I obtain,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{116}$$

which is the well-known Lorentz factor and is the multiplication constant connecting $|\overline{SO}|$ to $|\overline{SO'}|$.

3.3. *1D-Holographic principle (law of inertia)*

First, let us derive a relation between dS and dN.

Theorem 117. $dS = (\ln 2)k_B dN$

Proof.

$$S = Nk_B \ln(2) \qquad \text{from 96} \tag{118}$$

$$\implies dS = (\ln 2)k_B dN \qquad \text{(binary entropy)}$$

□

Second, let us look at the implications of the first term, Fdx in the $S \propto L$ regime.

Theorem 119. The $S \propto L$ scale implies the law of inertia, $F = ma$.

Proof. First, consider that as the entropy S, is related to the bits, then $dS = (\ln 2)k_B dN$ where N is the number of bits. Second, I will derive the equation for an entropic force.

Under the approximation $F \gg (kx + px^2 + ...)$, the state equation is,

$$dE = \frac{1}{\ln 2}TdS - 2\pi\mathcal{S}df - Fdx \qquad \text{state equation approx.} \tag{120}$$

$$0 = (\ln 2)^{-1}TdS - Fdx \qquad \text{posing } dE = df = 0 \tag{121}$$

$$Fdx = (\ln 2)^{-1}TdS \qquad \text{add } Fdx \tag{122}$$

$$F = (\ln 2)^{-1}T\frac{dS}{dx} \qquad \text{divide } dx \tag{123}$$

$$F = (\ln 2)^{-1}T\frac{(\ln 2)k_B dN}{dx} \qquad \text{binary entropy} \tag{124}$$

$$F = Tk_B\frac{dN}{dx} \qquad \text{entropic force} \tag{125}$$

As my goal is to recover $F = ma$, I must link T to the acceleration. To do so, I will use the Unruh temperature (Fulling(1973); Davies(1975); Unruh(1976)) experienced by a body undergoing constant acceleration as suggested by Erik Verldinde in (Verlinde(2011)). The existence of a well-defined temperature allows me to conclude that the system is described at thermodynamic equilibrium, hence the state equation holds.

$$F = \left(\frac{\hbar a}{2\pi c k_B}\right) k_B \frac{dN}{dx} \qquad \text{Unruh temperature} \tag{126}$$

$$F = \left(\frac{1}{2\pi}\frac{\hbar}{c}\frac{dN}{dx}\right) a \qquad \text{clean up} \tag{127}$$

Finally, the equation $F = ma$ can be recovered provided that the ratio dx/dN is the reduced Compton wavelength within one multiplication constant.

$$\implies 2\pi\frac{dx}{dN} = \frac{\hbar}{mc} \tag{128}$$

What does this means from the algorithmic perspective and why is $(2\pi)dx/dN$ the reduced Compton wavelength? dx/dN is the ratio between the position of an object and the number of bits required to express such a position. It implies that each increment of an object's position by its reduced Compton wavelength must use one additional bit of entropy. The algorithm to encode position used by the UTM is of the form $x = n\lambda$, where n indicates the number of times its reduced Compton wavelength is repeated to reach its position. The entropy usage is optimized as the position of an object does not need to be specified more accurately than its reduced Compton wavelength. □

3.4. *Note on the Schwarzschild radius*

As we have seen, the first term of the Taylor expansion is associated with the inertial mass as it implies $F = ma$. We have also seen that the first term of the Taylor expansion implies a one dimensional version of the holographic principle. As a result, we would expect that the mass in the universe is bounded linearly. Is that the case?

Consider the Schwarzschild radius,

$$R = \frac{2GM}{c^2} \tag{129}$$

As we can see, the radius grows linearly with the mass. Hence, the one dimensional holographic principle associated with the inertial mass holds.

3.5. *2D-Holographic principle (General relativity)*

In this section, I will show how the state equation (47) suggests that general relativity is an emergent entropic phenomenon attributable to the second term, $kxdx$, of the Taylor expansion of $d|p|$.

Theorem 130. The area-dominant regime implies general relativity.

Proof. My goal in this proof is to derive the Einstein field equation of general relativity starting from the holographic principle

$$\frac{1}{\ln 2}TdS = kxdx \tag{131}$$

$$\implies S \propto A \tag{132}$$

$$\implies dE = \gamma dA \tag{133}$$

This has indeed been done before in the literature, notably by Ted Jacobson in (Jacobson(1995)), then later (and differently) by Erik Verlinde in (Verlinde(2011)). Furthermore, Christoph Schiller in (Schiller(2005)) argues that a maximum power (90) implies the Field equation. Here, I will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

Jacobson, starting from $dE = TdS$, first connects dE to an arbitrary coordinate system and energy flow rates,

$$dE = \int T_{ab}k^a d\Sigma^b \tag{134}$$

Here T_{ab} is a energy-momentum tensor, k is a killing vector field and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that, assuming that the holographic principle holds (and in here it does according to 64), the right part of (133) can be rewritten to

$$dA = \frac{c^2}{a}\int R_{ab}k^a d\Sigma^b \tag{135}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab}k^a d\Sigma^b = \gamma\frac{c^2}{a}\int R_{ab}k^a d\Sigma^b \tag{136}$$

which can only be satisfied if

$$T_{ab} = \gamma\frac{c^2}{a}\left[R_{ab} - \left(\frac{R}{2} + \Lambda\right) g_{ab}\right] \tag{137}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant).

□

3.6. *Dark energy*

Associating dark energy to a volumetric entropy has been suggested and discussed by other authors before(Easson et al.(2011)Easson, Frampton, and Smoot; Easson et al.(2012)Easson, Frampton, and Smoot). Here, I suggest that dark energy provides the physical interpretation for the third term of the Taylor expansion.

Posing $dE = df = dx = dA = 0$, I get

$$0 = (\ln 2)^{-1} TdS - pdV \qquad \text{state equation} \tag{138}$$

$$TdS = (\ln 2) pdV \qquad \text{dark energy} \tag{139}$$

To determine the value of the pressure p associated with volumetric entropy, we consider the case of an entropic force. In this case, the pressure relates to the force as

$$F = -pA \tag{140}$$

$$\implies p = -\frac{F}{A} = -\frac{F}{4\pi x^2} \tag{141}$$

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward the enclosing area.

To determine x, it suffices to notice that $(F + kx + px^2)dx$ encodes the informational content of the universe up to a boundary given by x, which is common to all terms of the Taylor expansion. Physically, it makes sense to connect this bound to the Hubble horizon as it defines an event horizon applicable to the ”instantaneous system”. As it is an event horizon, its temperature is given by De Sitter’s temperature and is constant at the horizon. Therefore, an entropic force is expected. To obtain the magnitude of the force, it suffices to calculate the entropic force as per the Bekenstein-Hawking entropy and the De Sitter temperature, both applicable to event horizons.

$$dS = 2\pi \frac{k_B c^3}{G\hbar} x dx \qquad \text{Bekenstein-Hawking entropy} \tag{142}$$

$$T = \frac{\hbar}{k_B} \frac{H}{2\pi} \qquad \text{De Sitter temperature} \tag{143}$$

$$F = T \frac{dS}{dx} \qquad \text{entropic force} \tag{144}$$

$$\implies F = \left(\frac{\hbar}{k_B} \frac{H}{2\pi}\right) \left(2\pi \frac{k_B c^3}{G\hbar} x\right) \tag{145}$$

$$= \frac{c^3}{G} Hx \qquad \text{clean up} \tag{146}$$

As x is the radius of the Hubble horizon $x = c/H$, we obtain the final value of the force $F = c^4/G$, the Planck force. Finally, the pressure is given by;

$$F = \frac{c^4}{G} \qquad \text{Planck force} \tag{147}$$

$$\implies p = -\frac{F}{A} = -\left(\frac{c^4}{G}\right) \left(\frac{1}{4\pi (c/H)^2}\right) \tag{148}$$

$$p = -\frac{c^2 H^2}{4\pi G} \qquad \text{(negative pressure)}$$

This is close to the current measured value for the negative pressure associated with dark energy (Easson et al.(2011)Easson, Frampton, and Smoot). As we can see, the entropic derivation of dark energy they suggest applies to the third term of the Taylor expansion.

4. Characteristic units

My goal in this section is to show how the definition of the Planck units naturally follows from the state equation (47). To do so, I must first obtain definitions for G, c and $\hbar$ by deriving from (47) known laws of physics that contain them. I start by obtaining the gravitational constant G from Newton's law of gravitation.

Theorem 149. The gravitational constant G is defined as $c^3L^2/\hbar$.

Proof.
A derivation of Newton's law of gravitation from the entropic perspective has been done before in (Verlinde(2011)). I work in the area-dominant regime where $kx \gg \left(F + px^2 + ...\right)$. This regime contains the holographic principle and, as a result, the entropy of the system grows via x^2, an area law. I further consider that the entropy of this area law is given by bits exclusively occupying a small area L^2 on the surface. In this case, the total number of bits on the surface is given by

$$N = \frac{4\pi x^2}{L^2} \tag{150}$$

The equipartition theorem applies to energy terms of the partition function, which are quadratic. The term $kxdx$ is $\frac{1}{2}kx^2$ in the partition function. As a result its average energy is $E = \frac{1}{2}Nk_BT$ as per the equipartition theorem.

$$E = \frac{1}{2}\left(\frac{4\pi x^2}{L^2}\right)k_BT \tag{151}$$

$$\implies T = \frac{L^2}{2\pi k_B}\frac{E}{x^2} \tag{152}$$

I obtain a constant temperature throughout the system indicating that it is at thermodynamic equilibrium. As my goal is to recover the gravitational constant, I inject this temperature in the entropic force relation.

$$F = Tk_B\frac{dN}{dx} \qquad \text{entropic force (125)} \tag{153}$$

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)k_B\frac{dN}{dx} \qquad \text{derived temperature} \tag{154}$$

I then replace the ratio dx/dN by the reduced Compton wavelength.

$$F = \left(\frac{L^2}{2\pi k_B} \frac{E}{x^2} \right) k_B \left(2\pi \frac{mc}{\hbar} \right) \tag{155}$$

$$F = \left(\frac{L^2 c}{\hbar} \right) \frac{Em}{x^2} \qquad \text{clean up} \tag{156}$$

I then convert E to its rest mass via $E = mc^2$.

$$F = \left(\frac{L^2 c^3}{\hbar} \right) \frac{Mm}{x^2} \tag{157}$$

I obtain the Newton's law of gravitation along with a definition for G.

$$F = G \frac{Mm}{x^2} \tag{158}$$

$$\implies G = \frac{L^2 c^3}{\hbar} \tag{159}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{(Planck's length)}$$

□

Theorem 160. The speed of light c is defined by P/F.

Proof. I refer to the proof for theorem 80 where P/F is a characteristic speed associated with an inversion in the direction of the second law of thermodynamics. Then, under the principle that the second law is irreversible, the speed P/F is a boundary and defines c. □

Theorem 161. The action $\mathcal{S}$ is defined by $\hbar$.

Proof.

$$dE = \frac{1}{\ln 2} T dS - 2\pi \mathcal{S} df + \left(F + kx + px^2 + ... \right) dx \qquad \text{state equation} \tag{162}$$

$$dE = -2\pi \mathcal{S} df \qquad \text{posing } dS = dx = 0 \tag{163}$$

Switching to the angular frequency,

$$dE = -\mathcal{S} d\omega \qquad df = d\omega/(2\pi) \tag{164}$$

$$\int dE = -\int \mathcal{S} d\omega \tag{165}$$

$$E = -\mathcal{S}\omega + C \tag{166}$$

Posing $C = 0$ and flipping the axis for ω, this is the photon angular-frequency to energy relation $E = \hbar\omega \implies \mathcal{S} = \hbar$. □

I have now obtained a definition for three of the fundamental constants.

$$\hbar = \mathcal{S} \qquad c = \frac{P}{F} \qquad G = \frac{L^2 c^3}{\hbar} \tag{167}$$

I can now definite characteristic units applicable to the UTM,

$$G = \frac{L^2 c^3}{\hbar} \implies L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{(Planck's length)}$$

$$t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} \qquad \text{(Planck's time)}$$

$$E = \mathcal{S}/t \implies E = \sqrt{\frac{\hbar c^5}{G}} \qquad \text{(Planck's energy)}$$

$$P = t^{-2}\mathcal{S} = \frac{c^5}{G} \qquad \text{(Planck's power)}$$

$$\frac{P}{F} = c \implies F = \frac{c^4}{G} \qquad \text{(Planck's force)}$$

which agrees with the physical Planck units.

5. Discussion

I have shown a method in which many laws of physics can come out of a purely informational system. Indeed, any partition function that includes $2\pi\mathcal{S}f$, entropic time, and $D|p|$, entropic space, will imply such laws regardless of the microscopic system that it encodes. So why pick the halting probability of a prefix-free universal Turing machine as the microscopic interpretation of the entropy and not, well, anything else?

Normally, the entropy connects to a physical system. For example, the entropy of an ideal gas connects to the position of the molecules in the gas itself. In contrast, for a system described by the calculation of Ω over time, the *entropy is the system.* Many authors have suggested that the universe may be informational at its most fundamental level. The derivation herein shows that for a purely informational system, the laws of physics still come out. The fact that a system is encodable as provably *incompressible information* is apparently enough to recover them. As a result, the laws of physics herein derived may be connected to information itself.

Second, when $t \to \infty$, the partition function indeed becomes the Tadaki D-random number, which is connected to the halting probability Ω via a multiplication constant D applicable to program lengths. For each possible micro-state of the proposed partition function, we can associate to it a halting probability for a certain universal Turing machine whose D-random number is recovered at $t \to \infty$.

The entropy can be interpreted as encoding the answer to a series of yes/no equations of the form "Is this sentence [...] a theorem of the universe, yes or no?". As these are the most general questions that we can ask in a recursively enumerable axiomatic system, it follows that any formalizable system (including any formal theory of the universe) can be encoded as such. In the case of the universe, the number of such questions required to completely describe it is of course astronomical. It is most likely equal to the holographic entropy bound of the universe containing $\approx 10^{122}$ bits of information.

Hence, the laws of physics herein derived are a consequence of the fact that the universe can be encoded as a list of incompressible yes/no facts. What those facts are appears to have no bearing on these specific laws.

6. Conclusion

We note an affinity between an entropic UTM and the laws of physics. The affinity occurs when we consider a UTM calculating its Ω number in a manner so as to maximize the entropy throughout the calculation. When the entropy is maximized, the halting probability becomes a Gibbs ensemble. As a result, additional program observables can be added to the halting probability while preserving its connection to Ω. The laws of physics come out when a single observable is added, $\mathcal{S}df$, as it is sufficient to make the calculation converge towards Ω over time.

Understanding physics from the perspective of an entropic UTM holds several conceptual advantages. For one, we can now define a non-computable future with a computable singular past, where the halting entropy is 0. This provides us with an arrow of time closely matching human experience. The entropy of the complete system (which includes future possibilities, as well as an encoding scheme for the past) does remain constant over time as the change of entropy of one is offset by the other. The second law of thermodynamics, understood as an increase in entropy over time, is perceived in the exfoliation variables, while the larger system, made to include future possibilities, has a constant entropy over time. In this system, future possibilities are consumed to produce space-encoding possibilities.

The decomposition of the program encoding scheme used by the UTM via a Taylor expansion produces terms that can be linked to a scale where a specific entropic force is dominant. For the first Taylor expansion term, we recover special relativity (speed of light (80), light-cones (figure 1) and the Lorentz's factor (figure 2)) and the law of inertia (119). For the second term, we recover general relativity (130) and the holographic principle (64). Finally, the third term might be related to an entropic explanation of dark energy (139). This agrees with our prior conception of the laws of physics.

Provided that the isomorphism between the laws of physics and the "entropic" universal Turing machine described herein holds, it would appear that we now have discovered the computer program that runs the universe.

References

John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. . URL `http://dx.doi.org/10.1017/S0960129511000521`.

C. H. Bennett, P. Gacs, Ming Li, P. M. B. Vitanyi, and W. H. Zurek. Information distance. IEEE Transactions on Information Theory, 44(4):1407–1423, July 1998. ISSN 0018-9448. .

Cristian S. Calude and Michael A. Stay. Natural halting probabilities, partial randomness, and zeta functions. Inf. Comput., 204(11):1718–1739, November 2006. ISSN 0890-5401. . URL `http://dx.doi.org/10.1016/j.jc.2006.07.003`.

Gregory J. Chaitin. A theory of program size formally identical to information theory. J. ACM, 22(3):329–340, July 1975. ISSN 0004-5411. . URL `http://doi.acm.org/10.1145/321892.321894`.

P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4):609, 1975. URL `http://stacks.iop.org/0305-4470/8/i=4/a=022`.

Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3):273 – 277, 2011. ISSN 0370-2693. . URL `http://www.sciencedirect.com/science/article/pii/S0370269310014048`.

Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12):1250066, 2012. . URL `http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662`.

Edward Fredkin and Tommaso Toffoli. Conservative logic. International Journal of Theoretical Physics, 21(3):219–253, Apr 1982. ISSN 1572-9575. . URL `https://doi.org/10.1007/BF01857727`.

Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. . URL `https://link.aps.org/doi/10.1103/PhysRevD.7.2850`.

Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. . URL `https://link.aps.org/doi/10.1103/PhysRevLett.75.1260`.

Andrei Nikolaevich Kolmogorov. Three approaches to the definition of the concept "quantity of information". Problemy peredachi informatsii, 1(1):3–11, 1965.

Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986.

Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. . URL `https://doi.org/10.1007/s10773-005-4835-2`.

R.J. Solomonoff. A formal theory of inductive inference. part i. Information and Control, 7(1):1 – 22, 1964. ISSN 0019-9958. . URL `http://www.sciencedirect.com/science/article/pii/S0019995864902232`.

Leo Szilard. On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings. Behavioral Science, 9(4):301–310, 1964. ISSN 1099-1743. . URL `http://dx.doi.org/10.1002/bs.3830090402`.

Kohtaro Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. Hokkaido Math. J., 31(1):219–253, 02 2002. . URL `http://dx.doi.org/10.14492/hokmj/1350911778`.

Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic information theory. In Local Proceedings of the Computability in Europe 2008 (CiE 2008), pages 425–434. University of Athens, Greece, Jun 2008. URL `http://arxiv.org/abs/0801.4194`.

W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14:870–892, Aug 1976. . URL `https://link.aps.org/doi/10.1103/PhysRevD.14.870`.

Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. . URL `https://doi.org/10.1007/JHEP04(2011)029`.

A K Zvonkin and L A Levin. The complexity of finite objects and the development of the concepts of information and randomness by means of the theory of algorithms. Russian Mathematical Surveys, 25(6):83, 1970. URL `http://stacks.iop.org/0036-0279/25/i=6/a=R05`.