Special relativity: its inconsistency with the standard wave equation

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Abstract

By means of the Lorentz Transformation, Einstein's Special Theory of Relativity purports invariance of the standard wave equation. Counter-examples, satisfying the Lorentz Transformation, and hence Lorentz Invariance, prove that the Lorentz Transformation does not in fact produce invariance of the standard wave equation. Systems of clock-synchronised stationary observers are Galilean and necessarily transform by the Galilean Transformation. Einstein's insistence that inertial (i.e. Galilean) systems of clock-synchronised stationary observers transform, not by the Galilean Transformation, but by the non-Galilean Lorentz Transformation, is logically inconsistent. The Special Theory of Relativity is therefore logically inconsistent. Therefore, it is false. The Lorentz Transformation is meaningless.

I. INTRODUCTION

Engelhardt [1] recently proved that Einstein's clock-synchronisation is inconsistent with the Lorentz Transformation. I subsequently generalised his proof to all values of time [2] $t \geq 0$, in accordance with Einstein's time domain [3]. The æitology of this inconsistency is Einstein's false tacit assumption that he can construct systems of clock-synchronised stationary observers consistent with Lorentz Transformation. It has been proven elsewhere [4] that a system of stationary observers satisfying Lorentz Transformation cannot be clock-synchronised and that a system of clock-synchronised observers satisfying Lorentz Transformation cannot be stationary. In each case the set of observers is an infinite set. Only one element of each set has the appearance of being stationary and clock-synchronised. However, neither element (i.e. observer), being as each is singular and thereby privileged, can synchronise its clock with anything, and cannot determine simultaneity with anything, owing to its singularity. Einstein's 'system of clock-synchronised stationary observers' is actually the trivial case of a single observer, which Einstein erroneously allowed to speak for all observers, none of which are in fact equivalent. Although the systems of observers adduced in a previous paper[4] satisfy Lorentz Invariance, they do not make the standard wave equation form invariant, except in one particular case. This particular case constitutes Einstein's 'system of clock-synchronised stationary observers', and being so privileged, violates the fundamental tenet of Einstein's theory, that no observer is privileged.

Permitting any number of observers, as required by Einstein's theory, immediately reinstates the two inequivalent infinite sets of inequivalent observers. Thus, Special Relativity is logically inconsistent, and the Lorentz Transformation meaningless.

II. LORENTZ INVARIANCE

Einstein[3] presented the Lorentz Transformation thus:

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \\ &\xi = \beta(x - vt), \\ &\eta = y, \\ &\varsigma = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}. \end{aligned} \tag{1}$$

By mathematically constructing infinite sets (systems) of stationary observers consistent with Lorentz Transformation it has been shown [4] that the Lorentz Transformation between a system *K* with coordinates *x*, *y*, *z*, *t*, and a system *k* with coordinates $\xi, \eta, \varsigma, \tau$ respectively, has the co-ordinate relations[a],

$$\begin{aligned} &\tau = \beta\left(t_\sigma - \frac{vx_\sigma}{c^2}\right), \\ &x_\sigma = \sigma x_1, \\ &\xi_\sigma = \beta\left(x_\sigma - vt_\sigma\right) = \beta\left\{\left[\sigma\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2}{c^2}\right]x_1 - vt_1\right\}, \\ &t_\sigma = t_1 + \frac{(\sigma - 1)vx_1}{c^2}, \\ &\eta = y, \\ &\varsigma = z, \\ &\beta = 1\Big/\sqrt{1 - v^2/c^2}, \\ &\sigma \in \Re, \end{aligned} \tag{2}$$

where the real number σ labels an observer located at the stationary position x_σ reading a clock time t_σ at that position, and $x_1 \neq 0$ is arbitrary. Setting $\sigma = 1$ yields Einstein's privileged observer, which cannot speak for all observers. Interchanging the systems of coordinates and changing v to $-v$ yields the Inverse Stationary Lorentz Transformation. The system of stationary observers necessarily fixed by the co-ordinate relations (2) is not clock-synchronised.

According to Special Relativity, the 'spacetime interval' is the same for all coordinate systems. Thus,

$$x^2 + y^2 + z^2 - c^2t^2 = \xi^2 + \eta^2 + \varsigma^2 - c^2\tau^2. \tag{3}$$

By the Lorentz Transformation (1), $\eta = y$ and $\varsigma = z$. Therefore,

$$x^2 - c^2t^2 = \xi^2 - c^2\tau^2. \tag{4}$$

[a] Einstein [3] called *K* and *k* 'systems of co-ordinates': *K* his 'stationary system', *k* his 'moving system'.

Substituting into (4) the Stationary Lorentz Transformation (2) yields,

$$
\begin{aligned}
x_\sigma^2 - c^2 t_\sigma^2 &= \sigma^2 x_1^2 - c^2\left[t_1 - \frac{(\sigma-1)vx_1}{c^2}\right]^2 = \\
&= \beta^2\left\{\left[\sigma\left(1-\frac{v^2}{c^2}\right)+\frac{v^2}{c^2}\right]x_1 - vt_1\right\}^2 - c^2\beta^2\left(t_1 - \frac{vx_1}{c^2}\right)^2 \\
&= \xi_\sigma^2 - c^2\tau^2,
\end{aligned}
\tag{5}
$$

thus satisfying Lorentz Invariance.

By mathematically constructing infinite sets (systems) of clock-synchronised observers consistent with Lorentz Transformation it has been shown [4] that the Lorentz Transformation between a system K with coordinates x, y, z, t, and a system k with coordinates $\xi, \eta, \varsigma, \tau$, respectively, has the co-ordinate relations,

$$
\begin{aligned}
&\tau_\sigma = \beta\left(t - \frac{vx_\sigma}{c^2}\right) = \sigma\tau_1, \\
&\xi_\sigma = \beta(x_\sigma - vt), \\
&x_\sigma = \frac{(1-\sigma)c^2 t}{v} + \sigma x_1, \\
&\eta = y, \\
&\varsigma = z, \\
&\beta = 1\big/\sqrt{1 - v^2/c^2}, \\
&1 - \frac{v}{c} < \sigma < 1 + \frac{v}{c}, \\
&\sigma \in \Re,
\end{aligned}
\tag{6}
$$

where once again the real number σ labels an observer located at the position x_σ reading a common clock time t at that position, and $x_1 \neq 0$ is arbitrary. Interchanging the systems of coordinates and changing v to $-v$ yields the Inverse Clock-Synchronised Lorentz Transformation. Here the range on the real number (label) σ is determined by constraints imposed by Lorentz Transformation [4]. From (6),

$$
\frac{dx_\sigma}{dt} = \frac{(1-\sigma)c^2}{v} < c \quad \Rightarrow \quad 1 - \frac{v}{c} < \sigma. \tag{7}
$$

The inverse transformation yields,

$$
\frac{d\xi_\sigma}{d\tau} = \frac{(\sigma-1)c^2}{v} < c \quad \Rightarrow \quad \sigma < 1 + \frac{v}{c}. \tag{8}
$$

Combining (7) and (8) gives,

$$1-\frac{v}{c}<\sigma<1+\frac{v}{c}. \tag{9}$$

Since σ is a real number, (9) constitutes an infinite set of observers. If $\sigma=1$ then (6) reduces to (1), in which case $v=0$ only (no relative motion); for otherwise (6) holds for all other σ for all $v\neq 0$. Only $v\neq 0$ produces relative motion, in which case σ can take any value in the infinite set (9). The system of clock-synchronised observers (6) is not stationary since, in general, x_σ is a function of time *t*. Only the observer $\sigma=1$ is not dependent on time *t*: it is 'stationary'. Thus $\sigma=1$ is Einstein's latent privileged observer, which, again, to emphasise, he incorrectly allowed to speak for all observers. But a system that contains only one observer violates Special Relativity's requirement for any number of equivalent observers.

Substituting into (4) the Clock-Synchronised Lorentz Transformation (6) gives,

$$\begin{aligned} x_\sigma^2-c^2t^2 &= \left[\frac{(1-\sigma)c^2t}{v}+\sigma x_1\right]^2-c^2t^2 = \\ &= \beta^2\left[\frac{(1-\sigma)c^2t}{v}+\sigma x_1-vt\right]^2-c^2\beta^2\sigma^2\left(t-\frac{vx_1}{c^2}\right)^2 \\ &= \xi_\sigma^2-c^2\tau_\sigma^2, \end{aligned} \tag{10}$$

thus satisfying Lorentz Invariance.

Equations (5) and (10) are identical only when $\sigma=1$[b]:

$$\begin{aligned} &\sigma^2x_1^2-c^2\left[t_1-\frac{(\sigma-1)vx_1}{c^2}\right]^2 = \\ &= \beta^2\left[\frac{(1-\sigma)c^2t_1}{v}+\sigma x_1-vt_1\right]^2-c^2\beta^2\sigma^2\left(t_1-\frac{vx_1}{c^2}\right)^2. \end{aligned} \tag{11}$$

III. THE WAVE EQUATION

The wave equation for an electromagnetic wave polarised in the *y*-direction and travelling in the *x*-direction with speed *c* is,

$$\frac{\partial^2\Psi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}. \tag{12}$$

The Lorentz Transformation is purported to make the wave equation form invariant for systems of clock-synchronised stationary observers in constant rectilinear relative motion:

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[b] In (10), $t=t_\sigma\,\forall\sigma$ therefore $t=t_1$.

$$\frac{\partial^2\Psi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} \quad \leftrightarrow \quad \frac{\partial^2\Psi}{\partial \xi^2}=\frac{1}{c^2}\frac{\partial^2\Psi}{\partial \tau^2}. \tag{13}$$

But systems of clock-synchronised stationary observers are inconsistent with Lorentz Transformation. Consequently, the Lorentz Transformation does not make the wave equation form invariant. This fact was also proven by Thornhill [5-7], from a different perspective[c].

Applying the chain rule to equations (2), the differential operators are,

$$\begin{aligned}
\frac{\partial}{\partial x_1} &= \frac{\partial}{\partial \xi_\sigma}\frac{\partial \xi_\sigma}{\partial x_\sigma}\frac{\partial x_\sigma}{\partial x_1}+\frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial x_\sigma}\frac{\partial x_\sigma}{\partial x_1} \\
&= \sigma\beta\left(\frac{\partial}{\partial \xi_\sigma}-\frac{v}{c^2}\frac{\partial}{\partial \tau}\right), \\
\frac{\partial^2}{\partial x_1^2} &= \sigma^2\beta^2\left(\frac{\partial^2}{\partial \xi_\sigma^2}-\frac{2v}{c^2}\frac{\partial^2}{\partial \xi_\sigma \partial \tau}+\frac{v^2}{c^4}\frac{\partial^2}{\partial \tau^2}\right), \\
\frac{\partial}{\partial t_1} &= \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial t_\sigma}\frac{\partial t_\sigma}{\partial t_1}+\frac{\partial}{\partial \xi_\sigma}\frac{\partial \xi_\sigma}{\partial t_\sigma}\frac{\partial t_\sigma}{\partial t_1} \\
&= \beta\left(-v\frac{\partial}{\partial \xi_\sigma}+\frac{\partial}{\partial \tau}\right), \\
\frac{\partial^2}{\partial t_1^2} &= \beta^2\left(v^2\frac{\partial^2}{\partial \xi_\sigma^2}-2v\frac{\partial^2}{\partial \xi_\sigma \partial \tau}+\frac{\partial^2}{\partial \tau^2}\right).
\end{aligned} \tag{14}$$

Substituting (14) into (12) gives,

$$\left(\sigma^2-\frac{v^2}{c^2}\right)\frac{\partial^2\Psi}{\partial \xi_\sigma^2}-\frac{2v}{c^2}\left(\sigma^2-1\right)\frac{\partial^2\Psi}{\partial \xi_\sigma \partial \tau}=\frac{1}{c^2}\left(1-\frac{\sigma^2 v^2}{c^2}\right)\frac{\partial^2\Psi}{\partial \tau^2}. \tag{15}$$

Only for the observer $\sigma=1$ is the wave equation form invariant under the Stationary Lorentz Transformation: precisely Einstein's latent privileged observer.

Applying the chain rule to equations (6) the same differential operators (14) obtain, therefore leading again to (15). Thus, only for the observer $\sigma=1$ is the wave equation form invariant under the Clock-Synchronised Lorentz Transformation.

Hence, any $\sigma \neq 1$ appropriate for (2) or (6) is a specific counter-example to Einstein's theory. In fact, $\sigma=1$ is also a counterexample because then, by (6), $v=0$: there is no relative motion.

IV. CONCLUSIONS

Lorentz Invariance holds between systems of stationary observers and between systems of clock-synchronised observers. Systems of clock-synchronised stationary

[c] The theory of characteristics of linear partial differential equations.

observers however are inconsistent with Lorentz Transformation. Yet Special Relativity requires Lorentz Transformation between systems of clock-synchronised stationary observers.

Systems of clock-synchronised stationary observers consistent with Lorentz Transformation cannot be constructed. Einstein's tacit assumption that such systems of observers can be constructed is false. The Special Theory of Relativity is therefore logically inconsistent: It is therefore false.

The standard wave equation is not form invariant under Lorentz Transformation, except for one privileged observer, contrary to the requirements of Special Relativity.

The Lorentz Transformation is meaningless.

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